Advanced Physics 2 through Inquiry

Experiment Guide

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INTRODUCTION

PASCO scientific's *Advanced Physics 2 through Inquiry* manual includes investigations designed to move students from the low level task of memorization or confirmation of science facts, to higher-level tasks of experiment design, data analysis, concept construction, and application. For science to be learned at a deep level, it is essential to combine the teaching of abstract science concepts with "real-world" science investigations. Hands-on technology-based laboratory experiences serve to bridge the gap between the theoretical and the concrete, driving students toward a greater understanding of natural phenomena. Students also gain important science process skills that include: developing and using models, planning and carrying out independent investigations, interpreting data, and using mathematics.

The laboratory activities span the physics content as defined by the College Board® for AP® Physics 2. These activities introduce students to the tools of science and help develop conceptual understanding and link the academic content of college level physics to the experimental evidence that defines and supports this content. These activities have been selected and structured to support student achievement on the end-of-year AP® Physics 2 exam.

Three Levels of Scientific Inquiry

Sixteen laboratory activities cover topics in fluids, kinetic theory, gas laws, geometrical optics, electric and magnetic fields, electromagnetic induction, capacitors and RC circuits, and modern physics. Every activity is presented in three distinctly different formats, each with a varied level of inquiry-based content:

Structured: This traditional format provides students a concise background section and a formal step-by-step setup and procedure. The *Structured* format also includes a complete equipment list and data analysis procedure with prescribed data display forms and data manipulation techniques.

Guided Inquiry: With no prescribed setup or procedure, the *Guided Inquiry* format contains a series of questions designed to invoke inquiry in students that will guide them to a proper setup and execution. Students design their own setup and procedure while also deciding how they will present their data to properly fulfill the lab objective and correctly address the lab's driving question. This format does *not* include a background section.

Student Designed: This format includes simply a driving question, objective statement, and a suggested equipment list. Students are expected to design and execute their own setup and procedure with little or no guidance from the student handout. They choose how to present their data in a way that supports their answer to the driving question, while also fulfilling the lab objective. This format does *not* include a background section.

The three different formats for each lab activity support the instructional need to differentiate the level of scientific inquiry in the classroom. Teachers may choose first to provide activities in the *Structured* format where students receive full guidance while developing skills that include critical thinking (posing good questions, developing experimental strategies, finding and fixing operational errors), procedural expertise (calibrating equipment, collecting data), proficiency in design and construction (assembling apparatus, following safety procedures), and analytical skills (graphing, modeling, statistics).

Students can then progress to a more inquiry-based approach by carrying out subsequent activities in their *Guided Inquiry* formats. When students have formed the skills necessary to confidently design and build their own experiments without help from student handouts, the *Student Designed* format can be offered to provide students a nearly *open-inquiry* approach to the lab topics.

Each lab activity, regardless of the format, contains two assessment question sections—Analysis Questions and Synthesis Questions—that are identical and applicable to all of the three handout formats. These sections are explained in more detail in the Lab Activity Components section below. In addition to supporting the scientific inquiry process, the *Advanced Physics 2 through Inquiry* activities fulfill STEM education requirements by bringing together Science, Technology, Engineering, and Math in varying degrees in the lab activities. The use of sensors, data analysis and graphing tools, models and simulations, and work with instruments, all support scientific inquiry as implemented in a STEM-focused curriculum.

Manual Components

The *Advanced Physics 2 through Inquiry* lab manual offers five major components:

Student Lab Activity Handouts. Each of the lab activities has three independent student handouts, one for each of the three inquiry-based formats: *Structured*, *Guided Inquiry*, and *Student Designed*. All student handouts are available in Microsoft® Word format, allowing teachers to customize the labs for their curriculum, students, and equipment. All student handout files are available on the electronic storage device that comes with the printed *Advanced Physics 2 through Inquiry* manual. Refer to the Lab Activity Components section below for details on each handout.

Teacher Resources. Every lab activity has an accompanying *Teacher Resources* document that contains teacher-centered content specific to the activity, including alignments to the AP® Physics 2 Learning Objectives and Science Practices[1]; recommended time requirements for teacher preparation and student data collection; a procedural overview of the procedure in the *Structured* version of the lab activity; safety and lab preparation instructions (if applicable); teacher tips; sample data for the *Structured* version procedure; responses to the *Structured* version Data Analysis questions, *Guided Inquiry* version Guided Inquiry questions, and Analysis and Synthesis questions for all versions; and extended inquiry activity suggestions.

All *Teacher Resources* documents are available in PDF format on the electronic storage device that comes with every printed *Advanced Physics 2 through Inquiry* manual. Refer to the Lab Activity Components section below for further details on teacher resources.

Student Experiment Design Plan Handout. Students following the *Guided Inquiry* or *Student Designed* version of a lab activity can use this one-sheet handout to help design and implement their inquiry-based investigation. The handout provides students with a small amount of guidance and structure as they develop their own laboratory investigation, regardless of the lab topic. Students use this handout to identify important facets of their investigations: the objective of the lab activity; what variables should be part of their potential experiment; what variables should be manipulated and controlled; how these variables will be manipulated or controlled; and how they will structure their data analysis. This handout is available in PDF format on the electronic storage device that comes with the manual.

Probeware Resources Videos. Included in every lab activity handout are links (both URL and QR code) to short, equipment-specific videos that outline the functionality, specifications, and different use-cases of most of the PASCO hardware and probeware to be used. These information-rich videos will help students understand the functionality and applications of each piece of equipment before using it. The videos will also be a useful tool for students when they design their own inquiry-based investigations. These videos are hosted online and are also available on the electronic storage device that comes with the manual.

[1] From AP Physics 1 and 2 Course and Exam Description, Effective Fall 2014. Copyright © 2014 The College Board. Reproduced with permission. http://apcentral.collegeboard.com

PASCO Capstone™ and SPARKvue® Configuration Files. Although the *Structured* version of every lab activity in the manual includes instructions on how students should display and present their data (for example, in a table or a graph), students also have access to configuration files. Each SPARKvue and PASCO Capstone configuration file has been pre-configured to have the correct display type, sample rate, and other software components needed for each lab activity. With these configuration files, students can simply connect their sensors, open the corresponding configuration file, and begin recording data. These files are available on the electronic storage device that comes with the manual.

Lab Activity Components

Each lab activity consists of four documents: the *Teacher Resources* document and the three Student Handout formats: *Structured*, *Guided Inquiry*, and *Student Designed*.

Sections in each set of activity documents: Teacher Resources, Structured, Guided Inquiry, and Student Designed

SECTIONS	TEACHER RESOURCES	STRUCTURED FORMAT	GUIDED INQUIRY FORMAT	STUDENT DESIGNED FORMAT
Connections to the AP® Physics 1 Curriculum	■			
Time Requirement	■			
Prerequisites	■			
Driving Question | Objective	■	■	■	■
Procedural Overview	■			
Pre-Lab Discussion and Activity	■			
Design and Conduct Your Experiment			■	■
Materials and Equipment	■	■	**	**
Probeware Resources	■	■	**	**
Background		■		
Safety	*	*		
Lab Preparation	■			
Teacher Tips	■			
Procedure		■		
Sample Data	■			
Data Analysis	■	■		
Guided Inquiry/Guiding Questions	■		■	
Experimental Design			■	■
Assessment Questions: Sample Responses	■			
Analysis Questions		■	■	■
Synthesis Questions		■	■	■
Extended Inquiry Suggestions	■			

* This section is present if safety considerations are needed.

** These materials, equipment, and probeware resources are recommended, not required.

Teacher Resources Document

This document contains all of the teacher-centered information regarding the lab activity: preparation instructions, sample data, and sample responses to the questions in all student versions of the lab activity. Each *Teacher Resources* document contains the following sections:

- **CONNECTIONS TO THE AP® PHYSICS 2 CURRICULUM** – Every lab is correlated to one or more Learning Objective identified in the AP® Physics 2 curriculum framework from the *AP® Physics 1 and 2 Course and Exam Description*, Effective Fall 2014. This section lists each Big Idea, Enduring Understanding, Essential Knowledge, Learning Objective, and Science Practice applicable to the lab activity.
- **TIME REQUIREMENTS** – Two time frames are defined: the length of time needed for teacher preparation, and the recommended time allotment for students to complete the procedure outlined in the *Structured* version of the lab activity. If there is no specific lab preparation needed, ten minutes is designated to take into account the time required for gathering the materials, making copies of the student handout, and any other normal preparations. Note that more or less time may be required for students to finish data collection when using the *Guided Inquiry* or *Student Designed* versions.
- **PREREQUISITES** – This section details the concepts students should know before doing the activity. Use this section to gauge when to include this activity in lesson plans, in assessing requirements for prior learning, and as an outline for a review or discussion before starting the lab activity.
- **DRIVING QUESTION | OBJECTIVE** – This is the driving question and lab objective that students address when performing their laboratory investigation. This section is the same for all three student handout formats as well as the *Teacher Resources* document.
- **PROCEDURAL OVERVIEW** – This section is a summary of the procedure students follow using the *Structured* version of the lab activity, how they present their data, and the results and conclusions to be drawn from that data.
- **PRE-LAB DISCUSSION AND ACTIVITY** – The pre-lab discussion, activity, or both, are designed to accomplish some or all of the following: engage student attention; access prior knowledge; identify misconceptions; model correct lab technique; model procedures for mathematical computations required in the activity; generate student questions. This section may include pre-lab homework questions that help prepare students to carry out the lab activity.
- **MATERIALS AND EQUIPMENT** – This section lists all student materials and equipment needed per student group to carry out the procedure in the *Structured* version of each lab activity. Items that need to be prepared by the teacher or created using additional materials are called out in the Lab Preparation section of this document. Items included as part of a kit or as part of another product are highlighted with a number connecting them to the information provided in the Probeware Resources section.
- **PROBEWARE RESOURCES** – URLs and QR codes link to equipment-specific videos that outline the functionality, specifications, and different use-cases of most of the hardware and probeware that appear in each activity. Videos are hosted online and are also available on the electronic storage media that comes with the printed manual. The URL links and QR codes in the *Teacher Resources* document are found in all three student handout formats.
- **SAFETY** – This section lists the pertinent safety procedures, if any are needed, for each lab activity beyond a classroom's normal laboratory safety procedures. The Safety section found in the *Teacher Resources* document may include additional safety considerations the teacher should be aware of. This section is also found in the *Structured* version of the lab.
- **LAB PREPARATION** – If applicable, this section includes teacher-directed lab preparation instructions that are either required or suggested to help minimize preparation time.

- **TEACHER TIPS** – Depending on the activity, this section includes any or all of the following: 1) common misconceptions that students have regarding the lab topic; 2) skill requirements for using equipment; 3) difficulties students may encounter executing the lab and how to avoid or correct them; and 4) strategies and techniques for substituting items in the Materials and Equipment list that students may not have access to.
- **SAMPLE DATA** – This section is identical to the Procedure and the Data Analysis sections in the *Structured* version, with the addition of sample sensor data, tables, and graphs, answers to the questions in the Data Analysis section, and sample calculations used to process or manipulate the data.
- **GUIDED INQUIRY QUESTIONS** – This section includes sample responses and teacher-information pertaining to the questions in the Guiding Questions section of the *Guided Inquiry* version of the lab activity.
- **ASSESSMENT QUESTIONS: SAMPLE RESPONSES** – This section includes sample or correct responses to the questions in the Analysis Questions and Synthesis Questions sections in all three versions of the student handouts.
- **EXTENDED INQUIRY SUGGESTIONS** – These suggestions are natural extensions of the activity and can be used for further student inquiry. They include ideas for further experimentation and hands-on exploration, classroom debates, field trips, or research papers.

Structured Format Student Handout

This is the traditional "cookbook" version of each lab activity containing the least amount of student-directed learning. Each *Structured* student handout contains the following sections:

- **DRIVING QUESTION | OBJECTIVE** – Each lab activity begins with a driving question and objective statement on which students will base their investigation of the scientific topic. This section is the same for all three student handout formats.
- **MATERIALS AND EQUIPMENT** – This section lists all student materials and equipment needed per student group to carry out the procedure outlined in the *Structured* version of the lab activity. In this section also are URL and QR code links to equipment-specific videos that outline the functionality, specifications, and different use-cases of the PASCO hardware and probeware used in the activity. Videos are hosted online and are also available on the electronic storage media that comes with the manual. The same Materials and Equipment list, URL links, and QR codes are found in all student handout formats.
- **BACKGROUND** – The Background section appears only in the *Structured* version of each lab activity and contains information related to the scientific topic being investigated. The information frames the activity for students in the context of related curriculum materials. For broader and deeper information on a topic, students should refer to textbooks or other reference materials.
- **SAFETY** – If applicable, this section lists the pertinent safety procedures for each lab activity beyond a classroom's normal laboratory safety procedures. The Safety section found in the *Teacher Resources* document may include additional safety considerations the teacher should be aware of.
- **PROCEDURE** – This section directs the student hands-on portion of each lab activity. Students follow numbered tasks to complete the procedure. Depending on the lab activity, the procedure may be divided into parts. Each part in a lab activity has a Set Up section in which students are given instructions on assembling laboratory equipment, including hardware, sensors (probeware), and data collection systems (see Using Data Collection Technology). Each part also contains a Collect Data section with instructions on how and when to collect data, and where to record data in the Data Analysis section.

- **DATA ANALYSIS** – In this section, students are instructed to analyze and present their data in ways specific to the lab activity, such as completing a data table, making calculations to manipulate or process data, plotting or sketching graphs of data, or identifying key parts of the data plots. In addition, several of the activities in this manual employ a data linearization technique found in AP® Physics 1 and 2 exams.
- **ANALYSIS QUESTIONS** – These questions help students understand their collected data as it pertains to the lab topic and driving question. Students make comparisons, summaries, arguments, and conclusions regarding the scientific concept using their data for verification. This section is the same for all student handout formats.
- **SYNTHESIS QUESTIONS** – These questions help students integrate information and concepts explored in the lab activity with information from other topics using real-world scenarios. Students develop a deeper understanding of concepts as they transfer knowledge learned in the lab to other situations. Some questions may require students to consult available resources, such as textbooks, reference books, resources on the Internet, and local experts. This section is the same for all student handout formats.

Guided Inquiry Format Student Handout

The *Guided Inquiry* format, compared to the traditional version, is a more student-directed approach. The step-by-step Procedure and Data Analysis sections found in the *Structured* format are replaced with a set of guiding questions intended to help students design their own procedure and analysis strategies. The *Guided Inquiry* student handout contains the following sections:

- **DRIVING QUESTION | OBJECTIVE** – Each lab activity begins with a driving question and objective statement on which students will base their investigation of the scientific topic. This section is the same for all student handout formats.
- **DESIGN AND CONDUCT YOUR EXPERIMENT** – Instead of following a step-by-step procedure, students are directed to design an experiment that fulfills the lab objective, and whose data will support their answer to the driving question. Although the same materials and equipment list found in the *Structured* version also appears in this section, it is presented as suggested equipment. Students using the *Guided Inquiry* or *Student Designed* formats will have the freedom to choose any reasonable equipment at their disposal. This section is the same in the *Guided Inquiry* and *Student Designed* formats.
- **GUIDING QUESTIONS** – This section contains a series of questions designed to stimulate inquiry in students that will guide them to determine their experiment design. Although these questions will vary depending on the lab activity, most questions help students: identify variables that will be part of their experiment; define which variables to manipulate and which variables to control; determine how each variable can be measured, how data should be collected and in what order, and how to manipulate, process, and present data to isolate values of interest and identify unknowns. Along with each set of guiding questions, you may also choose to provide students with the *Experiment Design Plan* handout (refer to the Manual Components) to help facilitate the experimental design process.
- **EXPERIMENTAL DESIGN** – This section is divided into the Setup, Procedure, and Collect Data subsections in which students document the experimental setup and procedure they have chosen, and present any data that is part of their experiment. This section is the same in both the *Guided Inquiry* and *Student Designed* formats.
- **ANALYSIS QUESTIONS** – See the corresponding section in the *Structured Format Student Handout* above. This section is the same for all three student handout formats.
- **SYNTHESIS QUESTIONS** – See the corresponding section in the *Structured Format Student Handout* above. This section is the same for all three student handout formats.

Student Designed Format Student Handout

This format is the most student-directed version of a lab activity, containing the least amount of instruction and assistance. Students are responsible for designing and executing their own experimental setup and procedure with little or no guidance from the student handout. Students choose how to present their data in a way that supports their answer to the driving question, while also fulfilling the lab objective.

Each *Student Designed* handout contains the same sections found in the *Guided Inquiry* format except it has no Guiding Questions section.

Conducting Successful Inquiry-Based Lab Activities

Establish the Foundation

Preparing students to conduct their own scientific inquiry activities takes time and intention. Students need a foundation in conceptual development, laboratory bench skills, using electronic data collection and display equipment, and interpreting data. The following strategies help students build this foundation:

- Work with students to complete tutorials for equipment and software they will be using.
- Demonstrate the first few activities using the *Structured* or *Guided Inquiry* formats so you model the correct use of equipment and materials.
- Work with students to complete all sections of several lab activities until they understand your expectations.
- Create teams, giving defined responsibilities to members. (A key behavioral component of a STEM curriculum experience is that students work in teams and successfully solve problems as a team.) Devise a method to track the roles each student carries out, such as a team leader, a recorder, and a technician. Make sure each student has multiple opportunities to perform each role.
- Create opportunities for students to repeat activities that seemed beyond their grasp the first time through—perhaps with student-suggested modifications. You will see substantial improvement as students are given increased opportunities to work with the equipment and analyze the data.

Foster Inquiry Skills

Foster the growth and development of inquiry skills. Provide multiple opportunities for students to work with the equipment, analyze data, and communicate and discuss conclusions. The following strategies support development of laboratory and data analysis skills:

- Model the more complex technical tasks, such as mathematical computations.
- Provide multiple and varied opportunities for practice with hands-on activities using the data collection tools.
- Compile and compare class data whenever possible. Discuss the sources of variation in data and the best interpretation of the data.
- Challenge students to identify applications of the concept just studied.
- Have students brainstorm related questions they would like to explore in their own investigations.

Cultivate Student-Directed Inquiry

At the heart of an effective STEM curriculum is the cultivation of inquiry skills in students. As students complete instructor-directed activities in this manual (using the *Structured* and *Guided Inquiry* formats), their interest may be stimulated regarding one or more issues. Watch for these moments and provide students with assistance for generating their own driving questions and related objectives. For either students' own questions or for those provided in the *Student Designed* format of the lab activities, use the following strategies:

- Require a written plan with procedures. Review these plans and guide students accordingly. Make sure students define projects that are practical under the conditions of your classroom environment.
- Provide plenty of time, material, and equipment resources.
- Incorporate check points to assess progress.
- To guide the students, ask questions such as those in the *Guided Inquiry* version of each lab.

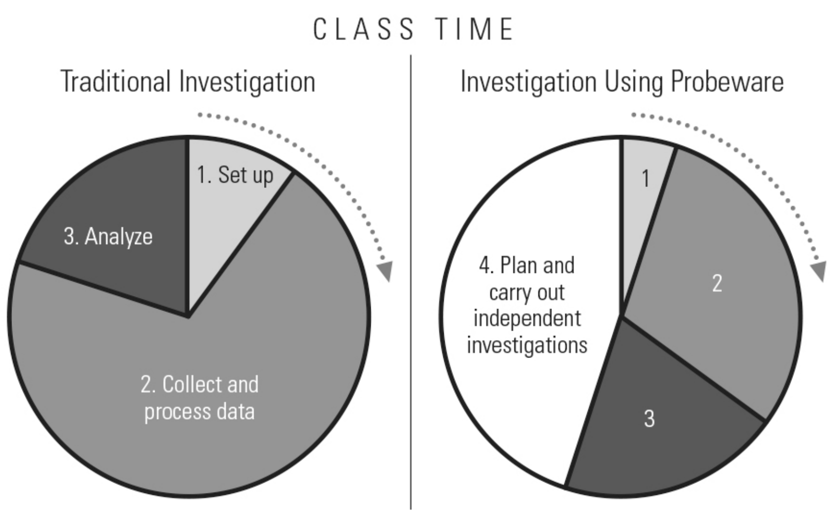
Communicate the Results of Student-Directed Inquiry

Provide opportunities for students to communicate the results of student-directed inquiry. Strategies include:

- Formal research papers, PowerPoint® presentations, video productions, and poster presentations are ways for students to share what they have learned.
- Student-directed inquiries related to community resources may be of interest to area news or conservation groups. Have students report on their findings in a community venue such as the school website or newspaper, local newspapers, or other publications.

Using Data Collection Technology

The use of electronic sensors (probeware) in investigations greatly reduces the class time required for set up and data collection, increases the accuracy of results, allows for richer analysis of data, and provides more time in the classroom for independent investigations.



Additionally, using electronic-sensor data collection, display, and analysis devices allows students to focus not on the tedium of collecting data, but rather on the trends, patterns, and relationships which become immediately discernible when gathering real-time data.

The Data Collection System

In this manual, *data collection system* refers to the system students use to record, visualize, and analyze sensor data during their experiments. The system consists of all components necessary to connect a sensor to a device containing the software that detects the sensor measurement and collects, records, and displays the data.

Some systems, such as the Xplorer GLX® or SPARK Science Learning System™, are stand-alone systems. These contain built-in software applications and students simply attach a sensor and begin collecting data. Other systems use a computer or tablet with downloaded software applications. In these systems, a USB or Bluetooth® interface is used to connect a sensor to the device. Software options for these include SPARKvue and PASCO Capstone software.

The activities are designed so that any PASCO scientific data collection system can be used to carry out the procedures.

Getting Started with Your Data Collection System

To become familiar with the many features of your data collection system, start with the tutorials and instructional videos available in the video library on the PASCO scientific website (www.pasco.com). Also, each system's software has a built-in help system.

There are free SPARKlab™ activities included in the SPARKvue software. Performing one of these activities can be a good starting place for students to familiarize themselves with connecting a sensor, viewing data, saving their work, and other tasks related to probeware use.

PASCO scientific also has a terrific technical and teacher support team. They pride themselves on providing timely and comprehensive help to teachers and students using PASCO scientific products.

Phone: 1-800-772-8700

Email: support@pasco.com

Web: www.pasco.com/support

Inside the Printed Manual

The printed *Advanced Physics 2 through Inquiry* lab manual includes the following documents:

- *Table of Contents*
- *Introduction*
- *Master Materials and Equipment List*
- *Experiment Design Plan* handout
- *Teacher Resources* for each of the lab activities
- *Structured* format student handout for each of the lab activities

Documents *not* printed but available on the accompanying electronic storage device are:

- *Guided Inquiry* format student handout for each of the lab activities
- *Student Designed* format student handout for each of the lab activities

Electronic Materials

The electronic storage device accompanying this manual contains the following:

- Complete *Advanced Physics 2 through Inquiry* manual in PDF format (Acrobat™ compatible)
- Each lab activity's *Teacher Resources* document in PDF format.
- Student handout versions of each laboratory activity in all three formats (*Structured*, *Guided Inquiry*, and *Student Designed*) in an editable Microsoft Word format. PASCO scientific provides editable files of the student lab activities so that teachers can customize activities to their needs.
- Student *Experiment Design Plan* handout in PDF format.
- A complete set of the Probeware Resources Videos used in the *Advanced Physics 2 through Inquiry manual*. Although these videos are hosted online, PASCO scientific provides them in MP4 format for those who may not have a reliable Internet connection, or cannot access videos due to internal system or website restrictions.
- PASCO Capstone and SPARKvue Configuration Files for every lab activity.

AP® Physics 2 Correlations

Below is a list of the 16 lab activities in the *Advanced Physics 2 through Inquiry* lab manual. In the right columns of the table are the correlations for each lab activity to the AP® Concept Outline found in the *AP Physics 1 and 2 Course and Exam Description*, Effective Fall 2014 curriculum framework published by the College Board®.

Each reference number indicates the Big Idea, Enduring Understanding, Essential Knowledge, and Learning Objective to which the activity is correlated. For example "3.B.1.2" indicates that the activity is correlated to Learning Objective 2 found within the first Essential Knowledge statement, which is within Enduring Understanding B, and in turn part of Big Idea 3. Shown in the column to the right of the Learning Objectives are the applicable Science Practices identified by the College Board® based on each correlated Learning Objective.

International Baccalaureate Organization (IBO) Support

The International Baccalaureate Organization (IBO) uses a specific science curriculum model that includes both theory and practical investigative work. While this lab guide was not produced by the IBO and does not include references to the IB internal assessment rubrics, the lab activities can be adapted easily to the IB classroom. The labs in this manual correlate closely to core and optional topics of the IB Physics standard level and higher level programs: fluid dynamics, optics and imaging, electromagnetic induction, quantum physics, and others. These correlations are listed in the table below.

By the end of the IB Diploma Program, students are expected to have completed a set number of practical investigative hours and are assessed using the specified internal assessment criteria. Students should be able to design a lab based on an original idea, carry out the procedure, draw conclusions, and evaluate their results. These scientific processes require an understanding of laboratory techniques and equipment as well as a high level of thinking, skills that are developed and sharpened by completing the investigations in this manual.

AP® Physics 2 and IBO correlations to the activities in this manual

Activity Number	Activity Name and Description	AP® Physics 2 Learning Objective	AP® Physics 2 Science Practice	IBO Standard
1	Hydrostatic Pressure *Students use a low-pressure sensor to measure the static pressure at different depths in a column of water and use their data to determine the mathematical relationship between static pressure and depth in a fluid.*	3.C.4.1 3.C.4.2	6.1 6.2	B.3
2	Buoyant Force *Students use a high-resolution force sensor to measure the buoyant force on a metal cylinder lowered into a fluid and then determine the relationship between the buoyant force on a submerged object and a) its volume and b) the weight of the fluid displaced by the submerged object.*	1.E.1.2 3.A.3.1 3.C.4.2	4.1 6.2 6.4 7.2	B.3
3	Fluid Dynamics *Students determine the relationship between the velocity of a water stream as it leaves the nozzle at the bottom of a water column and the height of the water column.*	5.B.10.1 5.B.10.3 5.B.10.4	2.2 6.2	B.3
4	Boyle's Law *Students use a low-pressure sensor and a syringe to determine the inverse proportionality between the pressure and volume of an enclosed gas.*	5.B.7.2 7.A.3.2 7.A.3.3	1.1 3.2 4.2 5.1	3.2
5	Spherical Mirror Reflection *Students use an optics light source, optics track, and half screen to measure the image and object distances associated with the real image formed by a concave spherical mirror and then use principles of reflection and the spherical mirror equation to determine the mirror's radius of curvature.*	6.E.4.1 6.E.4.2	3.2 4.1 4.2 5.1 5.2 5.3	C.1
6	Snell's Law *Students use an optics ray table to measure the incident and refraction angles of a light ray travelling from air into a material with unknown index of refraction, and then, using the principles of refraction and Snell's law, they determine the material's index of refraction.*	6.E.3.2 6.E.3.3	4.1 5.1 5.2 5.3 6.4 7.2	4.4
7	Focal Length of a Converging Lens *Students use an optics light source, optics track, and viewing screen to measure the image and object distances associated with the real image formed by a converging lens, and then determine the focal length of the lens.*	6.E.5.1 6.E.5.2	1.4 2.2 3.2 4.1 5.1 5.2 5.3	C.1
8	Interference and Diffraction *Students shine laser light through a double-slit aperture onto paper, measure the distances between the maxima of the resulting interference pattern, and use the principles associated with double-slit interference and diffraction to determine the spacing between the slits.*	6.C.3.1	1.4 6.4	4.4 9.2 9.3
9	Electric Field Mapping *Students use a DC power supply and semi-conductive paper to create dipole and parallel plate electrodes, and then use the principles of electric fields and electric potential energy to determine the shape and direction of the electric field lines in each configuration.*	2.E.2.1	6.4 7.2	5.1 10.1

Activity Number	Activity Name and Description	AP® Physics 2 Learning Objective	AP® Physics 2 Science Practice	IBO Standard
10	Magnetic Fields *Students use an AC/DC electronics laboratory, a power supply, and a Magnaprobe™ wand to detect and compare the magnetic field pattern surrounding a bar magnet and a current-carrying coil.*	2.D.2.1 2.D.3.1 2.D.4.1	1.1 1.2 1.4	5.4
11	Magnetic Field Strength *Students use a 2-axis magnetic field sensor and the AC/DC electronics laboratory to determine how the strength of the magnetic field at the center of a current-carrying coil depends on the coil current and radius.*	2.D.2.1	1.1	5.4
12	Electromagnetic Induction *Students use an induction wand, rotary motion sensor, variable gap magnet, and 2-axis magnetic field sensor to determine how the rate of change of magnetic flux through a coil affects the magnitude and direction of the average emf induced in it.*	4.E.2.1	6.4	11.1
13	Capacitor Fundamentals *Students use a digital capacitance meter and construct capacitors from aluminum foil and paper to determine how physical properties of a parallel-plate capacitor affect its ability to store electric charge.*	4.E.4.2 4.E.4.3	4.1 4.2 5.1	11.3
14	Series and Parallel Capacitors *Students use a capacitance meter to measure the equivalent capacitance in simple series and parallel circuits and determine the equivalent capacitance of capacitors connected in series and parallel.*	4.E.5.3 5.B.9.5	2.2 4.2 5.1 6.4	11.3
15	RC Circuits *Students use a voltage–current sensor and an AC/DC electronics laboratory to determine how the potential differences across the resistors and capacitor in a simple RC circuit differ when the capacitor is charging, discharging, and fully charged, and how these differences affect the current through each component in the circuit.*	4.E.5.1 4.E.5.2 4.E.5.3	2.2 4.2 5.1 6.1 6.4	11.3
16	Planck's Constant *Students use a voltage–current sensor and an AC/DC electronics laboratory to measure the turn-on voltage of various colors of LEDs and then plot the turn-on voltage versus LED frequency to determine the value of Planck's constant.*	6.F.3.1 6.F.4.1	6.4 7.1	12.1

MASTER MATERIALS AND EQUIPMENT LIST

This Master Materials and Equipment List shows the equipment required to perform the *Structured* version of each lab activity from the *Advanced Physics 2 through Inquiry* lab manual. Italicized entries indicate items not available from PASCO. The quantity indicated is per student or group.

Teachers can conduct some lab activities with sensors and probes other than those listed here. For assistance with substituting compatible sensors and probes for a lab activity, contact PASCO Teacher Support (800-772-8700 inside the United States or http://www.pasco.com/support).

Lab	Title	Materials and Equipment	PASCO Part Number	Qty
1	HYDROSTATIC PRESSURE	**FOR EACH STUDENT STATION**		
	Students use a low-pressure sensor to measure the static pressure at different depths in a column of water and use their data to determine the mathematical relationship between static pressure and depth in a fluid.	Data Collection System		1
		PASPORT Barometer/Low-Pressure Sensor	PS-2113A	1
		PASPORT Sensor Extension Cable*	PS-2500 or w/PS-2162	1
		Quick connector*	w/PS-2113A	1
		Tubing, 1/4" diameter*	w/PS-2113A	30 cm
		Four-Scale Meter Stick	SE-8695	1
		Water reservoir, transparent, over 30 cm high		1
		Distilled water, to fill the reservoir 3/4 full		2 L
2	BUOYANT FORCE	**FOR EACH STUDENT STATION**		
	Students use a high-resolution force sensor to measure the buoyant force on a metal cylinder lowered into a fluid and then determine the relationship between the buoyant force on a submerged object and a) its volume and b) the weight of the fluid displaced by the submerged object.	Data Collection System		1
		PASPORT High Resolution Force Sensor with hook	PS-2189	1
		PASCO Overflow Can	SE-8568	1
		PASCO Aluminum Table Clamp	ME-8995	1
		Brass cylinder[1]	w/ME-8569A	1
		Aluminum cylinder[1]	w/ME-8569A	1
		Rod, 45-cm	ME-8736	2
		Right angle clamp	SE-9444	1
		Four-Scale Meter Stick	SE-8695	1
		Thread	ME-9875	60 cm
		Beaker, 100-mL		1
		Beaker, 1-L		1
		Glass stir rod		1
		Felt-tipped pen with permanent ink		1
		Liquid dish soap		3 mL
		Distilled water		500 mL
		Paper towel		1 roll
		[1]Any two metal cylinders (of different metals) that can be suspended vertically above their center can be used.		
		FOR THE ENTIRE CLASS		
		Ohaus Scout Pro Balance 400-g	SE-8756A	1

Lab	Title	Materials and Equipment	PASCO Part Number	Qty
3	FLUID DYNAMICS Students determine the relationship between the velocity of a water stream as it leaves the nozzle at the bottom of a water column and the height of the water column.	**FOR EACH STUDENT STATION** Four-Scale Meter Stick *Water reservoir with a nozzle or hole at the bottom* *Support stand, 10 cm high* *Distilled water to fill the water reservoir* *Water catch basin* *Pen, felt marker*	 SE-8695	 1 1 1 2 L 1 1
4	BOYLE'S LAW Students use a low-pressure sensor and a syringe to determine the inverse proportionality between the pressure and volume of an enclosed gas.	**FOR EACH STUDENT STATION** Data Collection System PASPORT Barometer/Low-Pressure Sensor PASPORT Sensor Extension Cable* Quick connector* Tubing* Syringe, 60-mL* *Scissors*	 PS-2113A PS-2500 or w/PS-2162 w/PS-2113A w/PS-2113A w/SE-7562	 1 1 1 1 2 cm 1 1
5	SPHERICAL MIRROR REFLECTION Students use an optics light source, optics track, and half screen to measure the image and object distances associated with the real image formed by a concave spherical mirror and then use principles of reflection and the spherical mirror equation to determine the mirror's radius of curvature.	**FOR EACH STUDENT STATION** PASCO Optics Track[2] PASCO Basic Optics Light Source PASCO Concave Mirror Accessory PASCO Half-Screen Accessory* [2]or PASCO Dynamics Track with three Optics Carriages (OS-8472)	 OS-8508 OS-8470 OS-8457 w/OS-8457	 1 1 1 1
6	SNELL'S LAW Students use an optics ray table to measure the incident and refraction angles of a light ray travelling from air into a material with unknown index of refraction, and then, using the principles of refraction and Snell's law, they determine the material's index of refraction.	**FOR EACH STUDENT STATION** PASCO Basic Optics Ray Table PASCO Basic Optics Light Source D-shaped lens*	 OS-8465 OS-8470 w/OS-8465	 1 1 1
7	FOCAL LENGTH OF A CONVERGING LENS Students use an optics light source, optics track, and viewing screen to measure the image and object distances associated with the real image formed by a converging lens, and then determine the focal length of the lens.	**FOR EACH STUDENT STATION** PASCO Optics Track[2] PASCO Basic Optics Light Source PASCO Basic Optics Viewing Screen PASCO Adjustable Lens Holder Converging lens, 50-mm diameter * [2]or PASCO Dynamics Track with three Optics Carriages (OS-8472)	 OS-8508 OS-8470 OS-8460 OS-8474 w/OS-8466A	 1 1 1 1 1

Lab	Title	Materials and Equipment	PASCO Part Number	Qty
8	INTERFERENCE AND DIFFRACTION Students shine laser light through a double-slit aperture onto paper, measure the distances between the maxima of the resulting interference pattern, and use the principles associated with double-slit interference and diffraction to determine the spacing between the slits.	**FOR EACH STUDENT STATION**		
		PASCO Diffraction Plate	OS-8850	1
		PASCO Aluminum Table Clamp	ME-8995	2
		Rod, 45-cm	ME-8736	2
		Three finger clamp	SE-9445	2
		Stainless steel calipers	SE-8710	1
		Laser pointer with known wavelength	SE-9716B	1
		Four-Scale Meter Stick	SE-8695	1
		White paper		1 sheet
		Pencil		1
		Measuring tape		1
		FOR THE ENTIRE CLASS		
		Tape		1 roll
9	ELECTRIC FIELD MAPPING Students use a DC power supply and semi-conductive paper to create dipole and parallel plate electrodes, and then use the principles of electric fields and electric potential energy to determine the shape and direction of the electric field lines in each configuration.	**FOR EACH STUDENT STATION**		
		PASCO Field Mapper Kit	PK-9023	1
		Conductive paper*	w/PK-9023	2 sheets
		Conductive ink pen*	w/PK-9023	1
		Cork board*	w/PK-9023	1
		Pushpin, metal*	w/PK-9023	6
		Student power supply, 18 VDC, 3 A	SE-8828	1
		4-mm banana plug patch cord*	w/SE-9750 or w/PS-2115	4
		4-mm banana plug alligator clip*	w/SE-9756 or w/PS-2115	4
		Digital multimeter	SE-9786A	1
		T-pin, metal		1
		Felt-tip marker, silver		1
		Pencil		1
10	MAGNETIC FIELDS Students use an AC/DC electronics laboratory, a power supply, and a Magnaprobe™ wand to detect and compare the magnetic field pattern surrounding a bar magnet and a current-carrying coil.	**FOR EACH STUDENT STATION**		
		PASCO AC/DC Electronics Lab Kit	EM-8656	1
		Wire lead*	w/EM-8656	1
		Student power supply, 18 VDC, 3 A	SE-8828	1
		Magnaprobe™ wand	SE-7390	1
		Bar magnet	EM-8620	1
		4-mm banana plug patch cord*	w/SE-9750 or w/PS-2115	2
		Magnet wire or enameled wire, 22-gauge		4 m
		Sandpaper		1 sheet
		Scissors or wire cutters		1
		Beaker, 400-mL		1

Lab	Title	Materials and Equipment	PASCO Part Number	Qty
11	MAGNETIC FIELD STRENGTH	**FOR EACH STUDENT STATION**		
	Students use a 2-axis magnetic field sensor and the AC/DC electronics laboratory to determine how the strength of the magnetic field at the center of a current-carrying coil depends on the coil current and radius.	Data Collection System		1
		PASPORT 2-Axis Magnetic Field Sensor w/handle	PS-2162	1
		PASPORT Sensor Extension Cable*	w/PS-2162	1
		PASCO AC/DC Electronics Lab Kit	EM-8656	1
		Wire lead*	w/EM-8656	1
		Student power supply, 18 VDC, 3 A	SE-8828	1
		4-mm banana plug patch cord*	w/SE-9750 or w/PS-2115	2
		PASCO Aluminum Table Clamp	ME-8995	1
		Rod, 45-cm	ME-8736	1
		Right angle clamp	SE-9444	1
		Four-Scale Meter Stick	SE-8695	1
		Magnet wire or enameled wire, 22-gauge		10 m
		Beakers of different diameter		5
		Sandpaper		1 sheet
		Scissors or wire cutters		1
12	ELECTROMAGNETIC INDUCTION	**FOR EACH STUDENT STATION**		
	Students use an induction wand, rotary motion sensor, variable gap magnet, and 2-axis magnetic field sensor to determine how the rate of change of magnetic flux through a coil affects the magnitude and direction of the average emf induced in it.	Data Collection System		1
		PASPORT Voltage–Current Sensor	PS-2115	1
		PASPORT Rotary Motion Sensor	PS-2120A	1
		PASPORT 2-Axis Magnetic Field Sensor	PS-2162	1
		PASPORT Sensor Extension Cable*	w/PS-2162	1
		PASCO Variable Gap Magnet	EM-8618	1
		PASCO Induction Wand	EM-8099	1
		PASCO Aluminum Table Clamp	ME-8995	1
		Right angle clamp	SE-9444	1
		Rod, 45-cm	ME-8736	2
13	CAPACITOR FUNDAMENTALS	**FOR EACH STUDENT STATION**		
	Students use a digital capacitance meter and construct capacitors from aluminum foil and paper to determine how physical properties of a parallel-plate capacitor affect its ability to store electric charge.	4-mm banana plug patch cord*	w/SE-9750 or w/PS-2115	2
		4-mm banana plug alligator clip*	w/SE-9756 or w/PS-2115	2
		Four-Scale Meter Stick	SE-8695	1
		Digital capacitance meter, 0.01-nF resolution		1
		Aluminum foil sheet, 8 ½" × 11”		4
		Paper sheet, 8 ½" × 11”		6
		Scissors		1
		Heavy textbook		1
14	SERIES AND PARALLEL CAPACITORS	**FOR EACH STUDENT STATION**		
	Students use a capacitance meter to measure the equivalent capacitance in simple series and parallel circuits and determine the equivalent capacitance of capacitors connected in series and parallel.	PASCO AC/DC Electronics Lab Kit	EM-8656	1
		Wire lead*	w/EM-8656	6
		4-mm banana plug patch cord*	w/SE-9750 or w/PS-2115	2
		4-mm banana plug alligator clip*	w/SE-9756 or w/PS-2115	2
		Digital capacitance meter, 1-µF resolution		1
		Capacitor, 100-µF		5

Lab	Title	Materials and Equipment	PASCO Part Number	Qty
15	RC CIRCUITS Students use a voltage–current sensor and an AC/DC electronics laboratory to determine how the potential differences across the resistors and capacitor in a simple RC circuit differ when the capacitor is charging, discharging, and fully charged, and how these differences affect the current through each component in the circuit.	**FOR EACH STUDENT STATION**		
		Data Collection System		1
		PASPORT Voltage–Current Sensor	PS-2115	1
		4-mm banana plug patch cord*	w/PS-2115	2
		4-mm banana plug alligator clip*	w/PS-2115	4
		PASCO AC/DC Electronics Lab Kit	EM-8656	1
		Capacitor, 470-µF*	w/EM-8656	1
		Resistor, 33-Ω*	w/EM-8656	1
		Resistor, 100-Ω*	w/EM-8656	1
		Wire lead*	w/EM-8656	6
		D-cell Battery		2
16	PLANCK'S CONSTANT Students use a voltage–current sensor and an AC/DC electronics laboratory to measure the turn-on voltage of various colors of LEDs and then plot the turn-on voltage versus LED frequency to determine the value of Planck's constant.	**FOR EACH STUDENT STATION**		
		Data Collection System		1
		PASPORT Voltage–Current sensor	PS-2115	1
		PASCO AC/DC Electronics Lab Kit	EM-8656	1
		Wire lead*	w/EM-8656	5
		Resistor, 330-Ω*	w/EM-8656	1
		LED, blue (450–500 nm)		1
		LED, green (501–565 nm)		1
		LED, yellow/amber (566–620 nm)		1
		LED, red (621–750 nm)		1
		D-cell Battery		2

* These items are included with the specific kit, apparatus, or sensor used in the experiment.

ACTIVITY BY PASCO ITEM

This table indicates which lab activities use the PASCO scientific sensors or special equipment listed. The quantities shown indicate the number of each item required to complete all the activities that require the specified item.

Items Available from PASCO	PASCO Part Number	Qty	Activity Where Used
PASCO SENSORS			
PASPORT Barometer/Low-Pressure Sensor	PS-2113A	1	1, 4
PASPORT High Resolution Force Sensor with hook	PS-2189	1	2
PASPORT 2-Axis Magnetic Field Sensor	PS-2162	1	11, 12
PASPORT Sensor Extension Cable*	w/PS-2162	1	1, 4, 11, 12
PASPORT Rotary Motion Sensor	PS-2120A	1	12
PASPORT Voltage-Current Sensor	PS-2115	1	12, 15, 16
PASCO LABWARE			
PASCO AC/DC Electronics Lab Kit	EM-8656	1	10, 11, 14, 15, 16
PASCO Adjustable Lens Holder	OS-8474	1	7
PASCO Aluminum Table Clamp	ME-8995	1	2, 8, 11, 12
PASCO Basic Optics Light Source	OS-8470	1	5, 6, 7
PASCO Basic Optics Ray Table	OS-8465	1	6
PASCO Basic Optics Viewing Screen	OS-8460	1	7
PASCO Concave Mirror Accessory	OS-8457	1	5
PASCO Diffraction Plate	OS-8850	1	8
PASCO Field Mapper Kit	PK-9023	1	9
PASCO Induction Wand	EM-8099	1	12
PASCO Optics Track	OS-8508	1	5, 7
PASCO Overflow Can	SE-8568	1	2
PASCO Variable Gap Magnet	EM-8618	1	12
OTHER LABWARE			
Brass cylinder	w/ME-8569A	1	2
Aluminum cylinder	w/ME-8569A	1	2
Bar magnet	EM-8620	1	10
Converging lens, 50-mm diameter*	OS-8466A	1	7
Digital multimeter	SE-9786A	1	9
Four-Scale Meter Stick	SE-8695	1	1, 2, 3, 8, 11, 13
Laser pointer with known wavelength	SE-9716B	1	8
Magnaprobe™ wand	SE-7390	1	10
Right angle clamp	SE-9444	1	2, 11, 12
Rod, 45-cm	ME-8736	2	2, 8, 11, 12

Items Available from PASCO	PASCO Part Number	Qty	Activity Where Used
Stainless steel calipers	SE-8710	1	8
Student power supply, 18 VDC, 3 A	SE-8828	1	9, 10, 11
Syringe, 60-mL*	w/SE-7562	1	4
Thread	ME-9875	60 cm	2
Three finger clamp	SE-9445	2	8

* These items are included with the specific kit, apparatus, or other sensor.

EXPERIMENT DESIGN PLAN HANDOUT

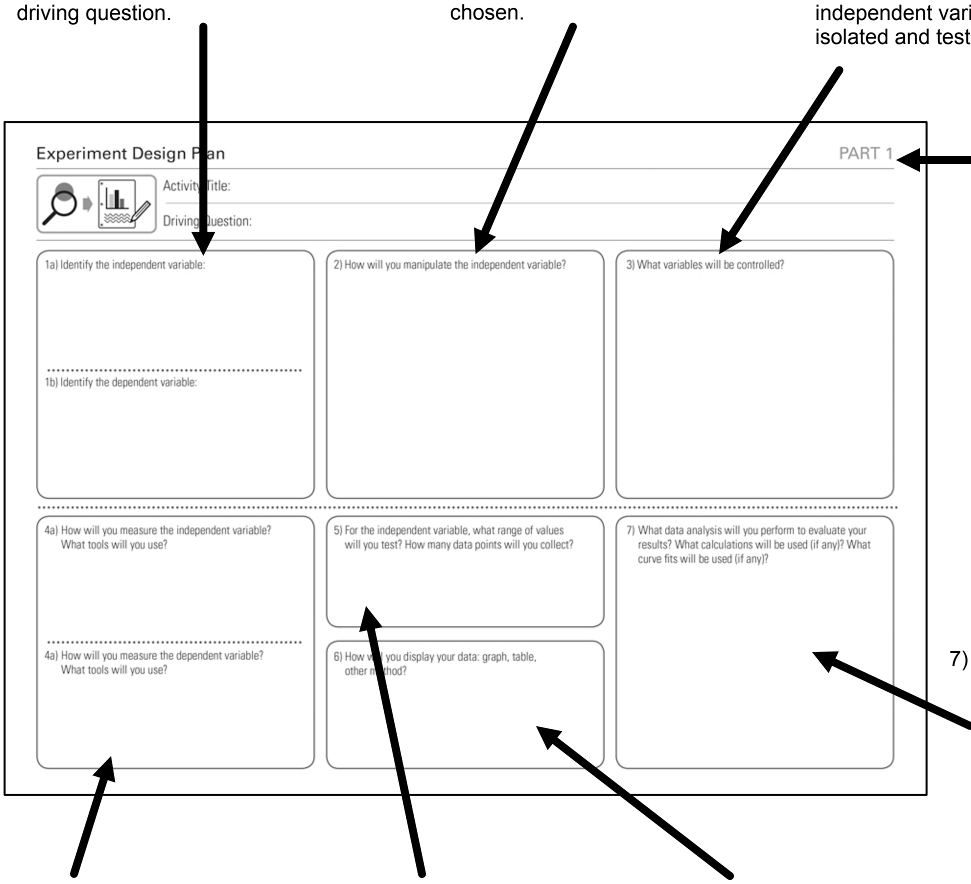
Students following the *Guided Inquiry* or *Student Designed* version of a lab activity can use the one-sheet handout on the following page to help design and implement their inquiry-based investigation. The handout provides students with a small amount of guidance and structure as they develop their own laboratory investigation, regardless of the lab topic. The different sections and their roles are described below:

Components of the Handout

1) Students first identify the independent and dependent variables in their proposed experiment, based on the driving question.

2) Students then determine how to physically manipulate the independent variable they've chosen.

3) Students identify other variables in their proposed experiment that must be controlled so the independent variable can be isolated and tested.



The same content is repeated on the document's backside for students whose proposed experiment requires more than one configuration.

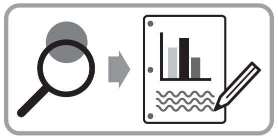
7) Students determine how they will analyze their data: Will there be any additional data manipulation (this includes linearization), calculations, curve fits, smoothing, et cetera.

4) Students determine how they will measure the independent and dependent variables in their proposed experiment, what sensors they will use, and what steps they will take to make the measurements.

5) Students determine the values for testing their independent variable. Results of too small of a range of values may prove inconclusive; using too large of a range may exceed the capabilities of their measurement tools.

6) Students then determine how best to display the data (such as a graph or table) they will collect in their proposed experiment. Students who choose to plot graphs should identify which variable to plot on the *x*-axis, and which to plot on the *y*-axis.

Experiment Design Plan



Activity Title:

Driving Question:

1a) Identify the independent variable:

1b) Identify the dependent variable:

2) How will you manipulate the independent variable?

3) What variables will be controlled?

4a) How will you measure the independent variable? What tools will you use?

4a) How will you measure the dependent variable? What tools will you use?

5) For the independent variable, what range of values will you test? How many data points will you collect?

6) How will you display your data: graph, table, other method?

7) What data analysis will you perform to evaluate your results? What calculations will be used (if any)? What curve fits will be used (if any)?

Experiment Design Plan

1a) Identify the independent variable:

1b) Identify the dependent variable:

2) How will you manipulate the independent variable?

3) What variables will be controlled?

4a) How will you measure the independent variable? What tools will you use?

4a) How will you measure the dependent variable? What tools will you use?

5) For the independent variable, what range of values will you test? How many data points will you collect?

6) How will you display your data: graph, table, other method?

7) What data analysis will you perform to evaluate your results? What calculations will be used (if any)? What curve fits will be used (if any)?

1. HYDROSTATIC PRESSURE

Connections to the AP® Physics 2 Curriculum*

The lab activity correlates to the following pieces of the AP® Physics 2 framework:

Big Idea	3	Enduring Understanding	C	Essential Knowledge	4

Learning Objective 1: The student is able to make claims about various contact forces between objects based on the microscopic cause of those forces.
Science Practices: 6.1

Learning Objective 2: The student is able to explain contact forces (tension, friction, normal, buoyant, spring) as arising from interatomic electric forces and that they therefore have certain directions.
Science Practices: 6.2

Time Requirement

Preparation Time: 15 minutes

Lab Activity: 30 minutes

Prerequisites

Students should be familiar with the following concepts:

- Pressure is the force applied to a fluid per unit area.
- Gauge pressure is the absolute pressure minus the atmospheric pressure.
- Density is the mass per unit volume of a fluid.

Driving Question | Objective

How is static pressure related to depth in a column of water? Experimentally determine the mathematical relationship between static pressure and depth in a column of water.

Procedural Overview

In the Structured version of this lab activity, students measure the pressure in a water reservoir at five different depths and record the pressure and height data in a table. After collecting data, the students graph pressure versus depth to discover a linear relationship in the data. A best-fit line is applied to the data; students learn that its slope is used to determine an experimental value for the density of the water.

Pre-Lab Discussion and Activity

Unless you are using 1000-mL graduated cylinders, it is very likely your students will be using water reservoirs that do not have a uniform shape, such as a 2-liter soda bottle. Students may question whether this introduces an additional variable. To demonstrate that the shape of the reservoir does not have any effect on the pressure measurement, take pressure measurements in water reservoirs of different shapes (see the Teacher Tips for examples), always at the same water depth. You may also want to consider having student groups use differently shaped water reservoirs for their investigations so they may compare their results.

Materials and Equipment

- Data collection system
- PASCO Barometer/Low Pressure Sensor[1]
- PASCO Sensor Extension Cable
- Quick connector[1]
- Tubing, 1/4" diameter, longer than 30-cm[1]
- Water reservoir, transparent, over 30 cm high
- Ruler
- Distilled water, to fill the reservoir 3/4 full

Probeware Resources

Below are web-link and QR codes that will direct you to instructional video resources for individual pieces of PASCO probeware, sensors, and other hardware used in the lab activity. These same links and codes are provided to students in their activity handouts.

[1]www.pasco.com/ap24

PASCO Barometer/Low Pressure Sensor

Safety

Follow this important safety precaution in addition to your regular classroom procedures:

- Make necessary arrangements to your workstation to avoid getting water on any electronic equipment.

Teacher Tips

Tip 1 – Water Reservoir

- Any transparent reservoir will work for this experiment, regardless of shape, since pressure is not dependent on volume. Examples of reservoirs: 1000-ml graduated cylinders, 2-liter soda bottles, vases, and fish tanks.

Tip 2 – Using an Absolute Pressure Sensor

- The PASCO Barometer/Low Pressure Sensor is recommended for this activity based on its measurement resolution. However, an absolute pressure sensor can also be used for this activity, although it may provide slightly less accurate results.

Sample Data

Below are sample data, acquired using the experimental setup and procedure outlined in the Structured version of the lab activity, and answers to questions in the Data Analysis section.

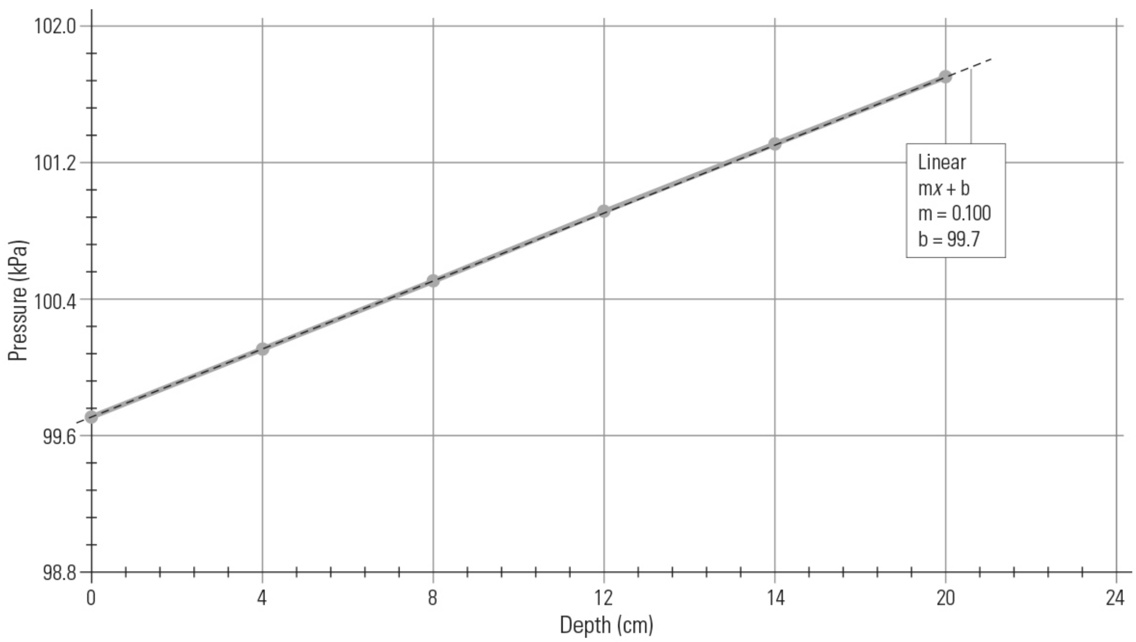
Data Analysis

Table 1: Pressure versus depth in a water column

Depth (cm)	Pressure (kPa)
0	99.71
4	100.10
8	100.50
12	100.91
16	101.31
20	101.70

1. Plot a graph of *pressure* versus *depth* in the blank Graph 1 axes. Be sure to label both axes with the correct scale and units.

Graph 1: Pressure versus depth in a water column



2. Draw a line of best fit through your data in Graph 1. Determine and record the equation of the line here:

Best fit line equation: $P = (0.100\ \text{kPa/cm})\,h + 99.7\ \text{kPa}$

Guided Inquiry Questions

Below are sample responses to the Guiding Questions found in the Guided Inquiry version of this lab activity.

❷ 1. Since the pressure sensor cannot be submerged in water, tubing attached to the pressure sensor must be placed in the water to take measurements. How long must your tubing be in order to take the necessary pressure measurements?

The tubing must be at least as long as the height of the water reservoir so that measurements can be made at the bottom of the water reservoir. Because the tubing will likely curve when it is in the reservoir, the tubing should actually be slightly longer than the height of the reservoir.

❷ 2. Assuming the tubing is sealed air tight to the sensor, if you submerge the tube from the low-pressure sensor into the water reservoir, how will the pressure reading change as the end of the tube is submerged deeper into the water? Explain your answer.

The pressure reading should increase as the end of the tube is submerged because the static pressure from the water, pushing on the air column, increases with depth.

❷ 3. If you submerge the entire length of tube into the water but the end of the tube remains exposed above the water's surface, what would the pressure read and why?

The pressure sensor would read atmospheric pressure because the exposed end allows the air inside the tube to equilibrate with the atmospheric pressure above the water.

❷ 4. When the pressure inside the submerged tubing increases, what will happen to the volume of air inside the tubing, assuming that the tubing does not change shape, and how will this affect your depth measurements? Will you still measure depth at the end of the tubing?

The volume of air will decrease because the increasing pressure causes the water level inside the tubing to rise. The depth will be measured from the water level inside the tube to the surface of the water because the air pressure in the tubing is balanced with the water pressure at this point.

Assessment Questions: Sample Responses

Sample responses to the Analysis and Synthesis questions found in each version of the lab activity:

Analysis Questions

❷ 1. Does a graph of pressure versus depth produce a linear relationship? If yes, what is the y-intercept of the relationship equal to, theoretically?

A graph of pressure versus depth will produce a linear relationship. The y-intercept is equal to the atmospheric pressure.

❷ 2. Static pressure is related to depth according to the equation,

$$P = P_0 + \rho g h$$

where P is pressure, P_0 is the initial pressure, ρ is fluid density, g is acceleration due to gravity, and h is depth. From a linear graph relating pressure to depth, extrapolate a value for the density of the fluid in the reservoir (water). Show your work.

$$P = \rho g h$$

$$\frac{P}{h} = \rho g$$

slope $= \rho g$; a typical result might be 10,000 Pa/m

$$\rho = \frac{\text{slope}}{g} = \frac{10{,}000\ \text{Pa/m}}{9.8\ \text{m/s}^2} = 1020\ \text{kg/m}^3$$

❷ 3. If the theoretical value of the density of water is 1,000 kg/m³, calculate the percent error between your experimental value and the actual value. Show your work.

$$\text{Percent error} = \left|\frac{\text{Actual} - \text{Experimental}}{\text{Actual}}\right| \times 100$$

Calculation using the typical result:

$$\text{Percent error} = \left|\frac{1000 \text{ kg/m}^3 - 1020 \text{ kg/m}^3}{1000 \text{ kg/m}^3}\right| \times 100 = 2.0\%$$

❷ 4. If you performed this same experiment using liquid iodine (density ≈ 4,900 kg/m³) instead of water, how would a graph of pressure versus depth be different?

The graph would have a slope that is approximately 5 times as steep as water, but the *y*-intercept would remain the same.

Synthesis Questions

❷ 1. Imagine you are submerged in a submarine 1,066 m below the surface of the ocean. On this submarine is a round window made of glass. How thick would the glass need to be if 4 mm of glass is needed per 197.5 kPa pressure difference to prevent the glass from shattering? Assume ocean water has a density of 1,030 kg/m³, and that the pressure inside the submarine is equal to atmospheric pressure.

$$P_{\text{water}} = \rho g h + P_0$$

$$P_{\text{submarine}} = P_0$$

$$\Delta P = P_{\text{water}} - P_{\text{submarine}} = (\rho g h + P_0) - P_0 = \rho g h$$

$$\Delta P = (1030 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(1066 \text{ m}) = 1.076 \times 10^7 \text{ Pa}$$

$$\Delta P = \frac{1.076 \times 10^7 \text{ Pa}}{1.975 \times 10^5 \text{ Pa}} = 54.48$$

$54.48 \times 4 \text{ mm} = 217.9 \text{ mm}$ thick glass is needed

❷ 2. A group of marine biologist researchers have found a rare form of sea life that exists only at great depth in the ocean. If these researchers measured the gauge pressure at the depth these creatures live and found it to be 377 atm, at what depth in meters do these creatures live? Assume ocean water has a density of 1,030 kg/m³.

$$\text{gauge pressure} = \Delta P = P - P_0 = \rho g h$$

$$h = \frac{\Delta P}{\rho g} = \frac{3.82 \times 10^7 \text{ Pa}}{(1030 \text{ kg/m}^3)(9.8 \text{ m/s}^2)} = 3784 \text{ m}$$

Extended Inquiry Suggestions

- Have the students investigate the relationship between pressure and fluid density. The students can measure pressure at the same fluid depth using different fluids. Here are some recommended fluids which are easy to obtain in local grocery or drug stores:
    - Cooking oil, 0.91 g/mL –0.92 g/mL
    - Isopropyl alcohol, 0.79 g/mL
    - Mineral oil, 0.80 g/mL
    - Glycerin, 1.26 g/mL
- The PASCO Barometer/Low Pressure sensor is sensitive enough to detect small changes in altitude. If you have access to a stairwell in a two-story building (or higher), the students can attempt to repeat this experiment using air as a fluid instead of water. Height measurements can be made by measuring the height of the steps in the stairwell.

1. HYDROSTATIC PRESSURE

STRUCTURED

Driving Question | Objective

How is static pressure related to depth in a column of water? Experimentally determine the mathematical relationship between static pressure and depth in a column of water.

Materials and Equipment

- Data collection system
- PASCO Barometer/Low Pressure Sensor[1]
- PASCO Sensor Extension Cable
- Quick connector[1]
- Tubing, 1/4" diameter, longer than 30-cm[1]
- Water reservoir, transparent, over 30 cm high
- Ruler
- Distilled water, to fill the reservoir 3/4 full

[1]www.pasco.com/ap24

PASCO Barometer/Low Pressure Sensor

Background

Anyone who has tried to swim to the bottom of a deep swimming pool has experienced the sensation of pressure on their body. The deeper you swim, the greater the pressure. So if you swim to twice the depth, would you feel twice the pressure? What is the mathematical relationship between the pressure on your body and the depth in the pool?

In this activity, you will measure the pressure at different depths using a pressure sensor to determine a mathematical relationship between pressure and depth.

Safety

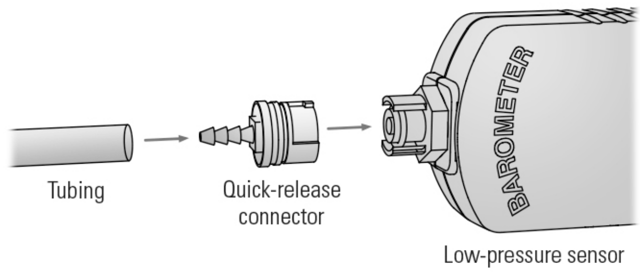
Follow this important safety precaution in addition to your regular classroom procedures:

- Make necessary arrangements to your workstation to avoid getting water on any electronic equipment.

Procedure

SET UP

1. Cut a length of 1/4" plastic tubing approximately as long as the water reservoir is deep, and connect the tubing to the valve on the low-pressure sensor using the corresponding barbed quick-snap tubing connector.



2. Connect the low-pressure sensor to the extension cable and connect the extension cable to the data collection system. Configure the data collection system to monitor data in a digits display.

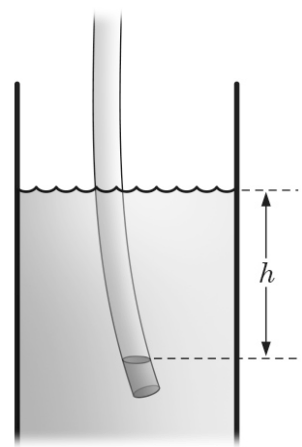
3. Fill the water reservoir approximately 3/4 full with distilled water.

COLLECT DATA

4. Hold the low-pressure sensor and the attached tubing above the water level in the reservoir, and then record the first pressure measurement and corresponding depth (0 cm) in Table 1.

5. Submerge the open end of the tubing from the low-pressure sensor into the water, lowering the tubing end to a depth of 4 cm.

   NOTE: The tubing may not be straight, which is not a problem as long as the depth is measured at the point of the open end of the tubing.



6. Record the pressure P and depth h in Table 1.

   NOTE: At greater depths, a small volume of water may creep up the tubing as the air inside the tubing compresses. In this case, measure the depth h from the surface of the water to the water level inside the tubing.

7. Lower the tube into the water four additional centimeters.

8. Repeat the previous steps until you have recorded the pressure at six depths: 0 cm, 4 cm, 8 cm, 12 cm, 16 cm, and 20 cm.

9. Empty your water reservoir.

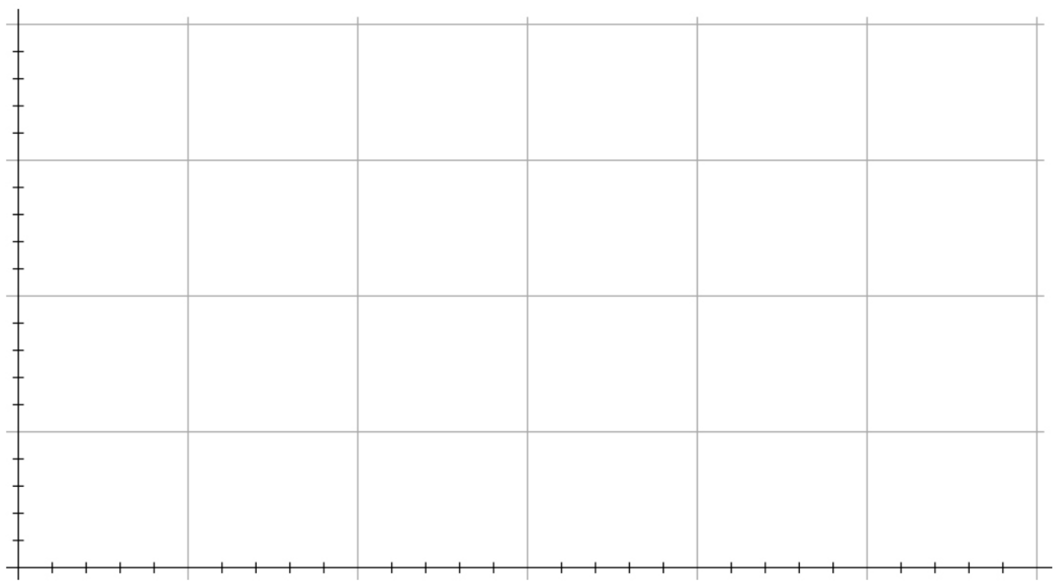
Data Analysis

Table 1: Pressure versus depth in a water column

Depth (cm)	Pressure (kPa)
0	
4	
8	
12	
16	
20	

1. Plot a graph of *pressure* versus *depth* in the blank Graph 1 axes. Be sure to label both axes with the correct scale and units.

Graph 1: Pressure versus depth in a water column



2. Draw a line of best fit through your data in Graph 1. Determine and record the equation of the line here:

   Best fit line equation: ____

Analysis Questions

1. Does a graph of pressure versus depth produce a linear relationship? If yes, what is the y-intercept of the relationship equal to (theoretically)?

2. Static pressure is related to depth according to the equation,

$$P = P_0 + \rho g h$$

where P is pressure, P_0 is the initial pressure, ρ is fluid density, g is acceleration due to gravity, and h is depth. From a linear graph relating pressure to depth, extrapolate a value for the density of the fluid in the reservoir (water). Show your work.

3. If the theoretical value of the density of water is 1,000 kg/m^3, calculate the percent error between your experimental value and the actual value. Show your work.

$$\text{Percent error} = \left|\frac{\text{Actual} - \text{Experimental}}{\text{Actual}}\right| \times 100$$

4. If you performed this same experiment using liquid iodine (density ≈ 4,900 kg/m^3) instead of water, how would a graph of pressure versus depth be different?

Synthesis Questions

1. Imagine you are submerged in a submarine 1,066 m below the surface of the ocean. On this submarine is a round window made of glass. How thick would the glass need to be if 4 mm of glass is needed per 197.5 kPa pressure difference to prevent the glass from shattering? Assume ocean water has a density of 1,030 kg/m^3, and that the pressure inside the submarine is equal to atmospheric pressure.

2. A group of marine biologist researchers have found a rare form of sea life that exists only at great depth in the ocean. If these researchers measured the gauge pressure at the depth these creatures live and found it to be 377 atm, at what depth in meters do these creatures live? Assume ocean water has a density of 1,030 kg/m^3.

2. BUOYANT FORCE

Connections to the AP® Physics 2 Curriculum*

The lab activity correlates to the following pieces of the AP® Physics 2 framework:

Big Idea	1	Enduring Understanding	E	Essential Knowledge	1

Learning Objective 2: The student is able to select from experimental data the information necessary to determine the density of an object and/or compare the densities of several objects.
Science Practices: 4.1, 6.4

Big Idea	3	Enduring Understanding	A	Essential Knowledge	3

Learning Objective 1: The student is able to analyze a scenario and make claims (develop arguments, justify assertions) about the forces exerted on an object by other objects for different types of forces or components of forces.
Science Practices: 6.4, 7.2

Big Idea	3	Enduring Understanding	C	Essential Knowledge	4

Learning Objective 2: The student is able to explain contact forces (tension, friction, normal, buoyant, spring) as arising from interatomic electric forces and that they therefore have certain directions.
Science Practices: 6.2

Time Requirement

Preparation Time: 20 minutes

Lab Activity: 50 minutes

Prerequisites

Students should be familiar with the following concepts:

- Newton's Second Law: the sum of the forces acting on an object is equal to the mass of the object multiplied by its acceleration.
- Free body diagrams and force vectors.

Driving Question | Objective

What are the relationships between the buoyant force on an object submerged in a fluid and a) the volume of the submerged object, and b) the weight of the fluid displaced by the submerged object? Perform an experiment whose data will help determine both relationships.

Procedural Overview

In the Structured version of this lab activity, students use thread to hang two metal cylinders (one brass, one aluminum) from a force sensor (one at a time) and measure the tension in the string as each cylinder is lowered into an overflow container filled with soapy water. Students capture the water that is displaced by the cylinder as it is lowered and simultaneously measure string tension and mass of the displaced water as the cylinder is submerged to five different depths.

Students use their tension values to calculate the buoyant force acting on the cylinder as it is submerged, and then plot a graph of buoyant force versus submerged volume showing two curves: one for each metal cylinder. Both curves in this graph should be nearly identical and should show a linear relationship. Students use their data to identify the proportionality between buoyant force and submerged volume and the extensibility of the relationship to objects of any density.

Students also calculate the weight of the five different volumes of water displaced by each submerged cylinder, and show the proportional relationship (and equality) between buoyant force and the weight of the displaced water, thus confirming Archimedes's principle.

Pre-Lab Discussion and Activity

Explain to students that objects will float in a fluid if its overall density is less than the density of the fluid, and sink if its density is greater. Whether an object will sink or float can also be determined by the buoyant force acting on it. An object will sink when the force from gravity acting on the object is greater than buoyant force, and float when the gravitational force is less.

Give students examples of materials and objects that float or sink in water (wood, plastic, rocks, aluminum, and ships). Explain why ships made from steel and iron are able to float; “Although a boat's hull is made from steel, which is denser than water, the boat will still float because much of the boat's volume is air, which is less dense than water.”

Demonstrate that by submerging an object, some volume of fluid is displaced by the volume of the submerged object; either partially of fully submerged. Express that the difference in pressure on the object submerged can be used to derive the buoyant force acting on that object: a simple net force calculation using the differences in pressures between top and bottom surfaces.

PRE-LAB QUESTIONS

❷ 1. Find the volume of a cylinder with a radius of 12.0 cm and height of 25.0 cm. Write the answer in cm^3 and m^3.

$$V_{cyl} = A_{cyl}h$$

$$V_{cyl} = \pi r^2 h = \pi (12.0\ \text{cm})^2 (25.0\ \text{cm}) = 11{,}300\ \text{cm}^3$$

$$V_{cyl} = \pi r^2 h = \pi (0.120\ \text{m})^2 (0.250\ \text{m}) = 0.0113\ \text{m}^3$$

❷ 2. Calculate the mass of 1,000 mL of water if the density of water is 1,000 kg/m^3.

$$\rho = 1{,}000\ \text{kg/m}^3$$

$$V = 1{,}000\ \text{mL} \times \frac{\text{cm}^3}{\text{mL}} \times \frac{10^{-6}\ \text{m}^3}{\text{cm}^3} = 0.001\ \text{m}^3$$

$$\rho = \frac{m}{V}$$

$$m = \rho V = \left(1{,}000\ \text{kg/m}^3\right)\left(0.001\ \text{m}^3\right) = 1\ \text{kg}$$

3. What is the buoyant force if an object experiences a net force of –50 N in air and –20 N in water?

$$\sum F_{\text{air}} = -F_{\text{g}} = -50\ \text{N}$$

$$\sum F_{\text{water}} = F_{\text{b}} - F_{\text{g}} = -20\ \text{N}$$

$$F_{\text{b}} = -20\ \text{N} + F_{\text{g}} = -20\ \text{N} + 50\ \text{N} = 30\ \text{N}$$

4. What is the buoyant force on an inflated 10.0 g balloon if the balloon experiences an upward acceleration of 0.75 m/s² the moment after it is released? Assume that air drag is negligible.

$$\sum F = ma$$

$$F_{\text{b}} - F_{\text{g}} = ma$$

$$F_{\text{b}} = ma + mg = m(a + g) = 0.0100\ \text{kg}\left(0.75\ \text{m/s}^2 + 9.8\ \text{m/s}^2\right) = 0.11\ \text{N}$$

Materials and Equipment

- Data collection system
- PASCO High Resolution Force Sensor with hook[1]
- PASCO Overflow Can[2]
- Brass cylinder[3]
- Aluminum cylinder[3]
- Balance, 0.01-g resolution (1 per class)
- Table clamp or large base
- Support rod, 45-cm (2)
- Right angle clamp
- Thread, 60 cm
- Beaker, 100-mL
- Beaker, 1-L
- Glass stir rod
- Felt-tipped pen with permanent ink
- Liquid dish soap, 3 mL
- Water, 500 mL
- Paper towel, several sheets
- Meter stick

[3] For information on other metal cylinders to use, see the Lab Preparation section below.

Probeware Resources

Below are web-link and QR codes that will direct you to instructional video resources for individual pieces of PASCO probeware, sensors, and other hardware used in the lab activity. These same links and codes are provided to students in their activity handouts.

[1]www.pasco.com/ap22

PASCO High Resolution Force Sensor

[2]www.pasco.com/ap25

PASCO Overflow Can

Safety

Follow this important safety precaution in addition to your regular classroom procedures:

- Make necessary arrangements to your workstation to avoid getting water on any electronic equipment.

Lab Preparation

1. The equipment list in this lab activity specifies two metal cylinders per lab group: one brass and one aluminum. It is recommended that groups use the brass and aluminum cylinders that are part of the ME-8569A PASCO Density Set. However, any two metal cylinders can be used as long the cylinders are made from different metals, and students have the ability to suspend each cylinder vertically above its center. This may require a hole, hook, or eyelet in the top (center) of each cylinder so that string can be attached to suspend it from a force sensor, similar to the setup in the Structured version of this lab activity.

   If students use cylinders made from metals other than brass and aluminum, the Materials and Equipment section in each student handout, as well as the Procedure and Data Analysis sections in the Structured student handout, should be altered to reflect the metals used.

2. Students following the procedure outlined in the Structured version of this lab activity use a solution of liquid dish soap and water inside the PASCO Overflow Can. According to the student handout, each group is responsible for mixing their own solution. If you choose, this solution can be mixed before class to help eliminate the first two procedural steps during the lab period.

   Each group needs approximately 500 mL of mixed solution for their experiment. To mix enough solution for the entire class, divide the number of lab groups in your class by two, and then mix that number in liters of water with about 6 mL of liquid dish soap to every 1 L of water. Avoid mixing the solution too quickly or vigorously as this will make the surface of the solution foamy.

Teacher Tips

Tip 1 – Add Soap to the Overflow Can

- Students using the PASCO Overflow Can in their experiments may choose to use any fluid inside the can; however, it is recommended that a solution with low viscosity and low surface tension be used to avoid clogs in the downspout on the can. Even surface tension in water can be too great to allow small amounts of water to travel into the downspout on the can.
- To avoid clogs in the downspout when using water in the overflow can, add a small amount of liquid dish soap to help disrupt the surface tension at the spout opening. Add approximately 3 mL of liquid dish soap to every 500 mL of water.

Tip 2 – Hang the Cylinder Vertically

- Students using cylinders to displace water, including those performing the Structured version of this lab activity, should make certain that the cylinders always hang in a perfectly vertical orientation. Lowering the cylinder into the overflow can at an angle may cause calculated values for the submerged volume to be different from the actual submerged volume of the cylinder.
- Each of the cylinders in the PASCO Density Set has a hole drilled through the top to facilitate hanging the cylinders from a force sensor. Students may find that the cylinders will hang from the hook on a force sensor at a very slight angle due to friction between the thread or string and the cylinder in, and just above, either side of the threading hole. This is easily corrected by small adjustments to the string and cylinder position.

Tip 3 – Using a Second High Resolution Force Sensor

- Students performing the Structured version of this lab activity use a centigram balance to measure the mass of the water displaced by their cylinder as it is lowered into the PASCO Overflow Can. This mass value is then used to calculate the weight (gravitational force in newtons) of the displaced water. If available, students can use a second high resolution force sensor with hook in place of the centigram balance to measure this gravitational force directly.

To do this, add an additional right angle clamp and rod to hang the second force sensor in an orientation similar to the original force sensor outlined in the Set Up section in the Structured student handout. Use thread to suspend a small cup from the hook on the second sensor and position the sensor with the cup so that the suspended cup will catch the water pouring from the spout on the overflow can, but will not impede the movement of the other force sensor as it is moved to lower the cylinder into the overflow can.

With the suspended cup in place, and the second sensor connected to the data collection system, press the Zero button on the second force sensor. On the data collection system, build a second digits display showing the Force (Inverted) measurement from the second force sensor. Now, as water pours into the suspended cup, the measurement from the second force sensor can be recorded as the weight (in newtons) of the displaced water collected in the suspended cup.

Sample Data

Below are sample data, acquired using the experimental setup and procedure outlined in the Structured version of the lab activity, and answers to questions in the Data Analysis section.

Data Analysis

PART 1 – BRASS CYLINDER

Brass cylinder length l (cm): 6.50 cm

Brass cylinder radius r (cm): 1.17 cm

Brass cylinder area A_{cyl} (cm²): 4.30 cm²

Table 1: Buoyant force and displacement values for a brass cylinder submerged in a fluid

Depth (cm)		V_{subm} (cm³)	Tension (N)		m_{disp} (g)	F_b (N)	w_{disp} (N)
0	0.00	0.00	T_1	2.05	0.00	0.00	0.00
$\frac{1}{4}l$	1.63	7.00	T_2	2.00	5.24	0.05	0.051
$\frac{1}{2}l$	3.25	14.0	T_3	1.94	11.33	0.11	0.11
$\frac{3}{4}l$	4.88	21.0	T_4	1.88	17.45	0.17	0.17
l	6.50	28.0	T_5	1.81	24.40	0.24	0.24

PART 2 – ALUMINUM CYLINDER

Aluminum cylinder length l (cm): 6.50 cm

Aluminum cylinder radius r (cm): 1.17 cm

Aluminum cylinder area A_{cyl} (cm^2): 4.30 cm^2

Table 2: Buoyant force and displacement values for an aluminum cylinder submerged in a fluid

Depth (cm)		V_{subm} (cm^3)	Tension (N)		m_{disp} (g)	F_b (N)	w_{disp} (N)
0	0.00	0.00	T_1	0.66	0.00	0.00	0.00
$\frac{1}{4}l$	1.63	7.00	T_2	0.61	5.27	0.05	0.05
$\frac{1}{2}l$	3.25	14.0	T_3	0.55	11.38	0.11	0.11
$\frac{3}{4}l$	4.88	21.0	T_4	0.49	17.72	0.17	0.17
l	6.50	28.0	T_5	0.42	24.55	0.24	0.24

1. Calculate the following using the measured values for each cylinder. Record the results into or above each cylinder's respective table.

   a. The depth in centimeters in Tables 1 and 2 using cylinder length l.

   Calculation using sample data for 3/4 l:

   $$\text{depth} = \frac{3}{4}l = \frac{3}{4}(6.50\ \text{cm}) = 4.88\ \text{cm}$$

   b. The cross-sectional area A, in cm^2, of each cylinder using radius r.

   $$A_{cyl} = \pi r^2$$

   Calculation using sample data for the brass cylinder:

   $$A_{cyl} = \pi r^2 = \pi(1.17\ \text{cm})^2 = 4.30\ \text{cm}^2$$

   c. The volume of the cylinder submerged V_{subm}, in cm^3, at each depth.

   $$V_{subm} = \text{depth} \times A_{cyl}$$

   Calculation using sample data for the brass cylinder at a depth of 4.88 cm:

   $$V_{subm} = \text{depth} \times A_{cyl} = 4.88\ \text{cm} \times 4.30\ \text{cm}^2 = 21.0\ \text{cm}^3$$

2. If the tension T_n measured when each cylinder was submerged is equal to the difference between the gravitational force and the buoyant force:

$$T_n = F_g - F_b$$

and the tension T_1 measured when each cylinder was suspended above the water is equal to the gravitational force:

$$T_1 = F_g$$

then the buoyant force on each cylinder is equal to:

$$F_b = T_1 - T_n \quad (1)$$

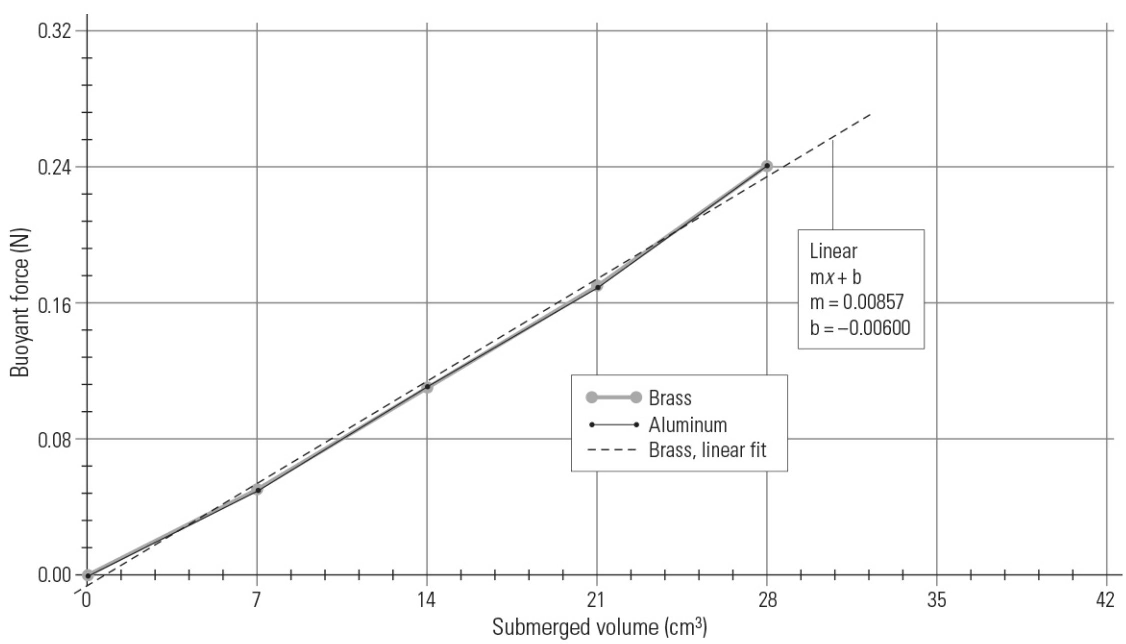
For both cylinders, use Equation 1 to calculate the buoyant force F_b at each depth. Record the results into each cylinder's respective table.

Calculation using sample data for the brass cylinder at a depth of 4.88 cm (T_4):

$F_b = T_1 - T_4 = 2.05\text{ N} - 1.88\text{ N} = 0.17\text{ N}$

3. In the blank Graph 1 axes, plot a graph of *buoyant force* versus *submerged volume* with two curves: one for the brass cylinder and one for the aluminum cylinder. Be sure to label both curves and both axes with the correct scale and units.

Graph 1: Buoyant force on a cylinder versus volume of cylinder submerged



4. Are the curves for the brass and aluminum cylinders in Graph 1 similar?

Because students use the same submerged volume values for each cylinder, both curves should be nearly identical. Discrepancies may arise if the actual submerged volume does not equal the V_{subm} values calculated in both tables. This is generally caused by human error when marking the sides of the cylinders, or determining when the cylinder is submerged up to each mark.

5. Based on your data, is it reasonable to assume that the relationship between buoyant force and submerged volume would be similar if you had used a third object with greater mass (greater density)? Explain your reasoning.

   Because the graphical relationship between buoyant force and submerged volume of an object is the same for both cylinders, and because the masses (and densities) of the brass and aluminum cylinders are different, students can reasonably assume that the relationship between buoyant force and volume of the submerged object is extensible to other objects with greater mass and density.

6. Calculate the weight w_{disp} (in newtons) of the displaced fluid at each depth for both cylinders. Record your results into their respective columns.

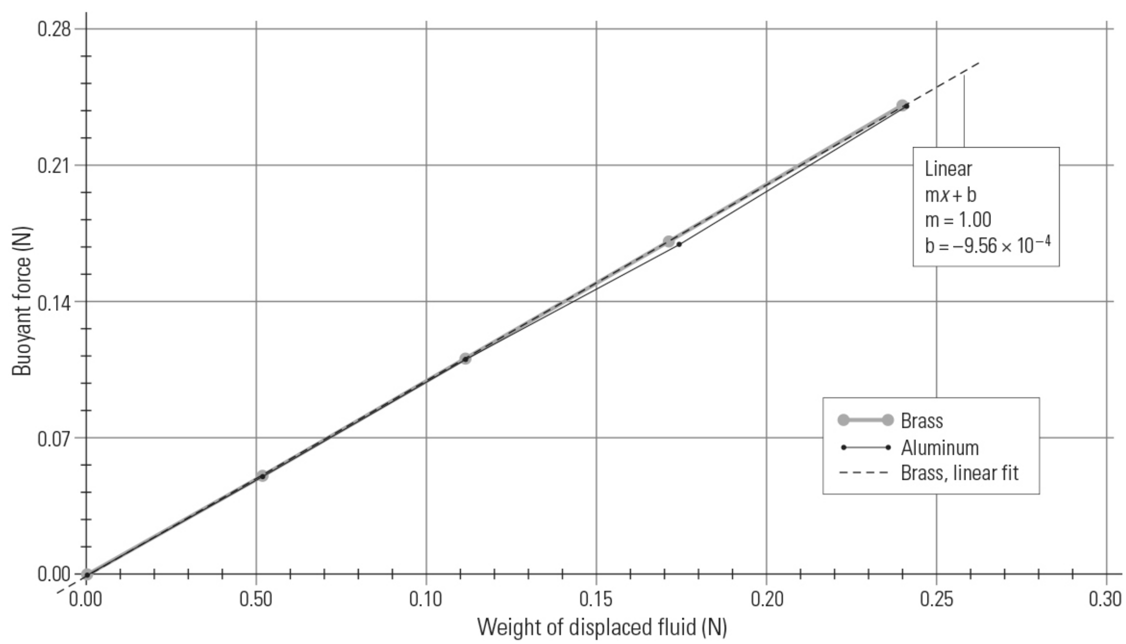
$$w_{disp} = m_{disp} g \times \frac{1\ \text{kg}}{1{,}000\ \text{g}}$$

   Calculation using sample data for the brass cylinder at a depth of 4.88 cm:

$$w_{disp} = m_{disp} g \times \frac{1\ \text{kg}}{1{,}000\ \text{g}} = (17.45\ \text{g})(9.8\ \text{m/s}^2) \times \frac{1\ \text{kg}}{1{,}000\ \text{g}} = 0.17\ \text{N}$$

7. In the blank Graph 2 axes, plot a graph of *buoyant force* versus *weight of displaced fluid* with two curves: one for the brass cylinder and one for the aluminum cylinder. Be sure to label both curves and both axes with the correct scale and units.

Graph 2: Buoyant force on a cylinder versus weight (in newtons) of fluid displaced by the cylinder



8. Are the curves for the brass and aluminum cylinders in Graph 2 similar?

   Both curves should be identical (or nearly identical), both with a proportionality constant equal to 1.00.

9. Based on your data, is it reasonable to assume that the relationship between buoyant force and the weight of the displaced fluid would be similar for a third object with greater mass (greater density)? Explain your reasoning.

   Because the graphical relationship between buoyant force and the weight of displaced fluid is the same for both cylinders, and because the masses (and densities) of the brass and aluminum cylinders are different, students can reasonably assume that the relationship between buoyant force and weight of displaced fluid is extensible to other objects with greater mass (and density).

Guided Inquiry Questions

Below are sample responses to the Guiding Questions found in the Guided Inquiry version of this lab activity.

❓ 1. If the objective of your experiment is to determine the relationships between the buoyant force on an object submerged in a fluid and a) the volume of the submerged object, and b) the weight of the fluid displaced by the submerged object, what should the dependent and independent variables be in your experiment?

The objective statement indicates three variables: buoyant force, volume of the object that is submerged, and the weight of the fluid displaced by the submerged object. Students explore how the submerged volume and weight of the displaced fluid affect the buoyant force acting on a submerged object. Students should divide their experiment into two parts, both with the same dependent variable, buoyant force, and different independent variables: a) volume of the submerged object and b) the weight of the fluid displaced by the submerged object.

❓ 2. What equipment do you have at your disposal to measure each variable, and how can you set up this equipment to measure each variable?

Buoyant force can be measured using (but is not limited to) the following tools and techniques:

- Students may choose to hang an object from a high resolution force sensor or high resolution spring scale, measuring the upward force required to suspend the object. As the object is lowered into a fluid, the buoyant force increases, thus decreasing the upward force required to suspend the object. The buoyant force equals the difference between the gravitational force on the object and the measurement by the sensor or scale when the object is submerged to specified depths.
- Students may choose to float an object in a fluid and use a high resolution force sensor or high resolution spring scale to measure the downward force required to press on the object to submerge it further (without fully submerging it). As the object is pressed into a fluid, the buoyant force increases, thus increasing the downward force required to submerge it further. The buoyant force equals the sum of the gravitational force acting on the object and the measurement by the sensor or scale.
- Students may choose to float an object in a fluid and use masses placed on the object to submerge it further (to specific depths, without fully submerging it). As masses are placed on the object, the downward force acting on the object increases, counteracting the buoyant force on the object. The buoyant force equals the sum of the gravitational force acting on the masses added to the gravitational force acting on the object.

The volume of the submerged object should be measured as directly as possible using spatial measurement tools such as a meter stick, and calculations of volume based on the shape of the object, similar to the technique outlined in the Structured version of this lab activity.

If students choose, they can also measure volume by submerging the object into a graduated cylinder or other beaker with graduations: as the object is submerged, fluid will be displaced whose volume is equal to the submerged volume of the object. Students can measure this displaced volume using the graduations on their beaker or graduated cylinder.

This same technique can be used to determine the mass of the displaced fluid: using the density of the fluid ρ, students can calculate the weight of the fluid using weight = ρgV, where V is the volume of displaced fluid. Otherwise, students should measure the weight of the displaced fluid directly with a balance and PASCO Overflow Can using a technique similar to that outlined in the Structured version of this lab activity.

❓ 3. How will you change each independent variable while collecting data? What steps will you take, and should you change more than one variable at a time? Justify your answer.

Each independent variable should be changed one at a time while the others are held constant: measure buoyant force and vary the volume of the submerged object while keeping object density constant; measure buoyant force and vary the weight of the displaced fluid while keeping the density of the object constant.

❓ 4. Should you measure all of the variables in your experiment at the same time? Justify your answer.

Only the dependent and independent variables need to be measured and recorded in each trial. If students choose to draw graphs on *x*–*y* axes, only two variables can be represented at a time. All other controlled (constant) variables need only be measured once as long as students are confident that they remain constant in each trial.

Because the volume of fluid displaced by a submerged object is equal to the submerged volume of the object, students may choose to measure only the volume of fluid displaced or the submerged volume and use either to calculate the weight of the displaced fluid using the equation weight = ρgV where V is the volume of displaced fluid and ρ is the density of the fluid.

5. What other variables in your experiment must remain unchanged to help isolate each variable of interest? Justify your answer.

The measuring devices and techniques that students use should remain constant throughout the experiment. This helps maintain consistency in resolution and accuracy in measurements. Students measuring buoyant force as their dependent variable must avoid any variable changes, other than those to the independent variable being investigated, that may affect the measured buoyant force, submerged volume, or displaced fluid weight, such as the density of the fluid.

Assessment Questions: Sample Responses

Sample responses to the Analysis and Synthesis questions found in each version of the lab activity:

Analysis Questions

1. What type of mathematical relationship (proportional, squared, inverse, inverse squared, et cetera) between buoyant force and submerged volume is implied by your data?

Student data should show that buoyant force is proportional to the submerged volume of an object. Graphically, this will appear as a linear curve with constant slope.

2. Based on your data, express the relationship between buoyant force F_b and submerged volume V_{subm} by completing this proportionality statement:

$F_b \propto$ V_{subm}

3. Convert the proportionality statement from the previous question into an equation by introducing a proportionality constant k:

$F_b =$ kV_{subm}

4. The buoyant force F_b acting on an object that is partially or completely submerged in a fluid is described by the equation:

$$F_b = \rho V g \qquad (2)$$

where V is the submerged volume of the object and ρ is the density of the fluid in which the object is submerged. Which terms from this equation would be represented in your equation's proportionality constant k?

$k =$ ρg

5. Use your data to determine an experimental value for the proportionality constant k. How does this value compare to the theoretical value of the constant in Equation 2? If the experimental value is different from the theoretical value, what caused the difference?

Students can use many different techniques to determine an experimental value for this proportionality constant, including using the slope of a best fit line applied to a graph of buoyant force versus submerged volume, similar to that shown in Graph 1 in the Sample Data section above.

From the Sample Data Graph 1:

Slope of best fit line = 0.0086 N/cm^3, which equals 8,600 N/m^3

Theoretically, the proportionality constant is equal to the product of the fluid density of the fluid used by students (for students carrying out the Structured version, this is 1,000 kg/m^3 for the water with dish soap) and the acceleration due to gravity:

$$\rho g = \left(1{,}000\ \text{kg/m}^3\right)\left(9.8\ \text{m/s}^2\right) = 9{,}800\ \text{N/m}^3$$

The sample data experimental proportionality constant is less than the theoretical. This is likely due to error in the V_{subm} data used; V_{subm} is a calculated quantity that may not equal the actual submerged volume of the object used.

6. Archimedes's principle states that an object completely or partially submerged in a fluid experiences an upward buoyant force equal in magnitude to the weight of the fluid displaced by the object. Does your data support this statement? If yes, explain how it supports it; if no, identify which data do not support it, and what may have caused this disagreement.

   Student data should support Archimedes' principle, showing that buoyant force is proportional to the weight of the fluid displaced by the object, regardless of the object and fluid used. A graph of buoyant force versus weight of displaced fluid will show a linear (proportional) relationship with a slope (proportionality constant) equal to 1.00. A proportionality constant of 1.00 implies that the weight of the displaced water is equal to the buoyant force regardless of the magnitude of the displaced volume.

Synthesis Questions

1. A wood salvage company is hoisting an old tree trunk off the bottom and out of a lake. The cable from the hoist is tied around the log above its center of mass. The hoist applies a force of 9,800 N to the cable to suspend the log in the lake water (T_{water}), and a force of 29,000 N to suspend the log above the lake surface (T_{air}). What are the volume and density of the log? Assume the lake water has a density of 1,007 kg/m³.

$$T_{air} = 29{,}000\text{ N}$$

$$T_{water} = 9{,}800\text{ N}$$

$$\sum F_{air} = T_{air} - F_g = 0$$

$$F_g = T_{air}$$

$$m_{log}g = T_{air}$$

$$m_{log} = \frac{T_{air}}{g} = \frac{29{,}000\text{ N}}{9.8\text{ m/s}^2} = 3{,}000\text{ kg}$$

$$\sum F_{water} = T_{water} + F_b - F_g = 0$$

$$F_b = F_g - T_{water}$$

$$F_b = T_{air} - T_{water}$$

$$\rho_{water}V_{log}g = T_{air} - T_{water}$$

$$V_{log} = \frac{T_{air} - T_{water}}{\rho_{water}g} = \frac{29{,}000\text{ N} - 9{,}800\text{ N}}{\left(1{,}007\text{ kg/m}^3\right)\left(9.8\text{ m/s}^2\right)} = \frac{19{,}200\text{ N}}{9{,}900\text{ N/m}^3} = 1.9\text{ m}^3$$

$$\rho_{log} = \frac{m_{log}}{V_{log}} = \frac{3000\text{ kg}}{1.9\text{ m}^3} = 1{,}600\text{ kg/m}^3$$

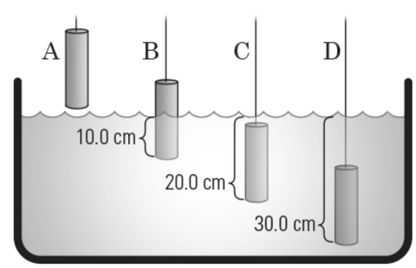
2. A cylinder with radius 5.00 cm and length 20.0 cm is lowered into a tank of glucose, which has a density of 1,385 kg/m³. The cylinder is lowered in four stages:

   A) Zero submersion

   B) Half-submerged to a depth of 10.0 cm

   C) Fully submerged to a depth of 20.0 cm

   D) Fully submerged to a depth of 30.0 cm



   a. What is the buoyant force on the cylinder at each stage?

   A) Zero volume of the cylinder is submerged, therefore the buoyant force is zero.

   B) $F_b = \rho V g$

$$F_b = \rho \pi r^2 (\text{depth})g = \left(1{,}385\text{ kg/m}^3\right)\pi\left(0.0500\text{ m}\right)^2\left(0.100\text{ m}\right)\left(9.8\text{ m/s}^2\right) = 11\text{ N}$$

   C) F_b = 22 N. The depth at stage C is twice as deep as stage B, therefore the magnitude of the submerged volume is twice as great. Because the buoyant force is proportional to the submerged volume, if the volume doubles, so does the buoyant force.

   D) F_b = 22 N. All of the cylinder's volume was submerged in stage C. Therefore, if the submerged volume doesn't change, neither does the buoyant force.

b. After being lowered to a depth of 30.0 cm, the string holding the cylinder is cut. If the net force on the cylinder after the string is cut is 1.00 N downward, what is the density of the cylinder material?

$$\sum F = F_b - F_g = -1.00 \text{ N}$$

$$\rho_{glucose}V_{cyl}g - m_{cyl}g = -1.00 \text{ N}$$

$$\rho_{glucose}V_{cyl}g - \rho_{cyl}V_{cyl}g = -1.00 \text{ N}$$

$$\rho_{cyl}V_{cyl}g = \rho_{glucose}V_{cyl}g + 1.00 \text{ N}$$

$$\rho_{cyl} = \rho_{glucose} + \frac{1.00 \text{ N}}{V_{cyl}g}$$

$$\rho_{cyl} = \rho_{glucose} + \frac{1.00 \text{ N}}{\pi r_{cyl}^2 l_{cyl} g} = 1{,}385 \text{ kg/m}^3 + \frac{1.00 \text{ N}}{\pi (0.0500 \text{ m})^2 (0.200 \text{ m})(9.8 \text{m/s}^2)} = 1{,}385 \text{ kg/m}^3 + \frac{1.00 \text{ N}}{0.015 \text{ N} \cdot \text{m}^3/\text{kg}} = 1{,}452 \text{ kg/m}^3$$

❷ 3. A crab fisherman has built a crab trap out of plastic pipe and wire mesh. The overall mass and volume of the trap are 5.59 kg and 6,213 cm^3, respectively. To catch crab, the trap must sink to the ocean floor. The fisherman has several lead weights to add to the trap to ensure it sinks. If sea water has a density of 1,021 kg/m^3, and each lead weight has mass of 113.4 g and volume of 10.0 cm^3, what is the minimum number of weights the fisherman must add so that the trap sinks to the ocean floor?

$$\sum F_g > \sum F_b$$

$$F_{g\text{ trap}} + nF_{g\text{ weight}} > F_{b\text{ trap}} + nF_{b\text{ weight}}$$

$$m_{trap}g + nm_{weight}g > \rho V_{trap}g + n\rho V_{weight}g$$

$$n(m_{weight} - \rho V_{weight}) > \rho V_{trap} - m_{trap}$$

$$n > \frac{\rho V_{trap} - m_{trap}}{m_{weight} - \rho V_{weight}} = \frac{(1{,}021 \text{ kg/m}^3)(6{,}213 \text{ cm}^3)(10^{-6} \text{ m}^3/\text{cm}^3) - 5.59 \text{ kg}}{0.1134 \text{ kg} - (1{,}021 \text{ kg/m}^3)(10.0 \text{ cm}^3)(10^{-6} \text{ m}^3/\text{cm}^3)} = \frac{6.343 \text{ kg} - 5.59 \text{ kg}}{0.1134 \text{ kg} - 0.01021 \text{ kg}} = \frac{0.75 \text{ kg}}{0.1032 \text{ kg}} = 7.3 \text{ weights}$$

$n > 7.3$ weights to sink the trap

$\therefore n = 8$ weights, the minimum number the fisherman must add

Extended Inquiry Suggestions

Extend this activity by challenging students to design an experiment that uses the relationships established in this lab activity to determine the density of the irregularly-shaped metal piece in the PASCO Density Set. Students can follow a procedure similar to that outlined in the Structured version of this lab activity in which the string tension needed to suspend the object and the submerged volume are measured as the object is lowered into a fluid.

Using the relationship:

$$\sum F = F_b + T - F_g = 0$$

$$T = F_g - F_b$$

$$T = \rho_{object}V_{object}g - \rho_{fluid}V_{object}g$$

$$T = V_{object}g(\rho_{object} - \rho_{fluid})$$

Students can then plot a graph of tension T versus submerged volume V_{object} (a linear relationship). The slope will equal the difference in densities between the fluid ρ_{fluid} and the object ρ_{object}, multiplied by earth's gravitational acceleration g. From this, students can calculate an experimental value for the density of the object.

2. BUOYANT FORCE

STRUCTURED

Driving Question | Objective

What are the relationships between the buoyant force on an object submerged in a fluid and a) the volume of the submerged object, and b) the weight of the fluid displaced by the submerged object? Perform an experiment whose data will help determine both relationships.

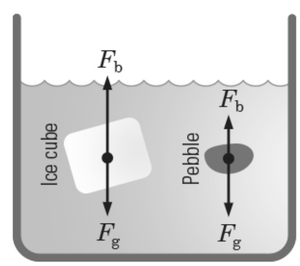
Materials and Equipment

- Data collection system
- PASCO High Resolution Force Sensor with hook[1]
- PASCO Overflow Can[2]
- Brass cylinder
- Aluminum cylinder
- Balance, 0.01-g resolution (1 per class)
- Table clamp or large base
- Support rod, 45-cm (2)
- Right angle clamp
- Thread, 60 cm
- Beaker, 100-mL
- Beaker, 1-L
- Glass stir rod
- Felt-tipped pen with permanent ink
- Liquid dish soap, 3 mL
- Water, 500 mL
- Paper towel, several sheets
- Meter stick

[1] www.pasco.com/ap22 — PASCO High Resolution Force Sensor

[2] www.pasco.com/ap25 — PASCO Overflow Can

Background



Fluids are generally thought of as liquids; however, this is a common misconception. A fluid is anything that can flow, which includes gasses as well as liquids. When an object is submerged in a fluid, it experiences an upward buoyant force F_b that opposes gravitational force F_g. This is the reason ice floats on the top of water, and a balloon filled with helium rises in air. If we define F_g in the negative direction, a submerged object will rise in the fluid if the net force is positive (the condition of ice rising in water, or a helium balloon rising in air), and sink if it is negative (the condition of a rock sinking in a pond).

The magnitude of the gravitational force acting on an object is proportional to its mass, but it is easily observable that the buoyant force acting on a submerged object is not proportional to the object's mass: a small rock may have the same mass as a tennis ball, but a tennis ball floats in water and the rock does not. So, what is different between these two objects? Their masses may be the same but their volumes are different, and so is the volume of water displaced by each once submerged.

In this activity you will explore the relationship between the buoyant force acting on an object and the volume of fluid displaced by the object, and draw conclusions that help establish the mathematical relationship between buoyant force and a) the volume of the submerged object, and b) the weight of the fluid displaced by the submerged object.

Safety

Follow this important safety precaution in addition to your regular classroom procedures:

- Make necessary arrangements to your workstation to avoid getting water on any electronic equipment.

Procedure

Part 1 – Brass Cylinder

SET UP

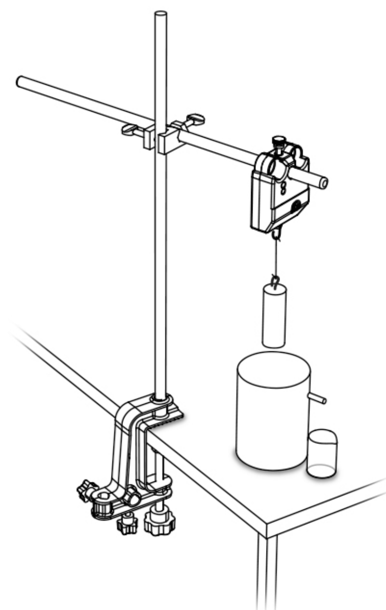
1. Fill the 1-L beaker with approximately 500 mL of water.

2. Slowly add approximately 3 mL of liquid dish soap to the water, and then use the stir rod to slowly mix the soap into the water being very careful not to make the soapy water foamy. Set the soapy water aside for a moment.

3. Use the meter stick to measure the length l and radius r of the brass and aluminum cylinders. Record the values for the brass cylinder in cm in the spaces above Table 1 in the Data Analysis section below. Record the values for the aluminum cylinder in cm above Table 2.

4. Measure and make small marks on the sides of both cylinders at $\frac{1}{4}l$, $\frac{1}{2}l$, and $\frac{3}{4}l$ locations.

5. Assemble your equipment similar to the diagram at right:
   - Use the thread to hang the brass cylinder from the force sensor hook so that it hangs vertically with its top surface approximately 5 cm to 10 cm from the sensor.
   - Place the 100-mL beaker under the spout of the overflow can so it will catch water as it pours out.



6. Connect the high resolution force sensor to the data collection system, and then create a digits display showing Force (Inverted) in newtons.

7. Remove the brass cylinder from the force sensor hook, and then press the Zero button on the force sensor. Rehang the cylinder after the sensor is zeroed.

8. Using the soapy water you just made, slowly fill the overflow can (being very careful not to make the soapy water foamy) until water starts to pour from its spout into the 100-mL beaker. The water will continue to drip into the beaker until it reaches the exact level of the spout inside the can.

9. Once the overflow can has finished dripping, empty the 100-mL beaker into the 1-L beaker, dry the inside of the 100-mL beaker, and place it on the balance.

10. Tare/zero the balance so that it reads zero with the dry 100-mL beaker on it, and then replace the beaker under the spout of the overflow can.

COLLECT DATA

11. Start recording data and observe the force measured by the force sensor. This measured force value is equal to the tension T in the string. Record the first tension value T_1 of the brass cylinder suspended above the water in the overflow can (corresponding to a Depth of 0 m) into Table 1.

12. Gently loosen the thumbscrew on the right angle clamp and slowly lower the cylinder into the water in the overflow can until the cylinder is submerged up to the first mark, $l/4$. Tighten the thumbscrew to hold the cylinder in place as water drips from the overflow can into the 100-mL beaker.

13. Once the overflow can has stopped dripping: in Table 1, record the new tension value T_2 from the force sensor, and then place the 100-mL beaker onto the balance and record the mass of the water that was displaced into the beaker.

14. Repeat the previous steps three additional times, lowering the cylinder to depths of $l/2$, $3l/4$, and l (cylinder completely submerged). Record the tension T_n measured by the force sensor and the total mass of the displaced water each time.

    NOTE: Each time the cylinder is lowered, be sure the overflow can has stopped dripping before recording any measurements.

15. Stop recording data.

Part 2 – Aluminum Cylinder

SET UP

16. Raise the brass cylinder out of the overflow can and remove it. Use the thread to hang the aluminum cylinder from the force sensor hook so that it hangs vertically with its top surface approximately 5 cm to 10 cm from the sensor.

17. Pour the soapy water from the 100-mL beaker into the 1-L beaker, place the 100-mL beaker back under the pour spout, and then slowly refill the overflow can from the 1-L beaker until water starts to pour from its spout into the 100-mL beaker.

18. Once the overflow can has finished dripping, empty the 100-mL beaker into the 1-L beaker, dry the inside of the 100- mL beaker, place it on the balance, and then tare/zero the balance so that it reads zero with the dry 100-mL beaker on it. Replace the dry beaker under the spout of the overflow can.

COLLECT DATA

19. Follow the same Part 1 data collection steps using the aluminum cylinder. Record tension T_n measured by the force sensor and the total mass of the displaced water at the same five depths: 0, $l/4$, $l/2$, $3l/4$, and l (cylinder completely submerged). Record all values using the aluminum cylinder into Table 2.

Data Analysis

Part 1 – Brass Cylinder

Brass cylinder length l (cm): ____________________________

Brass cylinder radius r (cm): ____________________________

Brass cylinder area A_{cyl} (cm²): ____________________________

Table 1: Buoyant force and displacement values for a brass cylinder submerged in a fluid

Depth (cm)		V_{subm} (cm³)	Tension (N)		m_{disp} (g)	F_b (N)	w_{disp} (N)
0	0.00	0.00	T_1		0.00	0.00	0.00
$\frac{1}{4}l$			T_2				
$\frac{1}{2}l$			T_3				
$\frac{3}{4}l$			T_4				
l			T_5				

Part 2 – Aluminum Cylinder

Aluminum cylinder length l (cm): ____________________________

Aluminum cylinder radius r (cm): ____________________________

Aluminum cylinder area A_{cyl} (cm²): ____________________________

Table 2: Buoyant force and displacement values for an aluminum cylinder submerged in a fluid

Depth (cm)		V_{subm} (cm³)	Tension (N)		m_{disp} (g)	F_b (N)	w_{disp} (N)
0	0.00	0.00	T_1		0.00	0.00	0.00
$\frac{1}{4}l$			T_2				
$\frac{1}{2}l$			T_3				
$\frac{3}{4}l$			T_4				
l			T_5				

1. Calculate the following using the measured values for each cylinder. Record the results into or above each cylinder's respective table.

   a. The depth in centimeters in Tables 1 and 2 using cylinder length *l*.

   b. The cross-sectional area *A*, in cm^2, of each cylinder using radius *r*.

   $$A_{cyl} = \pi r^2$$

   c. The volume of the cylinder submerged V_{subm}, in cm^3, at each depth.

   $$V_{subm} = \text{depth} \times A_{cyl}$$

2. If the tension T_n measured when each cylinder was submerged is equal to the difference between the gravitational force and the buoyant force:

   $$T_n = F_g - F_b$$

   and the tension T_1 measured when each cylinder was suspended above the water is equal to the gravitational force:

   $$T_1 = F_g$$

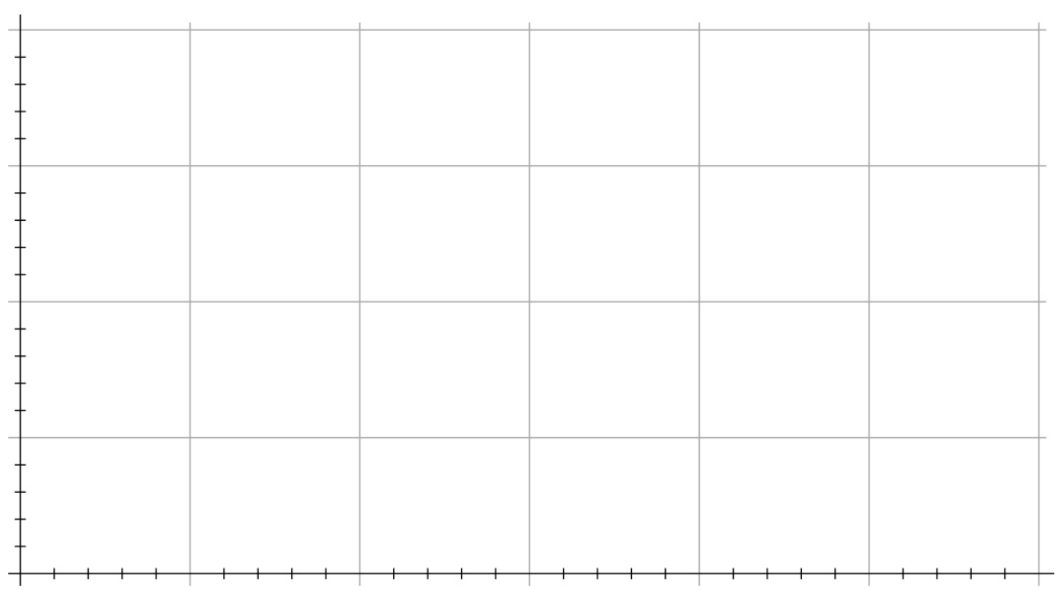
   then the buoyant force on each cylinder is equal to:

   $$F_b = T_1 - T_n \qquad (1)$$

   For both cylinders, use Equation 1 to calculate the buoyant force F_b at each depth. Record the results into each cylinder's respective table.

3. In the blank Graph 1 axes, plot a graph of *buoyant force* versus *submerged volume* with two curves: one for the brass cylinder and one for the aluminum cylinder. Be sure to label both curves and both axes with the correct scale and units.

Graph 1: Buoyant force on a cylinder versus volume of cylinder submerged



4. Are the curves for the brass and aluminum cylinders in Graph 1 similar?

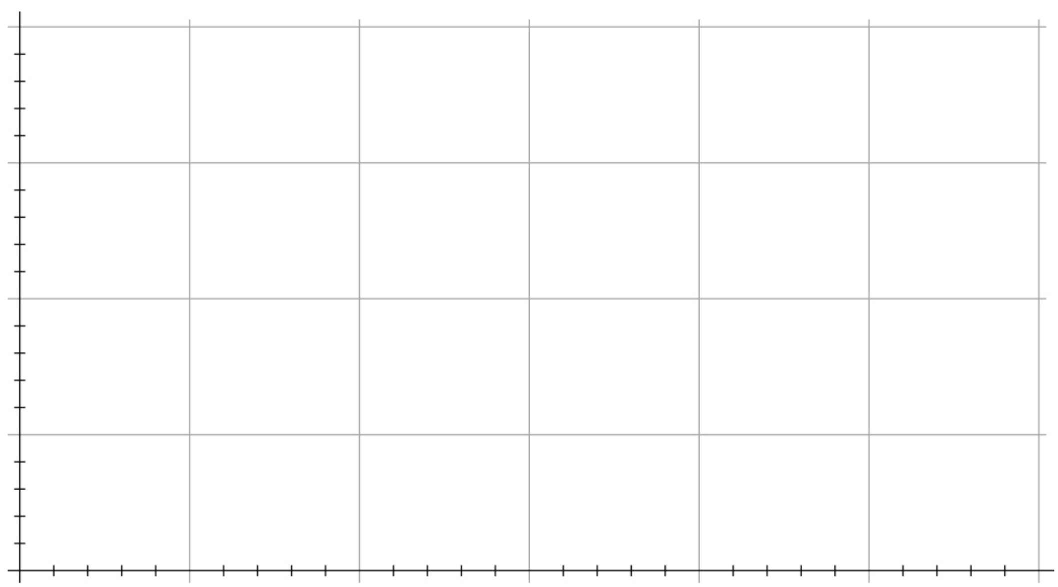
5. Based on your data, is it reasonable to assume that the relationship between buoyant force and submerged volume would be similar if you had used a third object with greater mass (greater density)? Explain your reasoning.

6. Calculate the weight w_{disp} (in newtons) of the displaced fluid at each depth for both cylinders. Record your results into their respective columns.

$$w_{disp} = m_{disp} g \times \frac{1\text{ kg}}{1{,}000\text{ g}}$$

7. In the blank Graph 2 axes, plot a graph of *buoyant force* versus *weight of displaced fluid* with two curves: one for the brass cylinder and one for the aluminum cylinder. Be sure to label both curves and both axes with the correct scale and units.

Graph 2: Buoyant force on a cylinder versus weight (in newtons) of fluid displaced by the cylinder



8. Are the curves for the brass and aluminum cylinders in Graph 2 similar?

9. Based on your data, is it reasonable to assume that the relationship between buoyant force and the weight of the displaced fluid would be similar for a third object with greater mass (greater density)? Explain your reasoning.

Analysis Questions

1. What type of mathematical relationship (proportional, squared, inverse, inverse squared, et cetera) between buoyant force and submerged volume is implied by your data?

____

2. Based on your data, express the relationship between buoyant force F_b and submerged volume V_{subm} by completing this proportionality statement:

   $F_b \propto$ ____________________

3. Convert the proportionality statement from the previous question into an equation by introducing a proportionality constant k:

   $F_b =$ ____________________

4. The buoyant force F_b acting on an object that is partially or completely submerged in a fluid is described by the equation:

   $$F_b = \rho V g \quad (2)$$

   where V is the submerged volume of the object and ρ is the density of the fluid in which the object is submerged. Which terms from this equation would be represented in your equation's proportionality constant k?

   $k =$ ____________________

5. Use your data to determine an experimental value for the proportionality constant k. How does this value compare to the theoretical value of the constant in Equation 2? If the experimental value is different from the theoretical value, what caused the difference?

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____

6. Archimedes's principle states that an object completely or partially submerged in a fluid experiences an upward buoyant force equal in magnitude to the weight of the fluid displaced by the object. Does your data support this statement? If yes, explain how it supports it; if no, identify which data do not support it, and what may have caused this disagreement.

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Synthesis Questions

1. A wood salvage company is hoisting an old tree trunk off the bottom and out of a lake. The cable from the hoist is tied around the log above its center of mass. The hoist applies a force of 9,800 N to the cable to suspend the log in the lake water (T_{water}), and a force of 29,000 N to suspend the log above the lake surface (T_{air}). What are the volume and density of the log? Assume the lake water has a density of 1,007 kg/m^3.

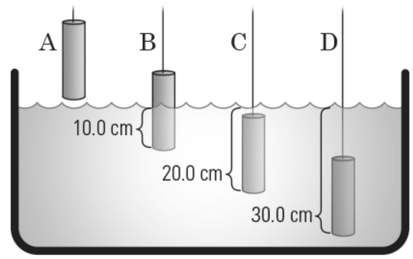
2. A cylinder with radius 5.00 cm and length 20.0 cm is lowered into a tank of glucose, which has a density of 1,385 kg/m^3. The cylinder is lowered in four stages:

   A) Zero submersion

   B) Half-submerged to a depth of 10.0 cm

   C) Fully submerged to a depth of 20.0 cm

   D) Fully submerged to a depth of 30.0 cm



   a. What is the buoyant force on the cylinder at each stage?

   b. After being lowered to a depth of 30.0 cm, the string holding the cylinder is cut. If the net force on the cylinder after the string is cut is 1.00 N downward, what is the density of the cylinder material?

3. A crab fisherman has built a crab trap out of plastic pipe and wire mesh. The overall mass and volume of the trap are 5.59 kg and 6,213 cm^3, respectively. To catch crab, the trap must sink to the ocean floor. The fisherman has several lead weights to add to the trap to ensure it sinks. If sea water has a density of 1,021 kg/m^3, and each lead weight has mass of 113.4 g and volume of 10.0 cm^3, what is the minimum number of weights the fisherman must add so that the trap sinks to the ocean floor?

3. FLUID DYNAMICS

Connections to the AP® Physics 2 Curriculum*

The lab activity correlates to the following pieces of the AP® Physics 2 framework:

Big Idea	5	Enduring Understanding	B	Essential Knowledge	10

Learning Objective 1: The student is able to use Bernoulli's equation to make calculations related to a moving fluid.
Science Practices: 2.2

Learning Objective 3: The student is able to use Bernoulli's equation and the continuity equation to make calculations related to a moving fluid.
Science Practices: 2.2

Learning Objective 4: The student is able to construct an explanation of Bernoulli's equation in terms of conservation of energy.
Science Practices: 6.2

Time Requirement

Preparation Time: 30 minutes

Lab Activity: 50 minutes

Prerequisites

Students should be familiar with the following concepts:

- Kinetic energy, gravitational potential energy, and conservation of energy.
- Deriving equations of motion for projectiles.

Driving Question | Objective

How is the height of a fluid column related to the exit velocity of the fluid at the bottom of the column? Experimentally determine the mathematical relationship between the height of a fluid column and the exit velocity of that fluid.

Procedural Overview

In the Structured version of this lab, students fill a tall reservoir with water which then flows out of a nozzle at the bottom of the reservoir. As the water flows through the nozzle, one student monitors the height of the water column as another student measures the range of the water flowing from the nozzle. The students take five data points of five separate height and range values, communicating with each other so they take both measurements simultaneously.

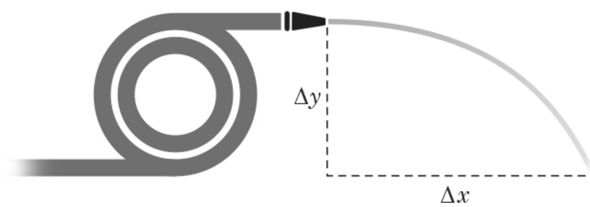
After collecting data, the students use the range value to calculate the velocity of the water exiting the nozzle. They plot velocity versus height to discover a mathematical relationship between the two quantities. In order to produce a linear relationship, the students then graph velocity squared versus height.

Pre-Lab Discussion and Activity

Since this lab makes use of the kinematics of projectile motion, it would be beneficial to review some of these concepts with the students. The question below would be a good exercise for the students to complete before starting the lab.

PRE-LAB QUESTION

1. Water is sprayed from a hose horizontally from a height Δy above the ground and travels a range of Δx before it hits the ground. Derive an equation in terms of Δy and Δx to determine the velocity of the water leaving the hose.



$$v_x = \frac{\Delta x}{\Delta t};\ \Delta y = v_{y0}t + \tfrac{1}{2}a_y t^2$$

Since $v_{y0} = 0$ and a_y is the acceleration due to gravity g, Δy can be simplified to

$$\Delta y = \tfrac{1}{2}gt^2$$

Solving for t:

$$t = \sqrt{\frac{2\Delta y}{g}}$$

Substituting this equation into the equation above to solve for v_x, noting that $\Delta t = t_f - t_i$, and $t_i = 0$, gives

$$v_x = \frac{\Delta x}{\sqrt{\frac{2\Delta y}{g}}}$$

$$v_x = \Delta x\sqrt{\frac{g}{2\Delta y}}$$

Materials and Equipment

- Support stand, 10 cm high
- Meter stick
- Water reservoir with a nozzle or hole at the bottom
- Water catch basin
- Pen, felt marker
- Distilled water to fill the water reservoir

Safety

Follow these important safety precautions in addition to your regular classroom procedures:

- Make necessary arrangements to your workstation to avoid getting water on any electronic equipment.

Lab Preparation

These are the materials and equipment to set up prior to the lab:

1. This lab is relatively easy to set up once you have a container with a bottom side outlet. We recommend using the PASCO ME-8594 Water Reservoir plus a narrowing water nozzle (PASCO Part Number 648-08434); however, any transparent water container taller than 30 cm (such as a 2-liter soda bottle) will work since pressure is not dependent on volume

2. If you choose to assemble your own water reservoirs, drill or poke a small (2 to 5 mm) hole into the side of the container approximately 2 cm from the bottom. For a consistent flow, the hole should have smooth edges. We recommend inserting a nozzle of some sort into the hole to produce a more uniform laminar flow. A compression fit drip-line connector (with or without 1/4" tubing) used in sprinkler systems will work.

3. The side outlet may have a valve or clamped tube attached to it or students may cover the outlet hole with a finger while filling the reservoir. This nozzle can have a valve or clamp of some sort as long as that valve does not impede the flow of water from the nozzle, except to stop the flow when needed.

4. The water reservoir must be placed on a short stand to elevate it above the catch basin. For this you can use any object that supports the reservoir properly and elevates it more than 10 cm above the catch basin. A small box or textbook will work as long as you are not concerned about either getting wet.

5. A cookie sheet lined with a sponge or cloth, or a sink can work as a catch basin for the water as it flows out of the reservoir.

6. An alternative to the cookie sheet or sink would be to move the experiment outside where students can make measurements on the grass or soil, allowing the water to pour freely onto the ground.

7. Good results depend on the accurate measurement of the fluid height and a smooth exit opening in the water reservoir. If using a tube, make sure the tube is straight with no kinks or constrictions.

Teacher Tips

Tip 1 – Using Video Analysis

- An alternative to using meter sticks to measure the water column height and projectile water range is video analysis. Since the Structured version of the lab requires at least two students to perform the lab, video analysis is a great option for students who were absent on the day of the lab. A single student can perform the lab using video analysis or you can also take the video ahead of time and give the video to the student to perform the analysis at home. The video analysis feature built into PASCO Capstone allows you to track two objects at one time, for this lab, the water level in the reservoir and the range of the water exiting the nozzle.

Tip 2 – Difficulties students may encounter

- Students measure height and range while the water level inside the reservoir is decreasing (approaching the nozzle near the bottom of the reservoir) which may be confusing considering that the first range measurement will correspond to the greatest depth and the final range measurement will correspond to the least depth. Be sure to stress the inverted scale with students using the Structured version of this lab.

Tip 3 – Controlling the Reservoir

- If the side outlet of the water reservoir doesn't have a valve or clamped tube attached to it, direct students to cover the outlet hole with a finger while filling the water reservoir.

Sample Data

Below are sample data, acquired using the experimental setup and procedure outlined in the Structured version of the lab activity, and answers to questions in the Data Analysis section.

Data Analysis

Nozzle height (m): 0.021 m

Table 1: Determining the relationship between the height of a column of water and the exit velocity of the water

Water Height (m)	Water Exit Range (m)	Velocity (m/s)	Velocity^2 (m^2/s^2)
0.200	0.128	2.0	4.0
0.160	0.112	1.7	2.9
0.120	0.097	1.5	2.3
0.080	0.074	1.1	1.2
0.040	0.046	0.70	0.49

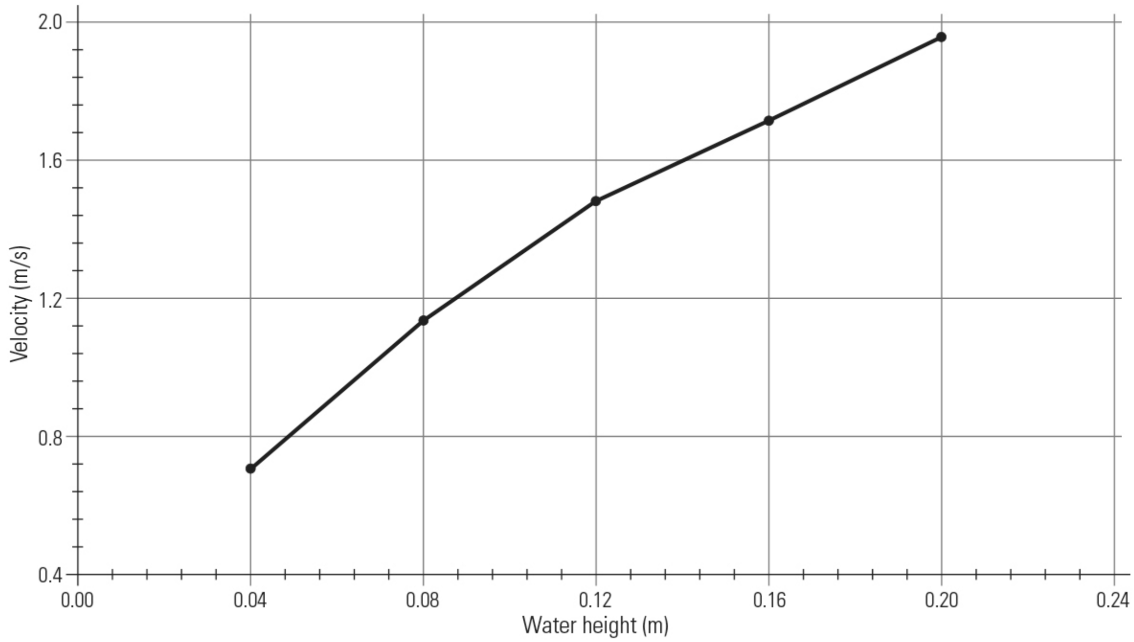
1. Using the water exit range measurements from Table 1 and the nozzle height, calculate the exit velocity for each range measurement using kinematic equations for projectile motion. Record the results into Table 1. Show your work for one of the calculations.

   Calculation using sample data for depth = 0.200 m:

   $$v_x = \Delta x\sqrt{\frac{g}{2\Delta y}} = 0.128\text{ m}\sqrt{\frac{9.8\text{ m/s}}{2(0.021\text{ m})}} = 2.0\text{ m/s}$$

2. Plot a graph of *velocity* versus *water height* in the blank Graph 1 axes. Be sure to label both axes with the correct scale and units.

Graph 1: Exit velocity versus height of a water column



3. How does the exit velocity change as the height of the water column decreases?

The exit velocity decreases as the water column height decreases.

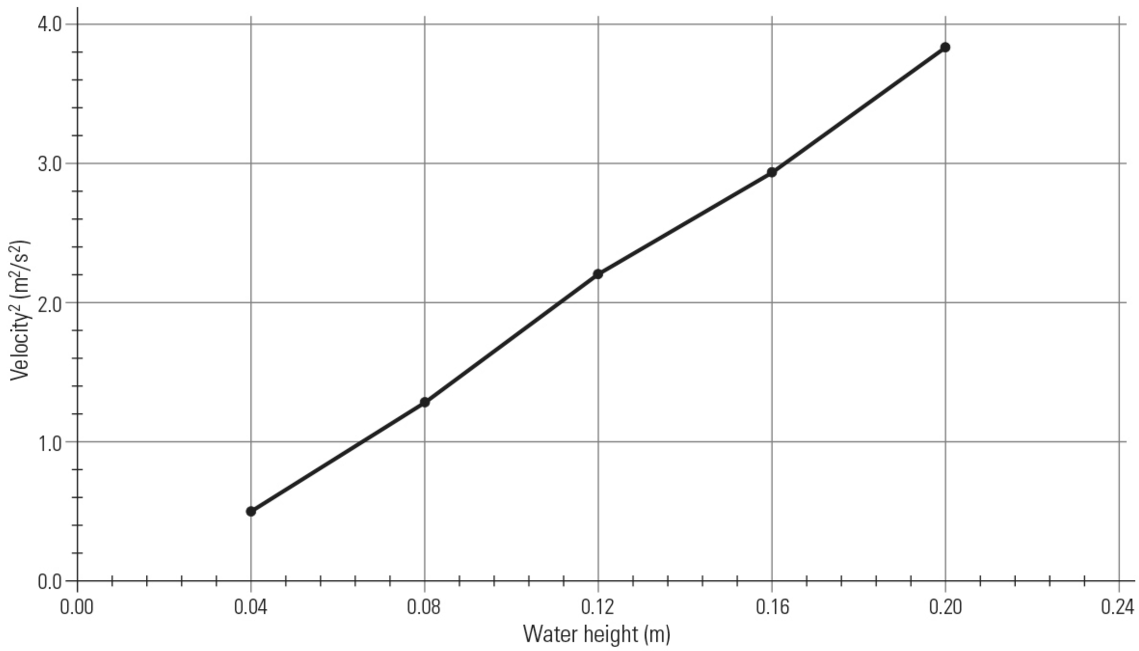
4. To produce a linear graph relating the height of the water column and the exit velocity, calculate velocity squared for all of your velocity values and record the results in Table 1.

Calculation using sample data for depth = 0.200 m:

$v_x^2 = (2.0\text{ m/s})^2 = 4.0\text{ m}^2/\text{s}^2$

5. Plot a graph of *velocity squared* versus *water height* in the blank Graph 2 axes. Be sure to label both axes with the correct scale and units.

Graph 2: Exit velocity squared versus water height of a water column



Guided Inquiry Questions

Below are sample responses to the Guiding Questions found in the Guided Inquiry version of this lab activity.

❷ 1. How should water height be measured in this experiment? What location would you consider to be zero height?

Water height should be measured from the surface of the water to the nozzle or valve where the water is being released. The location of the nozzle would be considered zero height.

❷ 2. If the stream of water that flows from the reservoir begins to "break up" the farther away from the reservoir it reaches, does it make more sense to measure at shorter ranges or longer, and how can you make the range shorter or longer without changing the water height in the reservoir?

It makes more sense to measure range at shorter distances so that the distance can be measured more accurately. The range can be changed by raising or lowering the reservoir from the point of measurement. Keeping the reservoir close to ground level will help to make measurements more accurate.

❷ 3. There are many sources of error that can be introduced into this experiment. Describe possible sources of error and what you could do to minimize that error.

Synchronizing the water column height and the projectile range measurements can be tricky depending on how fast the water is exiting the nozzle. Error can be minimized by using a slower flow of water. (Using a smaller nozzle opening will help slow the rate at which the water column is lowering.) One student needs to call out when a measurement is to be made in order to synchronize the measurements. Error could be minimized by using video analysis, which would help with synchronizing the measurements and could also provide more accurate measurements.

As mentioned earlier, the water flowing from the nozzle tends to "break up" the farther away from the reservoir it sprays. The solution to this is to measure range at shorter distances. However, this causes the stream to reach the meter stick on the basin at an angle which makes the range difficult to measure. Additionally, even if the stream does not disperse, the width of the stream makes it difficult to make an accurate measurement. It is recommended for students to record the mid-way point of the stream as the range measurement, being sure to measure the nozzle height from this point.

4. What method could you use to analyze your data in order to easily determine the mathematical relationship of the water exit velocity versus the water column height?

   Students should indicate that they need to graph their data to determine a linear relationship between variables since it is easiest to recognize a linear correlation using a graph.

Assessment Questions: Sample Responses

Sample responses to the Analysis and Synthesis questions found in each version of the lab activity:

Analysis Questions

1. How is the exit velocity mathematically related to the height of the water column? Use terms such as proportional, inversely proportional, linear, or quadratic in your response.

   The mathematical relationship is quadratic; the square of the velocity is proportional to the height of the water column.

2. The expression relating potential and kinetic energy in fluids is known as Bernoulli's equation

   $$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

   where P is the pressure in a fluid at height y above the bottom of the fluid container, ρ is the density of the fluid, and v is the velocity of the fluid. The term that includes the height is the potential energy per volume and the term that includes the velocity is the kinetic energy per unit volume. For this experiment, point 1 is at the top of the water column and point 2 is at the end of the nozzle.

   a. Assuming that points 1 and 2 are at the same pressure (atmospheric pressure) and approximating the speed of the water column to be 0 m/s, simplify Bernoulli's equation.

      Due to both points being at the same pressure and approximating the initial speed of the water column as 0 m/s, the equation simplifies to

      $$\rho g y_1 = \rho g y_2 + \frac{1}{2}\rho v_2^2$$

      The equation can be further simplified by setting the height of the nozzle to 0 m and canceling out the density, since the fluid is the same throughout.

      $$g y_1 = \frac{1}{2} v_2^2$$

   b. How does your data support the simplified equation?

      The equation shows that water height is proportional to the square of the velocity, which is what student data and graphs should show.

c. Using the assumptions mentioned above, calculate the potential energy per volume and the kinetic energy per volume for each data point. Also assume that the density of the water is 1000 kg/m³. Enter your results in Table 2.

Calculation using sample data for depth = 0.200 m:

Potential energy per unit volume $= \rho g y_1$

$$\rho g y_1 = \left(1{,}000\ \text{kg/m}^3\right)\left(9.8\ \text{m/s}^2\right)\left(0.200\ \text{m}\right) = 2{,}000\ \text{J/m}^3$$

Kinetic energy per unit volume $= \frac{1}{2}\rho v_2^2$

$$\frac{1}{2}\rho v_2^2 = \frac{1}{2}\left(1{,}000\ \text{kg/m}^3\right)\left(2.0\ \text{m/s}\right)^2 = 2{,}000\ \text{J/m}^3$$

Table 2: Determining the potential energy and kinetic energy of water exiting a nozzle

Water Height (m)	Velocity (m/s)	Potential Energy per Volume (J/m³)	Kinetic Energy per Volume (J/m³)
0.200	2.0	2,000	2,000
0.160	1.7	1,600	1,400
0.120	1.5	1,200	1,100
0.080	1.1	780	600
0.040	0.70	390	250

d. According to the simplified Bernoulli equation, the potential energy per volume at point 1 should be equivalent to the kinetic energy per volume at point 2. Do you find that to be the case? If not, what do you account for the discrepancy in your data?

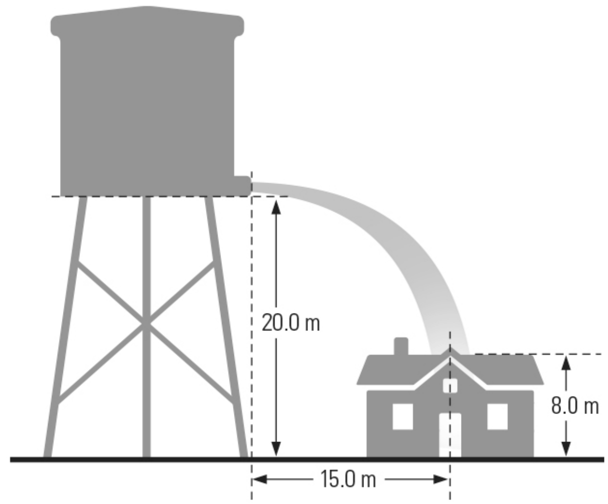
The energy values are not equivalent. The kinetic energy per volume tends to be lower than the potential energy per volume since there are energy losses at the nozzle. The small exit hole introduces turbulence which can account for the energy loss. This energy loss can be minimized by using a larger exit hole.

❷ 3. What variable in your experiment could you change in order to obtain a different velocity² versus height graph? Describe a possible way to change this variable (even if you do not have the resources to change this variable in your lab).

The only other variable in the simplified equation is acceleration due to gravity, which obviously cannot be changed in the classroom. To perform this experiment in a location with a different acceleration due to gravity value, students may mention performing this experiment on a different celestial body such as the moon.

Synthesis Questions

1. A city holding tank for water sits 20.0 m above the city. If a house near the holding tank was on fire, to what height would firefighters have to drain the tank so water sprayed from the bottom of the tank hits the top of the house at its center, 8.0 m above the ground and 15 m away? Show your work.



Applying the equation for the velocity of water leaving the nozzle derived earlier and the simplified Bernoulli equation:

$$v_x = \Delta x\sqrt{\frac{g}{2\Delta y}}\;;\quad gy_1 = \tfrac{1}{2}v_2^2 \rightarrow y_1 = \frac{v_2^2}{2g}$$

$$v_2 = \Delta x\sqrt{\frac{g}{2\Delta y}} = (15\text{ m})\sqrt{\frac{9.8\text{ m/s}^2}{2(20.0\text{ m} - 8.0)}} = 9.6\text{ m/s}$$

$$y_1 = \frac{v_2^2}{2g} = \frac{(9.6\text{ m/s})^2}{2(9.8\text{ m/s}^2)} = 4.7\text{ m}$$

2. Assume the tank from the previous question is sealed airtight with an air gap between the water's surface and the top of the tank. The air pressure in the air gap is 30.4 kPa, while atmospheric pressure outside the tank is 101.4 kPa. Assuming the velocity of the water level in the tank is effectively zero, to what height will firefighters now need to drain the tank so water hits the burning house?

$$P_1 + \rho g y_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \tfrac{1}{2}\rho v_2^2$$

Assuming that the velocity of the water level in the tank is effectively zero and setting the valve at zero height,

$$P_1 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2$$

$$y_1 = \frac{P_2 - P_1 + \tfrac{1}{2}\rho v_2^2}{\rho g} = \frac{101.4\times10^3\text{ Pa} - 30.4\times10^3\text{ Pa} + \tfrac{1}{2}(1000\text{ kg/m}^3)(7.42\text{ m/s})^2}{(1000\text{ kg/m}^3)(9.8\text{ m/s}^2)} = 10.1\text{ m}$$

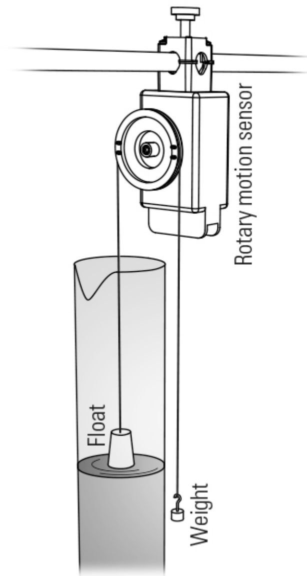
Extended Inquiry Suggestions

- A quick and effective demonstration to follow up this lab is to demonstrate how the exit velocity of the stream from the fluid column is only dependent on the height of fluid above the nozzle, and not the volume. Use two water reservoirs of different cross sectional area with identical spigots at the bottom. Fill both reservoirs to the same level (height) and open the spigots to let water flow out. Indicate that the range of each stream is identical, thus the static pressures are the same at that depth, even though the volume of water inside each reservoir is different.
- A rotary motion sensor can be used to measure the height of the water column and also the velocity of the water as it falls. Having this data, students can use the continuity equation to determine the theoretical exit velocity

  $$A_1v_1 = A_2v_2$$

  and compare it to the measured velocity using projectile motion.

  The cross sectional area of the water reservoir and the nozzle will need to be measured in order to determine the theoretical exit velocity. To make measurements using a rotary motion sensor, clamp the sensor above the reservoir and tie a float to one end of a string and a small weight to the other end. Place the float in the reservoir, run the string over the large step of the pulley, and let the weight hang freely. Students should find that the theoretical exit velocity is greater than the experimental exit velocity, supporting the claim that energy is lost at the nozzle.



3. FLUID DYNAMICS

STRUCTURED

Driving Question | Objective

How is the height of a fluid column related to the exit velocity of the fluid at the bottom of the column? Experimentally determine the mathematical relationship between the height of a fluid column and the exit velocity of that fluid.

Materials and Equipment

- Support stand, 10 cm high
- Meter stick
- Water reservoir with a nozzle or hole at the bottom
- Water catch basin
- Pen, felt marker
- Distilled water to fill the water reservoir

Background

As an object falls, it will continue to gain speed (as long as we neglect air resistance). The mathematical relationship between height and speed can be easily determined by using the concepts of gravitational potential energy and kinetic energy. According to the theory of conservation of energy, the total energy of the object will remain the same as the object falls since gravitational potential energy transfers to kinetic energy. We know that this theory works well for moving objects, but what about flowing fluids?

In this activity, you will use a tall reservoir, filled with water, with a nozzle at the bottom. As water exits the nozzle horizontally, the height of the water column decreases. Your goal is to determine the mathematical relationship between the velocity of the water flowing out of the nozzle and the height of the water column.

The height of the water column can be easily measured using a meter stick. Since the velocity of the water exiting the nozzle cannot be measured directly, the velocity will be determined by measuring the range of the projectile water and using kinematic equations.

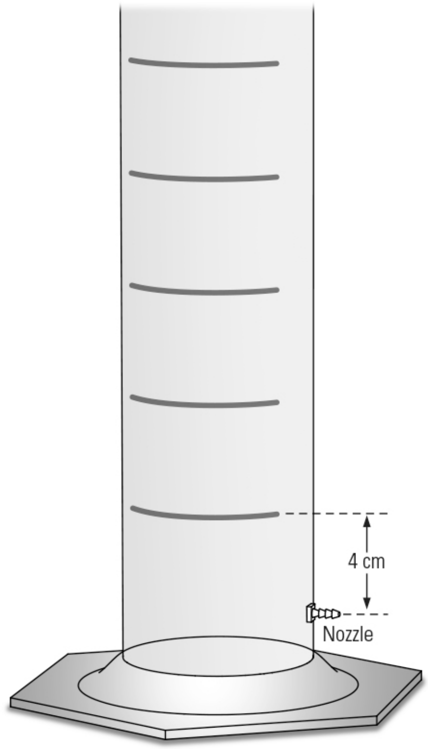
Safety

Follow these important safety precautions in addition to your regular classroom procedures:

- Make necessary arrangements to your workstation to avoid getting water on any electronic equipment.

Procedure

SET UP



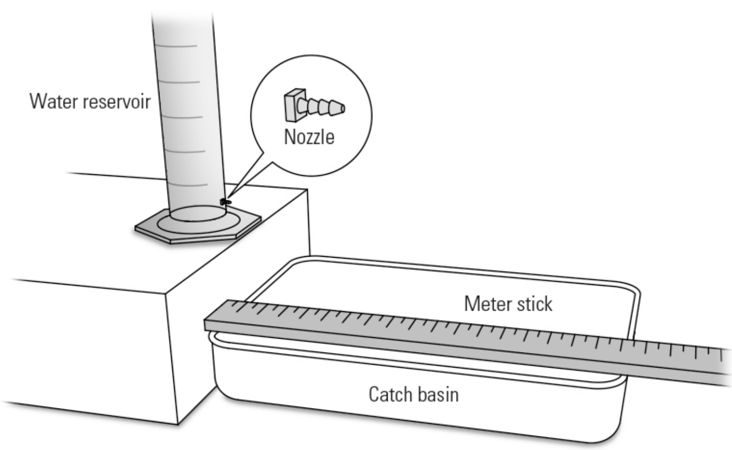
1. Using your meter stick and a felt-tipped pen, measure and mark five 4-cm graduations on the side of the water reservoir starting from zero at the nozzle or hole at the bottom of the reservoir.

2. Lay the catch basin flat on your lab table and set the water reservoir on a stand in front of and approximately 10 cm above the catch basin. Point the nozzle or hole towards the catch basin.

3. Use the meter stick to record the height Δy of the nozzle above the top of the catch basin. Record this value, "Nozzle height," above Table 1 in the Data Analysis section below.

4. Set your meter stick flat on or across the top of the water catch basin so that the length of the meter stick is aligned with the expected path of the stream and next to (but not under) the point where the water enters the catch basin. Align the zero on the meter stick with the end of the nozzle or hole.

5. Make certain the nozzle or hole on the bottom of the water reservoir is plugged or otherwise securely closed. Fill the water reservoir with distilled water 2 cm to 3 cm above the highest graduation mark.



NOTE: Unless you are absolutely certain about the path of the water stream, you may want to open the nozzle or hole and let a small amount of water out to properly align the meter stick with the stream; however, if you do so and the water level in the reservoir falls close to or below the highest graduation mark, add more distilled water to bring the water level 2 cm to 3 cm above the graduation.

COLLECT DATA

6. When the nozzle or hole is opened and water begins to stream out (DO NOT RELEASE THE WATER STREAM YET), perform the following steps to measure the range of the water flow:

   a. One student (student A) will watch the water level in the reservoir as it decreases while another student (student B) records the range of the water stream.

   b. When the water level reaches the first graduation on the side of the container, student A will tell student B to mark or record the range read from the meter stick.

   c. As the water level in the reservoir decreases, repeat the previous step for each graduation on the side of the reservoir.

7. Now open the nozzle or hole and allow the water to pour freely into the catch basin. Record data as described in the previous steps.

8. Once the water level has passed the final graduation mark, close the nozzle or hole and record your range measurements for each height in Table 1.

   NOTE: The highest graduation represents a height of 20 cm while the lowest represents a height of 4 cm.

Data Analysis

Nozzle height (m): ______________________________

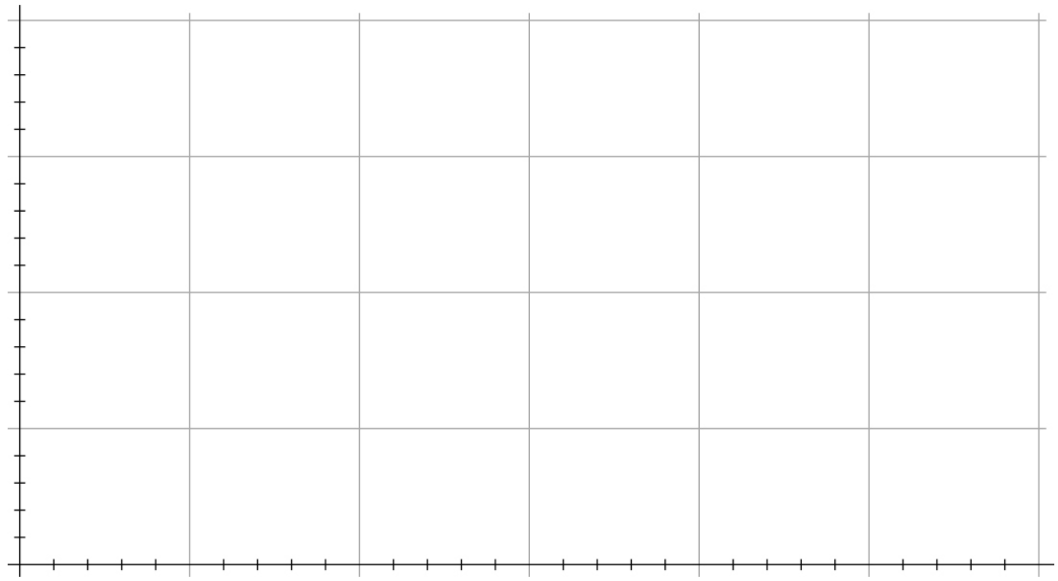
Table 1: Determining the relationship between the height of a column of water and the exit velocity of the water

Water Height (m)	Water Exit Range (m)	Velocity (m/s)	Velocity^2 (m^2/s^2)
0.200			
0.160			
0.120			
0.080			
0.040			

❓ 1. Using the water exit range measurements from Table 1 and the nozzle height, calculate the exit velocity for each range measurement using kinematic equations for projectile motion. Record the results into Table 1. Show your work for one of the calculations.

2. Plot a graph of *velocity* versus *water height* in the blank Graph 1 axes. Be sure to label both axes with the correct scale and units.

Graph 1: Exit velocity versus height of a water column

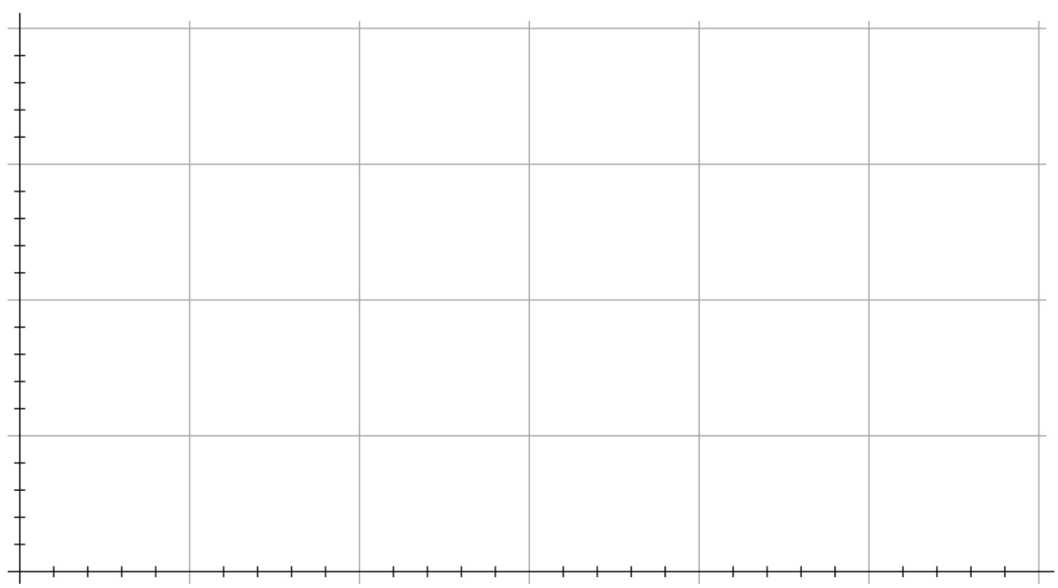


3. How does the exit velocity change as the height of the water column decreases?

4. To produce a linear graph relating the height of the water column and the exit velocity, calculate velocity squared for all of your velocity values and record the results in Table 1.

5. Plot a graph of *velocity squared* versus *water height* in the blank Graph 2 axes. Be sure to label both axes with the correct scale and units.

Graph 2: Exit velocity squared versus water height of a water column



Analysis Questions

1. How is the exit velocity mathematically related to the height of the water column? Use terms such as proportional, inversely proportional, linear, or quadratic in your response.

2. The expression relating potential and kinetic energy in fluids is known as Bernoulli's equation

$$P_1 + \rho g y_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \tfrac{1}{2}\rho v_2^2$$

where P is the pressure in a fluid at height y above the bottom of the fluid container, ρ is the density of the fluid, and v is the velocity of the fluid. The term that includes the height is the potential energy per volume and the term that includes the velocity is the kinetic energy per unit volume. For this experiment, point 1 is at the top of the water column and point 2 is at the end of the nozzle.

a. Assuming that points 1 and 2 are at the same pressure (atmospheric pressure) and approximating the speed of the water column to be 0 m/s, simplify Bernoulli's equation.

b. How does your data support the simplified equation?

c. Using the assumptions mentioned above, calculate the potential energy per volume and the kinetic energy per volume for each data point. Also assume that the density of the water is 1000 kg/m^3. Enter your results in Table 2.

Table 2: Determining the potential energy and kinetic energy of water exiting a nozzle

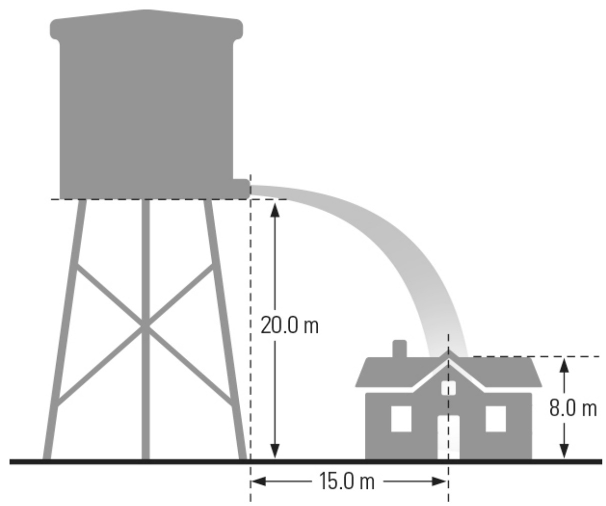
Water Height (m)	Velocity (m/s)	Potential Energy per Volume (J/m^3)	Kinetic Energy per Volume (J/m^3)
0.200			
0.160			
0.120			
0.080			
0.040			

d. According to the simplified Bernoulli equation, the potential energy per volume at point 1 should be equivalent to the kinetic energy per volume at point 2. Do you find that to be the case? If not, what do you account for the discrepancy in your data?

❓ 3. What variable in your experiment could you change in order to obtain different velocity^2 versus height graph? Describe a possible way to change this variable (even if you do not have the resources to change this variable in your lab).

Synthesis Questions

❓ 1. A city holding tank for water sits 20.0 m above the city. If a house near the holding tank was on fire, to what height would firefighters have to drain the tank so water sprayed from the bottom of the tank hits the top of the house at its center, 8.0 m above the ground and 15 m away? Show your work.



❓ 2. Assume the tank from the previous question is sealed airtight with an air gap between the water's surface and the top of the tank. The air pressure in the air gap is 30.4 kPa, while atmospheric pressure outside the tank is 101.4 kPa. Assuming the velocity of the water level in the tank is effectively zero, to what height will firefighters now need to drain the tank so water hits the burning house?

4. BOYLE'S LAW

Connections to the AP® Physics 2 Curriculum*

The lab activity correlates to the following pieces of the AP® Physics 2 framework:

Big Idea	5	Enduring Understanding	B	Essential Knowledge	7

Learning Objective 2: The student is able to create a plot of pressure versus volume for a thermodynamic process from given data.
Science Practices: 1.1

Big Idea	7	Enduring Understanding	A	Essential Knowledge	3

Learning Objective 2: The student is able to design a plan for collecting data to determine the relationships between pressure, volume, and temperature, and amount of an ideal gas, and to refine a scientific question concerning a proposed incorrect relationship between variables.
Science Practices: 3.2, 4.2

Learning Objective 3: The student is able to analyze graphical representations of macroscopic variables for an ideal gas to determine the relationships between these variables and to ultimately determine the ideal gas law $PV = nRT$.
Science Practices: 5.1

Time Requirement

Preparation Time: 10 minutes

Lab Activity: 30 minutes

Prerequisites

Students should be familiar with the following concepts:

- The kinetic molecular theory describes gases as a large number of molecules in constant motion, colliding with each other and with the walls of their container.
- Gas pressure P is a measure of the average force F exerted by these collisions on a given area A of the container wall: $P = F/A$.
- The first law of thermodynamics states that a change to the internal energy of a system is given by $\Delta U = Q + W$, where Q is the energy transferred to the system by heating and W is the work done on the system.
- The work done on a gas is defined by $W = -P\Delta V$, where P is the pressure and ΔV is the change in volume.

Driving Question | Objective

How is the pressure of a gas affected by changes in the volume of the container enclosing it? Experimentally determine the mathematical relationship between the pressure of a gas and the volume it occupies.

Procedural Overview

In the Structured version of this lab activity, students measure the pressure of air in a syringe at five different volumes. They use their data to plot a graph of gas pressure versus inverse volume. The linear relationship shown by this graph should lead them to determine that there is an inverse relationship between pressure and volume. The Analysis Questions ask students to relate this discovery to the more general Ideal Gas Law equation.

Pre-Lab Discussion and Activity

Before performing any version of this lab activity, students should understand the kinetic molecular theory's microscopic particulate explanation for the source of macroscopic gas pressure. Students performing the Structured version of this activity will find information on this in the Background section of their student handout. This background information does not exist in either the Guided Inquiry or Student Designed handouts. Students performing these versions should receive similar background information, if necessary. The Background section from the Structured student handout can serve as a guide.

Below is a question set that can be used as the basis for a pre-lab classroom discussion or assigned as pre-lab questions students can work on at home.

PRE-LAB QUESTIONS

❷ 1. The mass of air contained in a 60-mL cylinder is approximately 7.2×10^{-5} kg under normal room conditions. If the average molecular mass of air is 4.8×10^{-26} kg per molecule, approximately how many gas molecules are inside the container?

The total number of gas molecules is determined by dividing the total mass of air by the average mass per molecule:

$$\frac{7.2 \times 10^{-5}\ \text{kg}}{4.8 \times 10^{-26}\ \text{kg/molecule}} = 1.5 \times 10^{21}\ \text{molecules}$$

❷ 2. If the internal surface area of the above cylinder is 0.010 m^2 and the gas pressure is 1.0×10^5 Pa, what is the total force exerted on the cylinder's inner surface by gas molecules colliding with it?

Pressure is force per unit area, $P = F/A$, so the total force is given by

$$F = PA = \left(1.0 \times 10^5\ \text{Pa}\right)\left(0.010\ \text{m}^2\right) = \left(1.0 \times 10^5\ \text{N} \cdot \text{m}^{-2}\right)\left(0.010\ \text{m}^2\right) = 1.0 \times 10^3\ \text{N}$$

❷ 3. Air is primarily composed of nitrogen (N_2) and oxygen (O_2) molecules. In a container of air at constant temperature, which of these two molecules has the greater average speed? Explain your reasoning.

Temperature is proportional to the average translational kinetic energy of a molecule, given by

$$\overline{K} = \frac{1}{2}m\overline{v^2}$$

At a given temperature, both the nitrogen and oxygen molecules will have the same average translational kinetic energy, so

$$\frac{1}{2}m_{N_2}\overline{v_{N_2}}^2 = \frac{1}{2}m_{O_2}\overline{v_{O_2}}^2$$

Solving for the ratio of root mean square speeds,

$$\frac{v_{N_2\text{rms}}}{v_{O_2\text{rms}}} = \sqrt{\frac{m_{O_2}}{m_{N_2}}}$$

Since the molecular mass of N_2 (~28 u) is less than that of O_2 (~32 u), N_2 molecules will have a greater rms speed by a ratio of

$$\frac{v_{N_2\text{rms}}}{v_{O_2\text{rms}}} = \sqrt{\frac{32}{28}} = 1.07$$, so the speed of N_2 molecules is about 7% greater than the speed of O_2 molecules.

❷ 4. Imagine that a gas molecule traveling at its average speed for a given temperature experiences an elastic collision with and perpendicular to the wall of its container. Would the force on the wall be greater for a nitrogen or oxygen molecule? Explain your reasoning.

In an elastic collision perpendicular to the wall, the molecule's initial and final speeds will be the same before and after the collision, but its direction will be reversed. Therefore, the magnitude of its change in momentum is

$$\Delta p = m\Delta v = m\left(v_f - v_i\right) = m\left(v_f - \left(-v_f\right)\right) = 2mv_f \equiv 2mv$$

Since the change in momentum is the impulse $F\Delta t$, the ratio of the forces for the two molecules would be

$$\frac{F_{N_2}}{F_{O_2}} = \frac{2m_{N_2}v_{N_2}}{2m_{O_2}v_{O_2}} = \frac{m_{N_2}v_{N_2}}{m_{O_2}v_{O_2}}$$

Substituting the relationship determined in the previous problem for the ratio of rms speeds,

$$\frac{F_{N_2}}{F_{O_2}} = \frac{m_{N_2}v_{N_2\text{rms}}}{m_{O_2}v_{O_2\text{rms}}} = \frac{m_{N_2}}{m_{O_2}}\sqrt{\frac{m_{O_2}}{m_{N_2}}} = \sqrt{\frac{m_{N_2}}{m_{O_2}}}$$

This indicates that the less massive N_2 molecule, though traveling faster, would exert slightly less force on the wall than the more massive O_2 molecule. Specifically, the ratio of force exerted by N_2 compared to O_2 would be

$$\frac{F_{N_2}}{F_{O_2}} = \sqrt{\frac{m_{N_2}}{m_{O_2}}} = \sqrt{\frac{28}{32}} = 0.94$$, so the force exerted by N_2 is about 6% less than that of O_2.

Notice from this and the previous problem that the ratio of forces exerted by these two molecules is exactly the inverse of the ratio of their rms speeds.

❷ 5. A thirsty airplane passenger in flight drinks all of the water in a plastic bottle, retightens the bottle cap, and places the bottle in her backpack. Once landed, she retrieves the bottle for recycling at the airport and notices that it has collapsed. Explain why.

The air pressure in an airplane cabin is less than that on the ground. Having drunk the water in the bottle, the passenger tightened the cap on the bottle filled with air at the lower pressure, sealing it shut. As the plane descended, the air pressure outside the bottle became greater than that inside the bottle, resulting in an increasing net inward force on the bottle's surface. At some point, the structure of the bottle could no longer oppose that net inward force, and it collapsed. That caused the volume of the bottle to decrease and the pressure inside the bottle to increase, bringing it closer to that outside.

❷ 6. Show that the units of pressure are equivalent to those for energy per unit volume.

Pressure has units of Pa, or $N \cdot m^{-2}$. Since the unit N can be expanded to $kg \cdot m \cdot s^{-2}$, Pa is equivalent to $kg \cdot m^{-1} \cdot s^{-2}$, or $\frac{kg}{(m \cdot s^2)}$.

Multiplying the top and bottom by m^2,

$$\frac{kg}{(m \cdot s^2)} \cdot \frac{m^2}{m^2} = \frac{kg \cdot m^2}{(s^2 \cdot m^3)}$$

Since the unit of energy J is equivalent to $kg \cdot m^2 \cdot s^{-2}$ and the unit for volume is m^3, the above reduces to

$$\frac{kg \cdot m^2}{s^2} \cdot \frac{1}{m^3} = \frac{J}{m^3}$$, which is the units of energy per unit volume.

Materials and Equipment

- Data collection system
- PASCO Barometer/Low Pressure Sensor[1]
- Sensor extension cable
- Syringe, 60-mL
- Quick connector[1]
- Tubing[1]
- Scissors

Probeware Resources

Below are web-link and QR codes that will direct you to instructional video resources for individual pieces of PASCO probeware, sensors, and other hardware used in the lab activity. These same links and codes are provided to students in their activity handouts.

[1]www.pasco.com/ap24

PASCO Barometer/Low Pressure Sensor

Safety

Follow these important safety precautions in addition to your regular classroom procedures:

- Do not to exceed the measurement range of the sensor. The PASCO Barometer/Low Pressure Sensor's measurement range is 15 to 115 kPa. The sensor will not accurately report pressures outside this range and applying pressures outside this range could permanently damage the sensor.
- You will apply significant force on the syringe to change its volume. Grasp the syringe firmly with two hands and use caution to avoid slipping and potentially injuring yourself or those nearby.

Teacher Tips

Tip 1 – Pressure Sensor Considerations

- The procedure in the Structured version of this experiment involves setting an initial amount of air in the syringe and then increasing the syringe volume to ensure that the 15 kPa to 115 kPa measurement range of the PASCO Barometer/Low Pressure Sensor is not exceeded. Caution students designing their own measurement procedures to stay within their pressure sensor's measurement range.
- The PASCO Absolute Pressure Sensor (or other PASCO MultiMeasure™ sensors that include an absolute pressure measurement) can be used for this experiment. Its wider measurement range is especially appropriate for students designing their own procedure.
- In some versions of PASCO software, the PASCO Barometer/Low Pressure Sensor offers two measurements: *absolute pressure* and *barometric pressure*. These are effectively equivalent, so either can be used for this experiment.

Tip 2 – Choosing Volumes

- The nonlinear relationship between volume and pressure will be clearer to students who design their own procedures if they use a wide range of volumes. For best results, the volume should change by a factor of three or greater.

Tip 3 – Keeping Other Variables Constant

- Ensure that students who design their own experiment change the pressure of the gas by varying its volume and not by changing its temperature or the amount of gas.
- While the temperature of the gas in the syringe changes with each volume change, it quickly returns to room temperature by heat transfer through the thin plastic wall of the syringe. Students should wait a few seconds until the pressure reading stabilizes before recording it to ensure that the measurements are isothermal.

  A student group might investigate these temperature changes using a sensitive temperature sensor, ideally sealed inside the syringe. The following brief video uses the PASCO Ideal Gas Law Apparatus to demonstrate the quick return to temperature equilibrium with the surroundings after each small volume change: www.pasco.com/ap36.

Tip 4 – Fitting Tubing and Connectors

- A drop of glycerin can ease the insertion of the quick connector into the tubing.
- To properly fit the quick connector onto the pressure sensor port, turn the connector clockwise until you feel it click.

Sample Data

Below are sample data, acquired using the experimental setup and procedure outlined in the Structured version of the lab activity, and answers to questions in the Data Analysis section.

Data Analysis

Table 1: Gas pressure for various syringe volumes

Volume (mL)	Pressure (kPa)	1/Volume (mL^{-1})
10	101.6	0.10
20	50.9	0.050
30	33.5	0.033
40	24.4	0.025
50	18.9	0.020

❷ 1. In the table above, identify two volume measurements that represent a doubling of the volume. What is the relationship between the two corresponding pressure measurements? Does the same relationship hold for any other pairs of measurements that represent a doubling of the volume?

   The first and second measurements represent a doubling of the volume from 10 mL to 20 mL. For the sample data, the corresponding pressures of 101.6 kPa and 50.9 kPa show that the pressure was halved. Yes, for the sample data, the doubling of the volume from 20 mL to 40 mL also corresponds to another reduction in pressure by approximately half, from 50.9 kPa to 24.4 kPa.

❷ 2. What type of mathematical relationship (proportional, squared, inverse, inverse squared, et cetera) between pressure and volume is implied by your observations above?

   The doubling of one variable (the volume) resulted in the halving of the other (the pressure), which indicates an inverse relationship between pressure and volume.

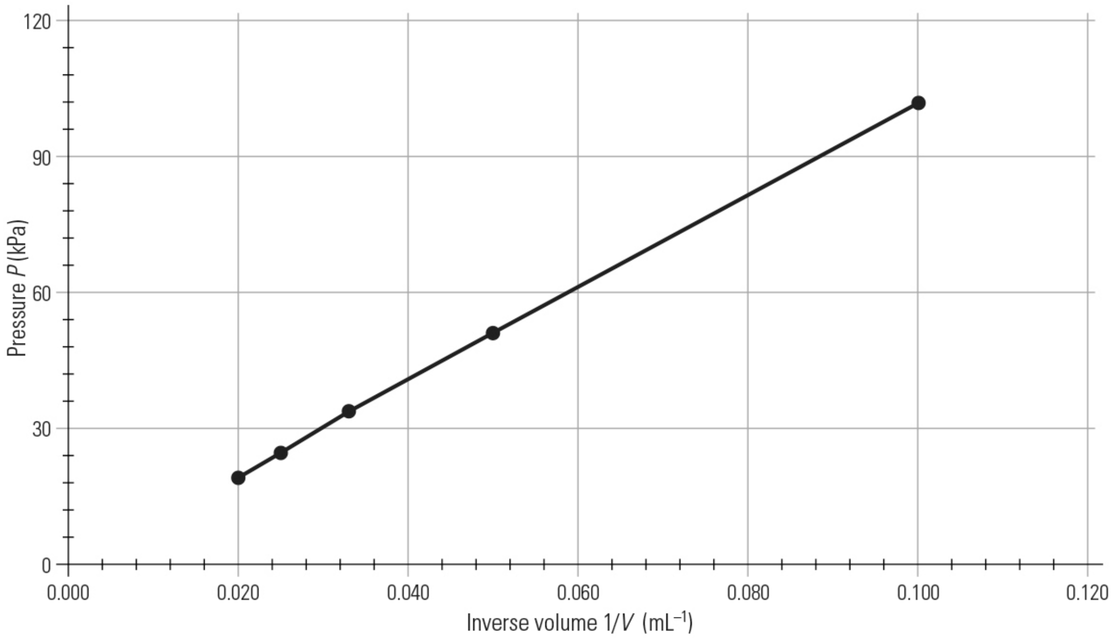
3. Calculate and record the inverse volume (1/volume) in units of inverse milliliters (mL^{-1}) for each volume measurement in Table 1.

   Calculation using the sample data for the first volume measurement:

   1/volume = 1/10 mL = 0.10 mL^{-1}

4. Plot a graph of *pressure* versus 1/*volume* in the blank Graph 1 axes below. Be sure to label both axes with the correct scale and units.

Graph 1: Gas pressure versus inverse volume



Guided Inquiry Questions

Below are sample responses to the Guiding Questions found in the Guided Inquiry version of this lab activity.

1. If the objective of this lab is to determine how the volume occupied by a gas affects its pressure, what should the independent and dependent variables be in your experiment?

   Students will explore how volume changes affect gas pressure, so volume should be their independent variable and absolute pressure should be their dependent variable.

2. What equipment will you use to measure each variable and how will you set it up?

   Students can read volume directly from the markings on the syringe, as long as the additional space in the tubing and connectors is insignificant in comparison. Students who use any other airtight, volume-adjustable container might determine gas volume by using the vessel's geometry or by temporarily filling it with known volumes of water and marking graduations on it.

   Pressure can be measured with any sensor with a quick connector port that reads absolute pressure over a wide enough range. Options include the PASCO Barometer/Low Pressure Sensor, PASCO Absolute Pressure Sensor, and other PASCO MultiMeasure sensors that include an absolute pressure measurement. See Teacher Tip 1, Pressure Sensor Considerations, for more details.

3. How will you change the independent variable while collecting data?

   Students can vary the volume of the syringe using its plunger. Ensure that students do not propose changing the pressure of their gas by varying other parameters that are not under investigation, such as the temperature or number of molecules.

   To avoid damaging the equipment, students should be aware of the pressure sensor's measurement range (refer to Teacher Tip 1).

4. What other variables might affect gas pressure? How will you keep these variables constant during your experiment?

   Temperature affects gas pressure. Although the temperature of the air inside the syringe will decrease slightly each time its volume is increased, the thin plastic wall of the syringe allows the gas inside to quickly return to room temperature. Students should wait a short time for the pressure reading to stabilize before recording results to ensure a constant temperature. (See Teacher Tip 3, Keeping Other Variables Constant, for details on temperature changes.)

   The number of molecules of a gas also affects its pressure. The number of molecules in the syringe is fixed upon its connection to the pressure sensor and will not change significantly as long as the system remains sealed throughout the experiment.

5. What are some potential sources of error in this experiment? What will you do to minimize error?

   There are two ways to contribute to systematic errors in the volume measurements. The most common is to use a long piece of tubing to connect the syringe to the pressure sensor without accounting for the additional volume of gas in the tubing. Either use a short length of tubing to make this volume small relative to the volume of gas in the syringe or account for the additional volume in the tubing when recording the total volume of gas.

   Misreading the syringe markings can also contribute to an error in volume measurements. Read the syringe volume marked at the plunger's front indicator ring, not at the rear ring or at the conical tip of the plunger.

   Changing the temperature of the gas in the syringe over the course of the experiment will also contribute errors. For example, a student might grasp the syringe in the palm of their hand as the syringe plunger becomes more difficult to hold, warming the syringe and causing those measurements to be taken at a different temperature than the others recorded at room temperature.

Assessment Questions: Sample Responses

Sample responses to the Analysis and Synthesis questions found in each version of the lab activity:

Analysis Questions

1. Based on your data, express the relationship between the pressure P and volume V by completing this proportionality statement:

   $P \propto$ $\frac{1}{V}$

2. Convert the proportionality statement above into an equation by introducing a proportionality constant k:

   $P =$ $\frac{k}{V}$

3. The Ideal Gas Law specifies the relationship between an ideal gas's absolute pressure P, volume V, absolute temperature T, and number of molecules N using Boltzmann's constant k_B:

   $$PV = Nk_BT$$

   We can rearrange the Ideal Gas Law equation as follows:

   $$P = \frac{Nk_BT}{V}$$

   Compare this form of the Ideal Gas Law to your equation in the previous step. If your equation were a reduced form of the Ideal Gas Law, which terms from the Ideal Gas Law would be represented in your equation's proportionality constant k?

   $k =$ Nk_BT

❷ 4. Is it reasonable to assume that the number of molecules of air in the syringe was constant during your experiment? Explain.

This is a reasonable assumption. Students seal a fixed amount of air in the syringe when they connect it to the sensor. If the connections are airtight, they should not observe any evidence that air leaks into or out of the syringe as they change the syringe volume. They may also notice upon releasing the plunger at the end of the experiment that the syringe returns to approximately its initial volume, further evidence that the number of molecules of air does not change significantly.

❷ 5. Is it reasonable to assume that the temperature of the air in the syringe was constant during your experiment? Explain.

Students may have prior experience that expanding or compressing a gas can change its temperature. For example, air let out of a bicycle tire rapidly expands and feels cool, while air in a compression igniter demonstration can get hot enough to light tissue paper on fire.

In this experiment, though, the changes in volume are typically small enough and the time between measurements long enough for the air in the syringe to return to room temperature. Observant students may notice that the pressure readings take a short time to settle after each volume change, corresponding to the time to return to temperature equilibrium. So while the temperature strictly does change during this experiment, pressure readings are effectively taken at close to the same temperature, that of the syringe's surroundings. (See Teacher Tip 3, Keeping Other Variables Constant, for more on temperature considerations in this experiment.)

Synthesis Questions

❷ 1. Boyle's Law is sometimes expressed as $P_1V_1 = P_2V_2$, which relates the pressure and volume of a given gas sample in two different situations at the same temperature. Show and explain how this expression can be derived from the form $P = k/V$, where k is a constant.

$P = k/V$ can be rearranged as $PV = k$. This indicates that the product of pressure and volume for a given gas sample is always constant (at a given temperature).

Thus, in two different situations, labeled 1 and 2, $P_1V_1 = k$ and $P_2V_2 = k$.

Eliminating k gives the desired expression, $P_1V_1 = P_2V_2$.

❷ 2. The volume of air in the lungs of a typical human is 6.0 L. Traditional free divers, known for their ability to hold their breath during long dives while gathering sponges or pearls, could descend underwater to a depth of 30 m, where the pressure is four times that at the surface. If such divers hold their breath during the descent and the volume of air in the lungs changed according to Boyle's Law, what would be the volume of air in their lungs at 30 m?

$$P_{surface}V_{surface} = P_{depth}V_{depth}$$

$$V_{depth} = \frac{P_{surface}}{P_{depth}}V_{surface}$$

$$V_{depth} = \left(\frac{1}{4}\right)(6.0\text{ L}) = 1.5\text{ L}$$, which is one-fourth the lung volume at the surface.

❷ 3. A hydrogen-filled weather balloon is roughly a sphere of radius 1.0 m when released from sea level. What would be the radius of the same balloon once it has risen to an altitude of 32 km, where the atmospheric pressure drops to a hundredth of that at sea level? Assume the temperature remains constant and the balloon does not significantly constrain the expansion of the gas it contains.

$$P_{\text{surface}}V_{\text{surface}} = P_{\text{altitude}}V_{\text{altitude}}$$

The volume V of a sphere is related to its radius r by $V = \frac{4}{3}\pi r^3$, so

$$P_{\text{surface}}r^3_{\text{surface}} = P_{\text{altitude}}r^3_{\text{altitude}}$$

$$r_{\text{altitude}} = \sqrt[3]{\frac{P_{\text{surface}}}{P_{\text{altitude}}}}\,r_{\text{surface}}$$

$$r_{\text{altitude}} = \left(\sqrt[3]{\frac{100}{1}}\right)(1.0\text{ m}) = 4.6\text{ m}$$

❷ 4. Describe at a molecular level why the gas pressure in a syringe is halved when you double its volume, keeping the temperature constant.

Model the syringe as a right circular cylinder with constant cross-section A, initial length L, and final doubled length $2L$.

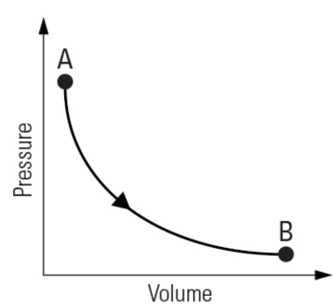
The two ends of the cylinder are initially distance L apart, and then their separation increased to $2L$. Since the gas temperature remains constant, the average speed of a given type of gas molecule is the same before and after. The component of that speed along the cylinder axis v_x also remains the same. When the separation between ends is L, the time between successive collisions with the opposite ends of the cylinder is given by $t_1 = L/v_x$.

When that separation doubles to $2L$, the time between successive collisions with the ends doubles to t_2: $t_2 = 2L/v_x$. With the number of collisions in any short time period cut in half, the average force is cut in half. Since pressure is given by $P = F/A$, the pressure at the cylinder ends is also halved.

But what about the pressure on the sides of the cylinder? The separation between opposite sides of the cylinder does not change, so the average time for a molecule with a speed component perpendicular to the cylinder axis to traverse from one side to another remains the same. The number of collisions with the sides is unchanged, so the average force on the sides is the same for both cylinder lengths. What has changed is the total surface area of the sides. As the cylinder length changes from L to $2L$, the total surface area of the sides doubles. With the same average force exerted across double the area, the pressure on the cylinder sides is halved.

Thus, the pressure on all cylinder surfaces (and throughout the gas) is cut in half when the volume doubles.

❷ 5. The change in pressure and volume of an enclosed gas taken from state A to state B at constant temperature is shown in the graph below. Was work done on the gas or by the gas? If the internal energy of the gas stayed the same during this process, did the gas absorb or release thermal energy? Explain your reasoning.



The work done *on* a system is defined as $W = -P\Delta V$. The graph shows that the volume increased, so ΔV was positive and thus the work on the system—the gas—was negative. This means that work was done *by* the gas on the environment.

The change in internal energy is given by $\Delta U = Q + W$, where Q is the energy transferred *to* the system by heating. Since the internal energy of the gas did not change, $\Delta U = 0$, and therefore $Q = -W$. Since the work W was determined above to be negative, Q must be positive. This means that the gas *absorbed* thermal energy from the environment.

Extended Inquiry Suggestions

Although each student group will need to hold constant the temperature and the number of molecules of air in their syringe during their pressure and volume measurements, consider varying the temperatures and number of molecules *between* lab groups. This will enable a class-wide exploration into the effects of temperature and the number of molecules on the value of the Boyle's Law proportionality constant k, leading to a more natural segue from Boyle's Law to the full Ideal Gas Law. To vary temperature, some lab groups might grasp the syringe in their warm hands throughout the experiment, while other groups can wrap it in an ice pack or keep it at room temperature. To vary the number of molecules of air, lab groups could begin with different initial volumes of air in their syringes (for example, 10 mL, 15, mL, or 20 mL) before connect them to the pressure sensor.

Also consider using one of the many Ideal Gas Law simulations available online to bolster your students' mental models of the molecular motion of gas molecules and their connections to the macroscopic measurements of the Ideal Gas Law.

NAME PERIOD DATE

4. BOYLE'S LAW

STRUCTURED

Driving Question | Objective

How is the pressure of a gas affected by changes in the volume of the container enclosing it? Experimentally determine the mathematical relationship between the pressure of a gas and the volume it occupies.

Materials and Equipment

- Data collection system
- PASCO Barometer/Low Pressure Sensor[1]
- Sensor extension cable
- Syringe, 60-mL
- Quick connector[1]
- Tubing[1]
- Scissors

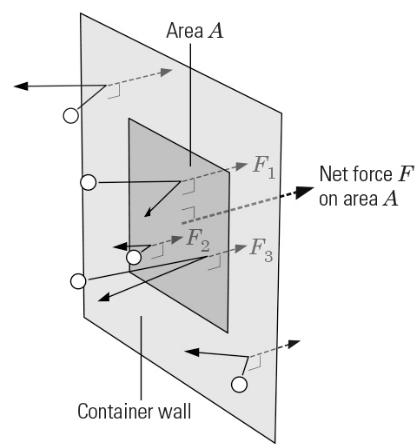
[1]www.pasco.com/ap24

PASCO Barometer/Low Pressure Sensor

Background

Relative to their size, the molecules in a gas are widely spaced. Because of this, the specific type of molecule has far less impact on the properties of a gas compared to its impact on the properties of a liquid or solid.

Using only a few macroscopic properties, we can develop a model that describes the behavior of an ideal gas. Since different gases behave similarly, this model applies quite accurately to a variety of real gases, including the mixture of molecules that constitutes the air around us.



Gas molecules are constantly in motion, colliding with each other and with the walls of the container that might enclose them. Each collision between a gas molecule and the container wall results in the wall exerting a force on the molecule that changes the molecule's momentum. According to Newton's Third Law, the molecule exerts a force equal in magnitude and opposite in direction on the wall.

The enormous number of molecules in even a small parcel of air ensures that a statistically large and relatively consistent number of molecules collide with the wall in any short time period. The average force F that these collisions exert on a given area A of the container wall is the average pressure P, which can be expressed as

$$P = \frac{F}{A}$$

We can measure this average pressure of the gas with a pressure sensor.

In this activity, you will investigate how this gas pressure varies as the volume of the container is changed.

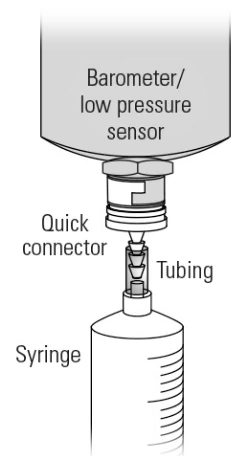
Safety

Follow these important safety precautions in addition to your regular classroom procedures:

- Do not to exceed the measurement range of the sensor. The PASCO Barometer/Low Pressure Sensor's measurement range is 15 to 115 kPa. The sensor will not accurately report pressures outside this range and applying pressures outside this range could permanently damage the sensor.
- You will apply significant force on the syringe to change its volume. Grasp the syringe firmly with two hands and use caution to avoid slipping and potentially injuring yourself or those nearby.

Procedure

Set Up



1. Use scissors to cut a section of tubing approximately 1 cm long.
2. Firmly attach that tubing to the end of the syringe.
3. Insert the barbed end of the quick connector into the open end of the tubing. Ensure that at least one of the barbs is fully covered by the tubing for an airtight fit.
4. Set the syringe volume to 10 mL.
5. Insert the quick connector into the port of the pressure sensor and turn it clockwise until it clicks.

NOTE: The sequence of the previous two steps is crucial for correctly setting the amount of air in the syringe.

6. Connect the pressure sensor to the data collection system, using the sensor extension cable if necessary to position the sensor and syringe close to you on the table top.
7. Create a digits display showing the barometric pressure measured by the sensor in units of kilopascals (kPa).

Collect Data

8. Record the current syringe volume (10 mL) and corresponding pressure in Table 1 of the Data Analysis section below.
9. Adjust the syringe volume to 20 mL and record the corresponding pressure in Table 1 once it has stabilized.

   NOTE: Since you'll be holding the syringe with two hands, have a lab partner record the measurements.

10. Repeat data collection for these additional volumes: 30 mL, 40 mL, and 50 mL. Record these volumes and the corresponding pressures in Table 1.

Data Analysis

Table 1: Gas pressure for various syringe volumes

Volume (mL)	Pressure (kPa)	1/Volume (mL^{-1})

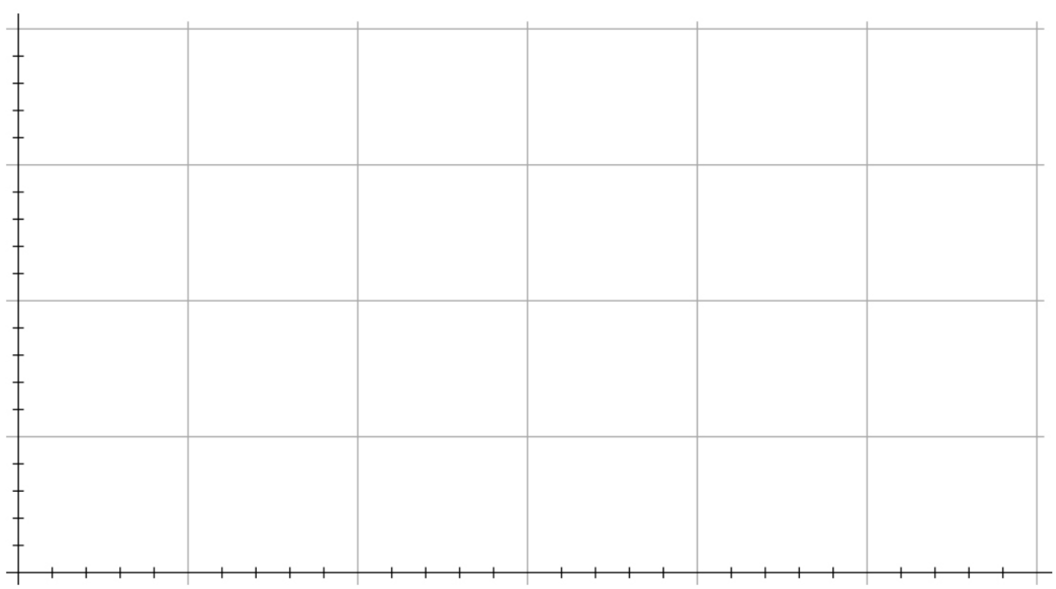
1. In the table above, identify two volume measurements that represent a doubling of the volume. What is the relationship between the two corresponding pressure measurements? Does the same relationship hold for any other pairs of measurements that represent a doubling of the volume?

2. What type of mathematical relationship (proportional, squared, inverse, inverse squared, et cetera) between pressure and volume is implied by your observations above?

3. Calculate and record the inverse volume (1/volume) in units of inverse milliliters (mL^{-1}) for each volume measurement in Table 1.

4. Plot a graph of *pressure* versus 1/*volume* in the blank Graph 1 axes below. Be sure to label both axes with the correct scale and units.

Graph 1: Gas pressure versus inverse volume



Analysis Questions

1. Based on your data, express the relationship between the pressure P and volume V by completing this proportionality statement:

   $P \propto$ ____________________________

2. Convert the proportionality statement above into an equation by introducing a proportionality constant k:

   $P =$ ____________________________

3. The Ideal Gas Law specifies the relationship between an ideal gas's absolute pressure P, volume V, absolute temperature T, and number of molecules N using Boltzmann's constant k_B :

   $$PV = Nk_BT$$

   We can rearrange the Ideal Gas Law equation as follows:

   $$P = \frac{Nk_BT}{V}$$

   Compare this form of the Ideal Gas Law to your equation in the previous step. If your equation were a reduced form of the Ideal Gas Law, which terms from the Ideal Gas Law would be represented in your equation's proportionality constant k?

   $k =$ ____________________________

4. Is it reasonable to assume that the number of molecules of air in the syringe was constant during your experiment? Explain.

   ____

   ____

   ____

5. Is it reasonable to assume that the temperature of the air in the syringe was constant during your experiment? Explain.

   ____

   ____

   ____

Synthesis Questions

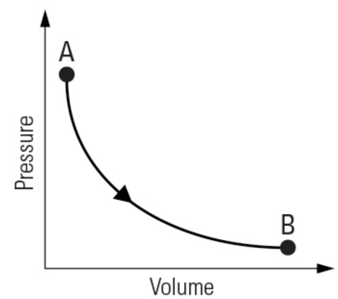
1. Boyle's Law is sometimes expressed as $P_1V_1 = P_2V_2$, which relates the pressure and volume of a given gas sample in two different situations at the same temperature. Show and explain how this expression can be derived from the form $P = k/V$, where k is a constant.

2. The volume of air in the lungs of a typical human is 6.0 L. Traditional free divers, known for their ability to hold their breath during long dives while gathering sponges or pearls, could descend underwater to a depth of 30 m, where the pressure is four times that at the surface. If such divers hold their breath during the descent and the volume of air in the lungs changed according to Boyle's Law, what would be the volume of air in their lungs at 30 m?

3. A hydrogen-filled weather balloon is roughly a sphere of radius 1.0 m when released from sea level. What would be the radius of the same balloon once it has risen to an altitude of 32 km, where the atmospheric pressure drops to a hundredth of that at sea level? Assume the temperature remains constant and the balloon does not significantly constrain the expansion of the gas it contains.

4. Describe at a molecular level why the gas pressure in a syringe is halved when you double its volume, keeping the temperature constant.

5. The change in pressure and volume of an enclosed gas taken from state A to state B at constant temperature is shown in the graph below. Was work done on the gas or by the gas? If the internal energy of the gas stayed the same during this process, did the gas absorb or release thermal energy? Explain your reasoning.



5. SPHERICAL MIRROR REFLECTION

Connections to the AP® Physics 2 Curriculum*

The lab activity correlates to the following pieces of the AP® Physics 2 framework:

Big Idea	6	Enduring Understanding	E	Essential Knowledge	4

Learning Objective 1: The student is able to plan data collection strategies and perform data analysis and evaluation of evidence about the formation of images due to reflection of light from curved spherical mirrors.
Science Practices: 3.2, 4.1, 5.1, 5.2, 5.3

Learning Objective 2: The student is able to use quantitative and qualitative representations and models to analyze situations and solve problems about image formation occurring due to the reflection of light from surfaces.
Science Practices: 4.1, 4.2, 5.1

Time Requirement

Preparation Time: 10 minutes

Lab Activity: 40 minutes

Prerequisites

Students should be familiar with the following concepts:

- *Specular reflection* is the reflection of light from a smooth reflecting surface where the incident light and reflected light form the same angle relative to a line normal to the surface. This is also known as the *Law of Reflection*. Specular reflection assumes that the point at which each light ray is incident is perfectly flat and the normal line is perpendicular to the surface at that point.
- The difference between *real images* and *virtual images* formed by mirrors.
- Drawing ray diagrams using curved and flat mirrors.
- The spherical mirror equation and its variables.

Driving Question | Objective

What is the radius of curvature of the concave spherical mirror provided by your instructor? Use the principles of reflection and the spherical mirror equation to experimentally determine the radius of curvature of a concave spherical mirror.

Procedural Overview

In the Structured version of this lab activity, students observe the formation distance of a real image produced by a concave spherical mirror and analyze its dependence on the distance between the mirror and the object (image source). Students identify and measure the image distance using five different object distances and use their data to plot a graph of inverse object distance versus inverse image distance. The shape of student graphs should be linear with the y-intercept equal to $2/R$, where R is the radius of curvature of their spherical mirror. From these relationships, students determine an experimental value for the radius of curvature of their spherical mirrors.

Pre-Lab Discussion and Activity

This lab is designed to precede the "Focal Length of a Converging Lens" activity in which students determine the focal length of a converging lens using a graphical method. Because the procedures outlined in the Structured versions of that and this lab activity are so similar, it is recommended to follow an open-inquiry progression in which the Structured or Guided Inquiry version of this activity is first performed by students, followed by the Guided Inquiry or Student Designed version, respectively, of the "Focal Length of a Converging Lens" activity. Doing this will help assess students' abilities to design and conduct an activity based on their experience using linearization and other science practices outlined in this activity.

Before performing this activity, students should be familiar with the spherical mirror equation:

$$\frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i} \tag{3}$$

where f is the mirror focal length (half the radius of curvature R of the mirror: $f = R/2$), s_o is the distance from the mirror to an object, and s_i is the distance from the mirror to the point at which the image of the object is in focus (image distance).

In the Structured version of this activity, the process of identifying the point at which the real image from a concave spherical mirror is formed is addressed step-by-step in the procedure section. However, students performing the Guided Inquiry and Student Designed versions of this activity may need a short demonstration showing this process, as this may be an important part of their experiment procedure.

Use the PASCO optics equipment listed in the Materials and Equipment section below to set up and perform the following demonstration:

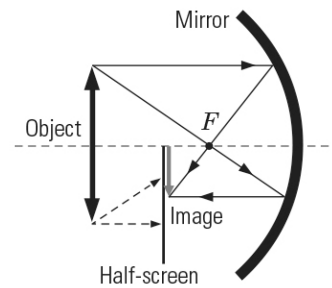
Begin by assembling the PASCO optics equipment as in the picture:

1. Lay the optics track flat on the lab table and mount the light source near the end of the track with the crossed-arrow image on the light source pointing down the length of the track.

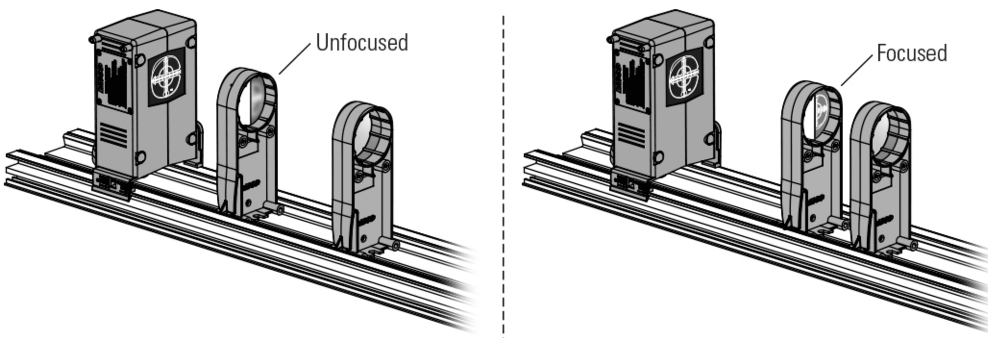
2. Mount the mirror to the track approximately 15 cm from the light source with the concave surface of the mirror facing it.

3. Mount the viewing screen to the track at some point between the light source and the mirror, and then plug in the light source to turn it on.

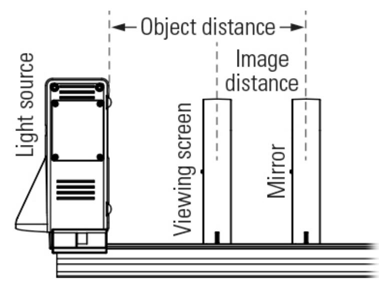
In this configuration, the illuminated crossed-arrow target on the light source will act as the object, and the image of the object will be formed on the half-screen. Explain to students that the equipment is designed to be used in a straight path to best emulate the linear configurations depicted in most ray diagrams, and to enable the accurate use of Equation 3 in various experiments; however, because the viewing screen must be placed between the object and the mirror in this configuration, half of the viewing screen is removed to allow light rays to pass through. These light rays are reflected off the mirror and converge on the remaining half of the viewing screen to form half of the image. This is outlined in the ray diagram at right.



With the light source on, slide the viewing screen along the track so that its position is about halfway between the light source and the mirror. Show students how the light rays reflected off the mirror appear after converging on the half-screen: the image should appear as a blur near the center of the half-screen.

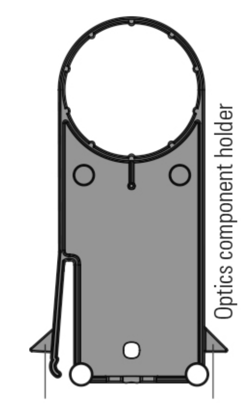


Have students observe the image on the half-screen as you slowly slide the screen closer to the mirror. The image should condense and become more focused as the screen approaches the mirror. Stop sliding the half-screen at the point the image is most sharply focused (you may need to slide the screen back and forth through the image location until you can determine where the image is the most sharply focused); this is the position of the real image formed by the concave spherical mirror.



Explain to students that with the screen in this position, measurements of the variables in Equation 3: object distance s_o and image distance s_i (where object distance is measured from the front of the light source to the mirror, and image distance is measured from the mirror to the half-screen), can now be accurately made.

Another helpful tool for students may be the small indicators molded onto the bottom of each PASCO Basic Optics component holder. These indicators show the position of each component in its holder so can be used to make measurements of object and image distance using the metric scale on the optics track.



Materials and Equipment

- PASCO Optics Track[1] or PASCO Dynamics Track with Optics Carriages[2]
- PASCO Basic Optics Light Source[3]
- PASCO Concave Mirror Accessory[4]
- PASCO Half-Screen Accessory[4]

Probeware Resources

Below are web-link and QR codes that will direct you to instructional video resources for individual pieces of PASCO probeware, sensors, and other hardware used in the lab activity. These same links and codes are provided to students in their activity handouts.

[1]www.pasco.com/ap28	[2]www.pasco.com/ap29	[3]www.pasco.com/ap26	[4]www.pasco.com/ap32
PASCO Optics Track	PASCO Dynamics Track Optics Carriages	PASCO Basic Optics Light Source	PASCO Concave Mirror Accessory

Lab Preparation

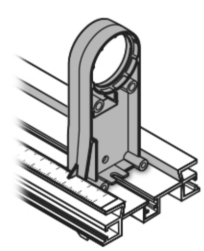
In this activity, students experimentally determine the radius of curvature of a concave spherical mirror, and then compare their experimental value to the actual value.

If your students are using the PASCO Concave Mirror Accessory, note that the mirror in the accessory package has its focal length shown on its component holder base: +50 mm focal length; 0.100 m radius of curvature with a tolerance of 3%. It is recommended that you cover this value with tape or other masking material so that students do not have a preconceived idea of what their results should show.

If you are using mirrors with unknown radii of curvature, you will need to identify these values or determine them empirically before the lab activity commences. Some third-party manufacturers will publish the accepted component values in the literature that accompanies them. If not, it is recommended that you follow the procedure outlined in the Structured version of this lab activity to empirically determine each mirror's radius of curvature.

Teacher Tips

Tip 1 – Using PASCO Basic Optics System Components

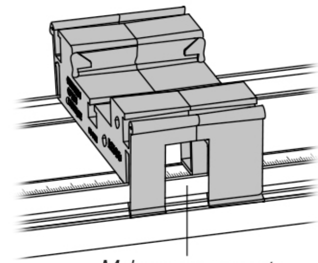


- PASCO Basic Optics system component such as the Concave Mirror Accessory and Half-Screen Accessory are held in component holders designed to fit in the wide central channel of the PASCO Optics Track or the PASCO Dynamics Track Optics Carriages (see Teacher Tip 2). Place the base of the component holder on the bench and push down firmly to snap it in place. To move it, squeeze the tab on the base of the component holder and slide it along the track.
- Use the metric scale on the optics track to make spatial measurements with these components. Each component holder has a small molded tab extending from its base that indicates the position of the optical component in its holder.

Tip 2 – Using PASCO Dynamics Track Optics Carriages

- PASCO Dynamics Track Optics Carriages, in conjunction with a PASCO dynamics track, can be used in place of a PASCO Optics Track. The carriages are designed to snap onto the top of the dynamics track, and are molded with a central channel to receive PASCO Basic Optics system component holders, similar to the PASCO Optics Track.

- To mount the carriages and basic optics component holders, place the carriage on top of a dynamics track and press down to snap the outer tabs of the carriage into the side slots of the track. Place the base of the component holder on the carriage and push down firmly to snap it in place. The component holder should snap into the recesses in the center of the carriage, locking it in place. You can then slide the carriage and component combination along the track to the desired position. To detach the carriage, squeeze the arms on either side and pull up.
- To make spatial measurements using the carriage-component holder combination, use the metric scale on the dynamics track. Each carriage has a molded opening through which the metric scale on the dynamics track is visible. The front square edge indicates the position of the optical component in its holder.



Tip 3 – Appropriate Choice of Object Distances

Students performing the Guided Inquiry or Student Designed version of this lab activity may choose an experiment procedure in which they measure image distance at varying object distances (similar to that outlined in the Structured version of this lab activity). It is important for these students to test a range of object distances that does not consist entirely of values much larger than the focal length of the mirror, or values that are tightly grouped.

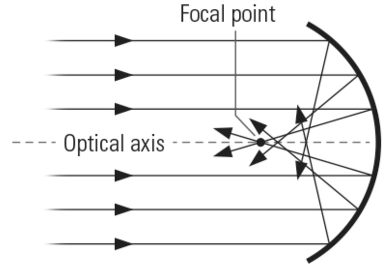
At object distances much larger than the focal length of the mirror (values > 6 × focal length), the resulting image distances have very small variance. This can make it difficult for students to measure differences in image distances accurately, especially if the difference between image distance values does not exceed the resolution of the measurement tool being used.

Similarly, choosing a range of object distances that are too tightly grouped may also produce very small variance in image distance values, resulting in added difficulty in making accurate making measurements.

Students should be encouraged and guided to choose a range of object distances that begins with a value as small as possible, but still permits the measurement of the image formation distance, and then extends to a value approximately six times the focal length of the mirror.

Tip 4 – Correcting Spherical Aberration

In most applications of concave spherical mirrors it is assumed that the shape and surface of the mirror is such that all incoming light rays parallel to the optical axis will reflect and converge at the mirror's focal point *F*. However, this assumption fails for those incident light rays not near the optical axis, which converge behind the focal point. As a result, images formed by concave spherical mirrors appear focused at their center, but blurry along the outer edge. This is known as *spherical aberration* and can confuse students when attempting to determine where the actual image formation from a concave spherical mirror is, spatially.



Students can correct for spherical aberration by doing the following:

1. Cut a small piece of paper just larger than the size of the mirror.
2. Cut a small round hole in the center of the paper about 1–2 cm in diameter.

3. Hold the paper over the mirror surface with the center of the hole aligned with the center of the mirror.

With the paper in place, the light rays not near the optical axis will be blocked, resulting in a more focused image.

Sample Data

Below are sample data, acquired using the experimental setup and procedure outlined in the Structured version of the lab activity, and answers to questions in the Data Analysis section.

Data Analysis

Table 1: Object distance and corresponding image distance using a concave spherical mirror

Trial	Object Distance s_o (m)	Image Distance s_i (m)	$1/s_o$ (m^{-1})	$1/s_i$ (m^{-1})
1	0.1400	0.0770	7.143	13.0
2	0.1700	0.0700	5.882	14.3
3	0.2000	0.0660	5.000	15.2
4	0.2300	0.0630	4.348	15.9
5	0.2600	0.0610	3.846	16.4

1. Calculate the inverse of each object distance and image distance in Table 1. Record your results in the $1/s_o$ and $1/s_i$ columns of Table 1.

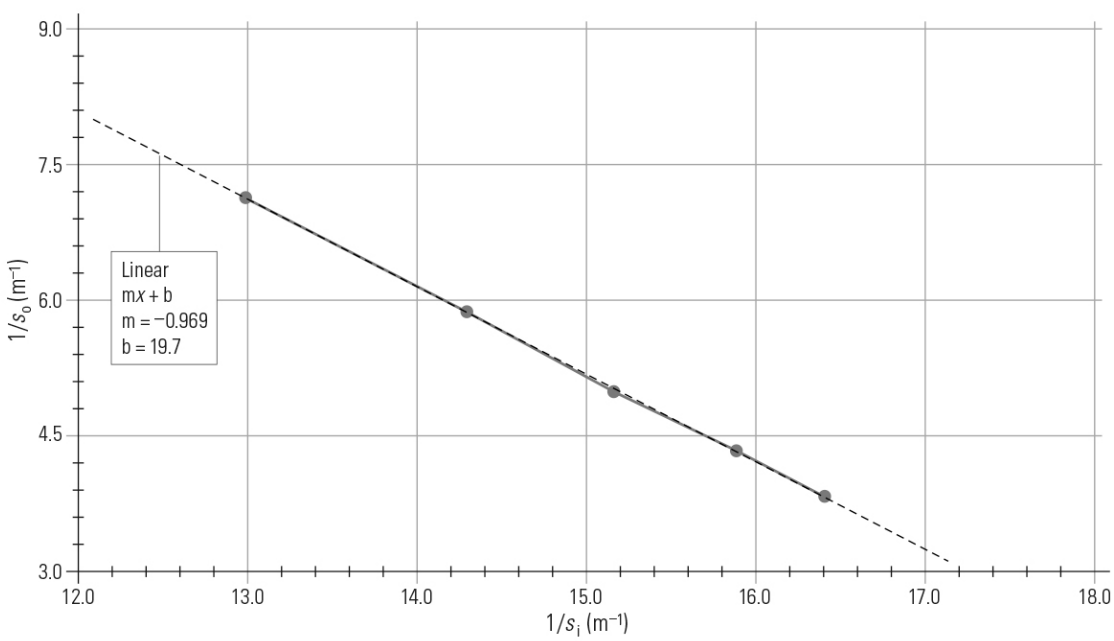
Calculation using sample data for Trial 1:

$$\frac{1}{s_o} = \frac{1}{0.1400\ \text{m}} = 7.143\ \text{m}^{-1}$$

$$\frac{1}{s_i} = \frac{1}{0.0770\ \text{m}} = 13.0\ \text{m}^{-1}$$

2. Plot a graph of $1/s_o$ versus $1/s_i$ in Graph 1. Be sure to label both axes with the correct scale and units.

Graph 1: Inverse object distance versus inverse image distance using a concave spherical mirror



3. Draw a line of best fit through your data in Graph 1. Determine and record the equation of the line here:

   Best fit line equation: $\frac{1}{s_o} = -0.969\frac{1}{s_i} + 19.7\ \text{m}^{-1}$

4. Use the y-intercept from the best fit line to determine an experimental value for the radius of curvature R of your mirror:

   $$y\text{-intercept} = \frac{2}{R}$$

   Radius of curvature R (m): 0.102 m

   Calculation using sample data from Graph 1:

   $$y\text{-intercept} = \frac{2}{R}$$

   $$R = \frac{2}{y\text{-intercept}} = \frac{2}{19.7\ \text{m}} = 0.102\ \text{m}$$

Guided Inquiry Questions

Below are sample responses to the Guiding Questions found in the Guided Inquiry version of this lab activity.

❓ 1. The spherical mirror equation defines the position of the real image formed by a spherical mirror:

$$\frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i} \qquad (3)$$

where f is the mirror focal length (half the radius of curvature R of the mirror: $f = R/2$), s_o is the distance from the mirror to an object (*object distance*), and s_i is the distance from the mirror to the point at which the image of the object is in focus (*image distance*).

a. Explain how Equation 3 helps inform which measurements you will make in your investigation?

   The primary objective in this lab activity is for students to determine the focal length of their concave mirror. Student responses should explain that Equation 3 identifies quantities (image distance and object distance) that can be measured and used to calculate a concave mirror's focal length, which in turn can be used to determine the radius of curvature. By focusing their experiment on these quantities, students can follow a straightforward path in their investigation.

b. Which variables in Equation 3 can be measured directly using measurement tools that are available to you?

   Object distance s_o and image distance s_i can be measured directly using any spatial measurement tool: meter stick, ruler, tape measure, et cetera. Focal length f cannot be measured directly and requires some form of indirect measurement, or unique measurement strategy. Students should be encouraged not to attempt to measure focal length directly and concentrate their efforts on identifying a simple strategy to determine it indirectly using the relationship outlined in Equation 3.

c. What tools would you use to measure each variable listed in your previous answer and how would you use them to make measurements?

   Students can use any spatial measurement tool to measure both object and image distance, but it should be recommended that they use a meter stick, metric ruler, or measuring tape. Object distance should be the measured distance from the object or light source being used to the deepest point of the concave spherical mirror. Image distance should be measured from the deepest point of the concave spherical mirror to the screen on which a focused image of the object is formed.

❷ 2. What equipment will you use and how will you set it up so that each variable can be measured as accurately as possible?

Although the Structured version of this activity identifies specific PASCO optics components for students to use, students can use any brand of optics components as long as those components include a concave spherical mirror, an “object” of which an image will be formed, and a viewing screen on which the image of the object will be formed. Students should be creative in their experiment design, but should follow these general rules for equipment setup:

- Students can choose any object as long as it is sufficiently illuminated or emits light so that a reasonable image of the object can be formed. It is recommended that students use a light source (light bulb, lamp, candle, et cetera) as their object.
- The “reflecting” side of the mirror must face the chosen object in such a way that the distance between the object and the mirror can be measured in a straight line.
- A viewing screen should be used, but the screen must not completely block the light emitted from the object so that it cannot reach the mirror. Students can achieve this by using a half-screen (similar to that shown in the Structured version of this lab activity) that blocks half of the incoming light but allows the unblocked light to be reflected back onto the half-screen to form an image. A viewing screen with a small hole cut in its center can also achieve the same result.
- Students should avoid using components in any configuration that is not a straight line. Students angling their mirrors to project the image onto a viewing screen not in line with the optical path can cause unnecessary aberration in their images, making it difficult to identify the image location and adding inaccuracy in their measurements.

❷ 3. If you place a concave spherical mirror at one end of a straight optics track and a light source (object source) at the other, how would you use a viewing screen to locate the image of the light source, and why can’t the viewing screen completely block the path between the mirror and the light source?

Students should indicate that in this configuration, to identify the position of an image formed from a concave mirror, the viewing screen should be placed directly between the light source and mirror and then the viewing screen should be slid up and down the track until a focused image of the light source is formed on the screen. Students should also note that the viewing screen should only partially block the light from the light source; otherwise, the light from the object cannot reach the mirror and no reflected image can be formed.

❷ 4. Do you think it is better to make one measurement of each variable and then use that one value to determine R for your mirror, or make several measurements using different values? Explain your reasoning.

Students will likely conclude that if their experiment is based on one measurement, it is likely that there will be error that can cause their results to be inaccurate or less representative. Making several measurements of the dependent variable based on a good variance of the independent variable will provide an averaged and more accurate (representative) value for R.

❷ 5. According to Equation 3, what happens to the image distance when the object distance approaches the focal length of the mirror? Should you measure the image distance when the object distance is near the focal length of the mirror? Justify your answer.

Students should recognize that the relationship outlined in Equation 3 indicates that the image distance approaches infinity as the object distance approaches the focal length of the mirror.

No, in this case, the image distance should not be measured, as it is greater than the length of the optics bench.

Assessment Questions: Sample Responses

Sample responses to the Analysis and Synthesis questions found in each version of the lab activity:

Analysis Questions

❷ 1. What is your experimental value for the radius of curvature R of your mirror, and how did you determine this value from your data?

Based on the data shown in the Sample Data section above, R = 0.102 m.

Students performing the Student Designed or Guided Inquiry version of this activity will acquire values that vary between groups based on the measurement tools used and the procedural strategies chosen. Students performing any of the three versions of this activity may also show variability in their values between groups due to the variability in the actual radius of curvature of each mirror. The PASCO Concave Mirror Accessory has a published radius of curvature of 0.100 m with a tolerance of 3%.

Students following the procedure outlined in the Structured version of this lab activity identify and measure the image distance using five different object distances and use their data to plot a graph of inverse object distance versus inverse image distance. The shape of student graphs should be linear with y-intercept equal to $2/R$. From this, students calculate an experimental value for R.

❷ 2. What are factors that might have caused error in your measured value for radius of curvature? Explain how each factor you list could be avoided or minimized.

The list of factors that could cause error may include, but is not limited to:

- Difficulty identifying the point at which the image was located. This may be caused by spherical aberration. See Teacher Tip 4 above.
- Components were not aligned in a straight line. Students can avoid this by using a fixed optics track or table to help accurately align components.
- Measurements of component positions were made incorrectly: for example, students may measure object and image distances based on the position of the component holders and not the actual components in the holders. Students should make spatial measurements based on the position of the components in the holders and not the holders themselves. See Teacher Tips 1 and 2 above.

❷ 3. Ask your teacher for the actual value of the radius of curvature of your mirror, and then calculate the percent error between your experimental value and the actual value.

$$\text{Percent error} = \left|\frac{\text{Actual} - \text{Experimental}}{\text{Actual}}\right| \times 100$$

Sample calculation:

$$\text{Percent error} = \left|\frac{0.100\text{ m} - 0.102\text{ m}}{0.100\text{ m}}\right| \times 100 = 2.00\%$$

❷ 4. What do you predict happens to the image distance from a concave spherical mirror as the object distance grows very large (much larger than the image distance)? Justify your answer: use mathematical reasoning or data from your experiment, or both, to support your answer.

Based on the mathematical relationship in Equation 3, as the object distance grows very large, the image distance approaches the focal length of the mirror.

Students may choose to justify their response using their data and a graph of object distance versus image distance. This graph should show a vertical asymptote where image distance is equal to focal length, thus indicating that as object distance increases, image distance will approach the focal length of the mirror.

Synthesis Questions

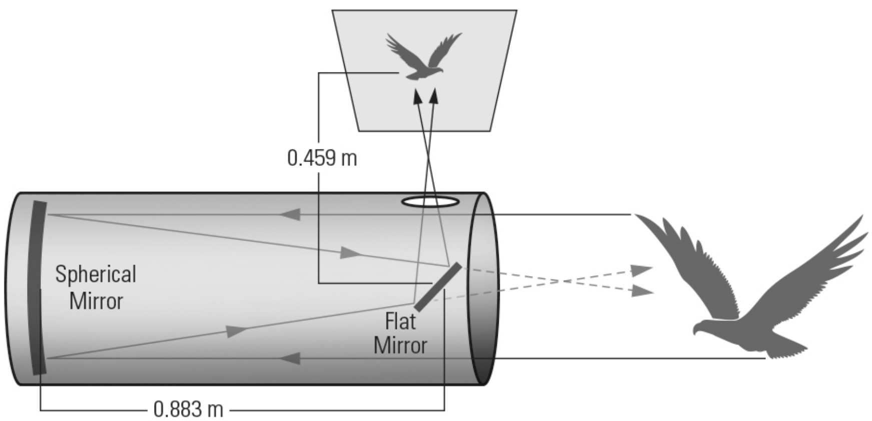
1. In 214 BC, Archimedes invented a large spherical-type mirror used to focus the sun's intense rays onto far away enemy boats, which would eventually light them on fire. If the boats were travelling in a nearby channel approximately 1,000 m from the river bank, what would the radius of curvature of his mirror need to be? Show your work.

   The focal point of a concave mirror is the point at which paraxial (parallel) light rays are focused. Because the sun is so far away, it can be assumed that all the rays from the sun, incident on the mirror are parallel. To focus the sun's rays onto a boat, the distance between the boat and the mirror must be equal to the focal length of the mirror:

   $f = 1{,}000 \text{ m}$

   $R = 2f = 2(1{,}000 \text{ m}) = 2{,}000 \text{ m}$

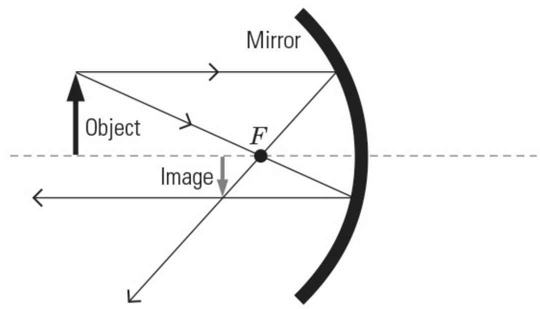
2. The image below shows the position of two mirrors inside a reflector-style telescope. Light from far-off objects enters the body of the telescope through the opening on the right. That light is reflected off a spherical mirror in the back of the telescope and then again off a flat mirror that redirects the light to a screen where the image of the object appears in focus. Given that the image of the eagle on the screen is in focus, and the actual eagle is 726 m away from the spherical mirror, what is the radius of curvature of the spherical mirror?



$$\frac{2}{R} = \frac{1}{s_o} + \frac{1}{s_i}$$

$$R = \frac{2}{\frac{1}{s_o} + \frac{1}{s_i}} = \frac{2}{\frac{1}{726 \text{ m}} + \frac{1}{(0.883 \text{ m} + 0.459 \text{ m})}} = \frac{2}{0.747 \text{ m}^{-1}} = 2.68 \text{ m}$$

❓ 3. Sketch a ray diagram showing the image produced by the concave spherical mirror below. Is the image real or virtual? How do you know?

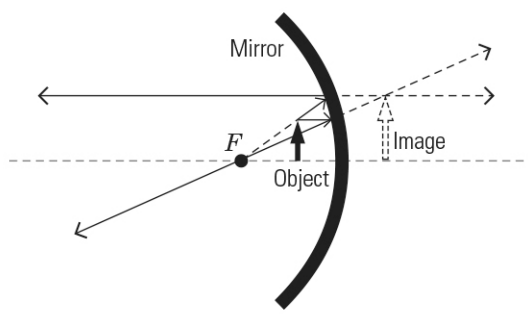


The image is a real image.

Student reasoning may include, but is not limited to:

- The image can be seen formed on a screen rather than appearing behind the mirror.
- The reflected light rays are converging to form the real image.
- Because in the ray diagram, the image appears on the same side of the mirror as the actual object.

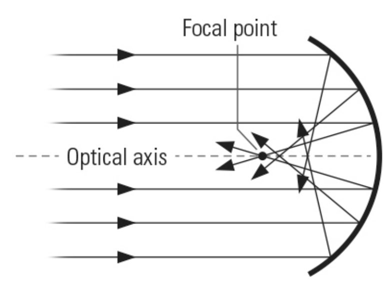
❓ 4. Is it possible for a concave spherical mirror to produce a virtual image of an object? Sketch a ray diagram that supports your answer.



It is possible to produce a virtual image using a concave mirror, but this is only achievable if the object distance is less than the focal length of the mirror. In a ray diagram, the object appears between the focal point and the actual mirror itself.

❓ 5. In most applications of concave spherical mirrors (including this activity) it is assumed that all light rays travelling parallel to the mirror's optical axis are reflected through the mirror's focal point; however, this is an approximation that applies only to light rays traveling near the mirror's optical axis. In a few sentences, explain why a concave spherical mirror does not reflect all light rays travelling parallel to its optical axis through its focal point.

Student responses should use complete sentences and convey that light rays incident on a concave spherical mirror that are not near the optical axis will not converge to the mirror's focal point due to the intrinsic shape of the mirror and the increasing incident angles of the light rays as they depart from the optical axis.



Light rays incident on the mirror's surface will obey the Law of Reflection such that their incident angle will equal the angle of reflection. For parallel light rays approaching a concave spherical mirror near its optical axis, the incident and reflected angles are small enough to approximate that the light rays will reflect and converge at the mirror's focal point. Parallel light rays approaching the mirror at a distance from its optical axis experience an incident angle that increases as the distance from the optical axis increases. Eventually the incident angle grows too large to make the approximation and the light rays converge at points behind the mirror's focal point, as in the diagram.

Extended Inquiry Suggestions

A natural extension to this activity is a discussion about convex mirrors, the formation of virtual images, and the sign convention describing the focal length of both mirror types (concave and convex). This discussion will prove useful to students when they begin the study of other topics in geometrical optics, including image formation using converging and diverging lenses, as both topics require a consistent sign convention.

In this activity, student focus was on concave spherical mirrors, the real images formed by those mirrors, and the spatial relationship outlined in Equation 3. Similar to *concave* spherical mirrors, *convex* spherical mirrors also form images whose position can be determined using the relationship in Equation 3, except for three important differences:

1. The image formed by a convex mirror is a virtual image appearing physically behind the mirror, unlike the real image formed by a concave mirror, which appears in front of the mirror.

2. The focal length value used in Equation 3 is negative for convex mirrors, compared to the positive focal length of concave mirrors (radius of curvature R is positive for both mirror types)

3. The focal point lies behind the mirror (negative) for convex mirrors, and in front (positive) for concave mirrors.

Before explaining these differences to students, engage them in a small group activity where they are given the chance to explore a convex mirror: how its shape, the behavior of light reflected from its surface, and the image formed by it are all different than that from a concave mirror. The goal of the activity is for students to discover the three differences listed above with subtle guidance from you and their classmates.

Break the class into groups, and then hand each group a convex mirror.

Tell students using the PASCO Concave Mirror Accessory that the reverse side of the concave mirror is a convex mirror with identical radius of curvature.

Ask students to explore the behavior of the light from the convex mirror as it relates to image formation. Have each group elect a presenter and be prepared to:

- Identify the obvious differences between the behavior of light from the concave mirror and the convex mirror.
- Make factual statements about the image formed by the mirror that can be supported with evidence.
- Explain how Equation 3 can be used to describe the position of the image, and explain what is different between its (Equation 3) use with concave mirrors and convex mirrors.

After groups have been given ample time to explore, have each group's presenter explain to the class their group's discoveries and supporting evidence. Record each model or claim (with evidence) on the board and ask other groups to offer evidence based on their exploration that supports or refutes them. After each group has presented, propose the three differences listed above and identify any evidence, observations, or claims on the board that supports these.

Finish by proposing the convention:

- Negative values in Equation 3 represent distances that are behind the mirror for both concave and convex mirror types.

Identify any evidence, observations, or claims on the board that supports the proposed convention, and then test the convention by answering these questions as a class:

1. Is the focal length of a convex mirror positive or negative?

   Negative

2. Assuming that object distance is always positive for mirrors, based on our convention, where should we expect to find the image formed by a convex mirror: in front of the mirror or behind it? Explain your answer.

   The image is formed behind the mirror. Based on Equation 3, if focal length is negative and object distance is positive, image distance will always be negative.

3. In your experience with this exploration, where did you physically find the image formed by a convex mirror: in front of the mirror or behind it? Does this support or refute our convention?

   Student exploration should show that the image formed by a convex mirror will always appear behind the mirror, which indicates that the image distance is negative. This supports the convention.

4. Is the focal length of a concave mirror positive or negative?

   Positive

5. Assuming that object distance is always positive for mirrors, and the object distance is greater than the focal length of the mirror, based on our convention, where should we expect to find the image formed by a concave mirror: in front of the mirror or behind it, and why?

   The image is formed in front of the mirror. Based on Equation 3, if focal length is positive and object distance is positive and greater than the focal length of the mirror, image distance will always be positive.

6. In your lab activity, where did you physically find the image formed by a concave mirror: in front of the mirror or behind it? Does this support or refute our convention?

   The image formed by a concave mirror will always appear in front of the mirror as long as the object distance is greater than the focal length of the mirror. This indicates that the image distance is positive, which supports the convention.

7. Based on our convention and the relationship shown in Equation 3, is there a situation where the image formed by a concave mirror will appear behind the mirror (virtual)?

   Based on Equation 3, if the focal length is positive and the object distance is positive but less than the focal length of the mirror, image distance will be negative, indicating that the (virtual) image appears behind the mirror.

8. Can we test our prediction from the previous question using our mirrors?

   Yes. Place any object in front of a concave mirror, closer to the mirror than its focal length, and a virtual image of that object will appear behind the mirror.

9. Does our test support or refute our convention?

   Students should find that the test supports the convention.

NAME PERIOD DATE

5. SPHERICAL MIRROR REFLECTION

STRUCTURED

Driving Question | Objective

What is the radius of curvature of the concave spherical mirror provided by your instructor? Use the principles of reflection and the spherical mirror equation to experimentally determine the radius of curvature of a concave spherical mirror.

Materials and Equipment

- PASCO Optics Track[1] or PASCO Dynamics Track with Optics Carriages[2]
- PASCO Basic Optics Light Source[3]
- PASCO Concave Mirror Accessory[4]
- PASCO Half-Screen Accessory[4]

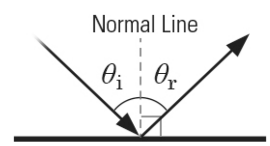
[1]www.pasco.com/ap28 — PASCO Optics Track

[2]www.pasco.com/ap29 — PASCO Dynamics Track Optics Carriages

[3]www.pasco.com/ap26 — PASCO Basic Optics Light Source

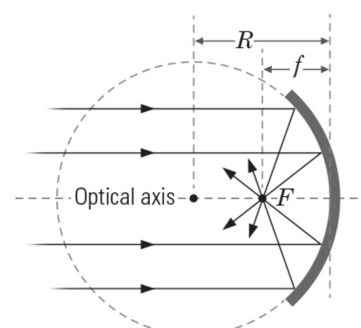
[4]www.pasco.com/ap32 — PASCO Concave Mirror Accessory

Background



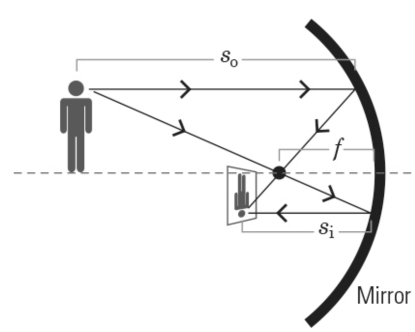
Light rays reflecting from a mirrored surface obey the Law of Reflection: the incident angle θ_i of a reflected light ray is equal to the reflected angle θ_r, where both angles are measured relative to a line normal to the reflecting surface.

$$\theta_i = \theta_r \qquad (1)$$



In the case of a concave spherical mirror, the surface of the mirror is curved in the shape of a section of a sphere with radius R. (R is also known as the mirror's *radius of curvature*). Because of this curvature, light rays incident on a concave mirror, parallel to its optical axis, have incident angles that increase as the distance between the optical axis and the light ray increases. These varying incident angles cause the parallel light rays to reflect from the mirror and converge to one point F along the optical axis known as the *focal point*. The distance f from the mirror's surface to the focal point is known as the *focal length* and is equal to half of the mirror's radius of curvature:

$$f = \frac{R}{2} \qquad (2)$$



If an object is introduced in front of a concave spherical mirror along its optical axis, beyond the mirror's focal length, an image of that object will form in front of the mirror. This image is said to be a *real image* because it forms where light rays converge onto a viewing screen. Images that are formed by diverging light rays (images that cannot be formed on a viewing screen) are said to be *virtual images*.

The following spherical mirror equation defines the position of the real image formed by a spherical mirror:

$$\frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i} \tag{3}$$

where f is the mirror focal length, s_o is the distance from the mirror to an object (*object distance*), and s_i is the distance from the mirror to the point at which the image of the object is in focus (*image distance*). Using the variables in this equation, you will perform an investigation to experimentally determine the radius of curvature of a concave spherical mirror.

RELEVANT EQUATIONS

$$f = \frac{R}{2} \tag{2}$$

$$\frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i} \tag{3}$$

Procedure

SET UP

1. Lay the optics track flat on your lab table and mount the light source to it so that the "screen zero" (see the bottom of the light source for the screen zero indicator) is aligned with the 6-cm mark on the track. Make sure the crossed-arrow image on the light source points down the length of the track.

2. Mount the mirror to the track at the 20-cm mark with the concave surface of the mirror facing the light source.

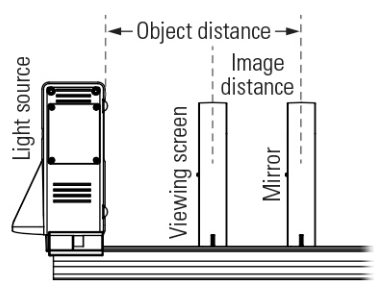
3. Mount the viewing screen to the track at some point between the light source and the mirror.

4. Plug in the light source to turn it on.

COLLECT DATA

5. Slide the viewing screen up and down the optics track between the mirror and the light source until the image of the crossed-arrow target is in focus on the half-screen.

   NOTE: Depending on your setup, you may not see the entire image on the viewing screen. Also, the image may not be in perfect focus; however, the image location *is where the image is most focused. You may need to slide the screen back and forth through the image location until you can determine where the image is the most sharply focused.*



6. Using the graduated scale on the optics track, determine the object distance s_o and the image distance s_i. Record these values in Table 1 in the Data Analysis section below.

   NOTE: Object distance is measured from the position of the mirror to the front of the light source (the object), and image distance is measured from the position of the mirror to the position of the viewing screen.

7. Slide the mirror 3 cm farther from the light source. (Do not change the light source position.)

8. Slide the viewing screen up and down the optics track between the mirror and the light source until the image of the crossed-arrow target is again in focus on the screen.

9. Record the new object distance and image distance next to Trial 2 in Table 1.

10. Repeat the data collection steps three more times, increasing the distance between the mirror and the light source by 3 cm in each trial. Record the object distance and corresponding image distance for each trial into Table 1.

Data Analysis

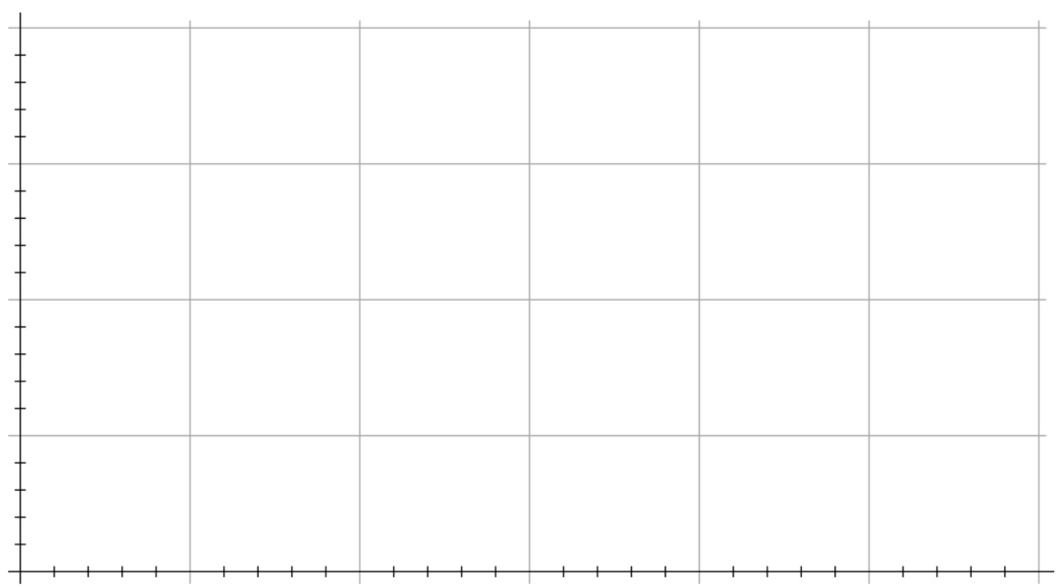
Table 1: Object distance and corresponding image distance using a concave spherical mirror

Trial	Object Distance s_o (m)	Image Distance s_i (m)	$1/s_o$ (m^{-1})	$1/s_i$ (m^{-1})
1				
2				
3				
4				
5				

1. Calculate the inverse of each object distance and image distance in Table 1. Record your results in the $1/s_o$ and $1/s_i$ columns of Table 1.

2. Plot a graph of $1/s_o$ versus $1/s_i$ in Graph 1. Be sure to label both axes with the correct scale and units.

Graph 1: Inverse object distance versus inverse image distance using a concave spherical mirror



3. Draw a line of best fit through your data in Graph 1. Determine and record the equation of the line here:

   Best fit line equation: ______________________________

4. Use the y-intercept from the best fit line to determine an experimental value for the radius of curvature R of your mirror:

$$y\text{-intercept} = \frac{2}{R}$$

Radius of Curvature R (m): ______________________

Analysis Questions

1. What is your experimental value for the radius of curvature R of your mirror, and how did you determine this value from your data?

2. What are factors that might have caused error in your measured value for radius of curvature? Explain how each factor you list could be avoided or minimized.

3. Ask your teacher for the actual value of the radius of curvature of your mirror, and then calculate the percent error between your experimental value and the actual value.

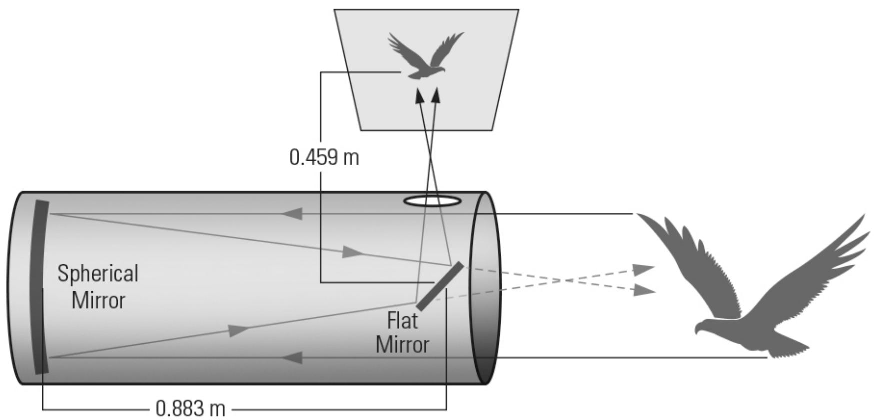
$$\text{Percent error} = \left|\frac{\text{Actual} - \text{Experimental}}{\text{Actual}}\right| \times 100$$

4. What do you predict happens to the image distance from a concave spherical mirror as the object distance grows very large (much larger than the image distance)? Justify your answer: use mathematical reasoning or data from your experiment, or both, to support your answer.

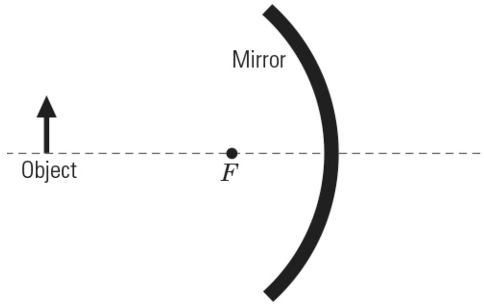
Synthesis Questions

1. In 214 BC, Archimedes invented a large spherical-type mirror used to focus the sun's intense rays onto far away enemy boats, which would eventually light them on fire. If the boats were travelling in a nearby channel approximately 1,000 m from the river bank, what would the radius of curvature of his mirror need to be? Show your work.

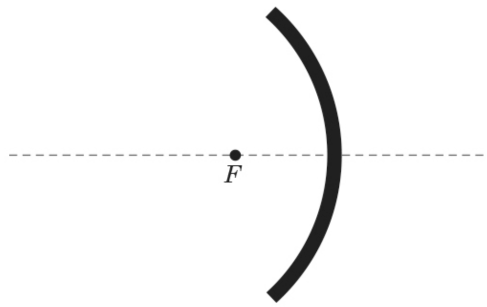
2. The image below shows the position of two mirrors inside a reflector-style telescope. Light from far-off objects enters the body of the telescope through the opening on the right. That light is reflected off a spherical mirror in the back of the telescope and then again off a flat mirror that redirects the light to a screen where the image of the object appears in focus. Given that the image of the eagle on the screen is in focus, and the actual eagle is 726 m away from the spherical mirror, what is the radius of curvature of the spherical mirror?



❓ 3. Sketch a ray diagram showing the image produced by the concave spherical mirror below. Is the image real or virtual? How do you know?



❓ 4. Is it possible for a concave spherical mirror to produce a virtual image of an object? Sketch a ray diagram that supports your answer.



❓ 5. In most applications of concave spherical mirrors (including this activity) it is assumed that all light rays travelling parallel to the mirror's optical axis are reflected through the mirror's focal point; however, this is an approximation that applies only to light rays traveling near the mirror's optical axis. In a few sentences, explain why a concave spherical mirror does not reflect all light rays travelling parallel to its optical axis through its focal point.

6. SNELL'S LAW

Connections to the AP® Physics 2 Curriculum*

The lab activity correlates to the following pieces of the AP® Physics 2 framework:

Big Idea	6	Enduring Understanding	E	Essential Knowledge	3

Learning Objective 2: The student is able to plan data collection strategies as well as perform data analysis and evaluation of the evidence for finding the relationship between the angle on incidence and the angle of refraction for light crossing boundaries from one transparent material to another (Snell's law).
Science Practices: 4.1, 5.1, 5.2, 5.3

Learning Objective 3: The student is able to make claims and predictions about path changes for light travelling across a boundary from one transparent material to another at non-normal angles resulting from changes in the speed of propagation.
Science Practices: 6.4, 7.2

Time Requirement

Preparation Time: 10 minutes

Lab Activity: 30 minutes

Prerequisites

Students should be familiar with the following concepts:

- The index of refraction n of a transparent material is defined as the ratio of the speed of light in a vacuum c and the speed of light inside the transparent material v:

$$n = \frac{c}{v} \tag{2}$$

- Light crossing a boundary between two transparent materials will experience a direction change if the speed of light within those media is different. This is known as *refraction*, and can be quantified using the equation known as Snell's law:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \tag{1}$$

Driving Question | Objective

What is the index of refraction of a transparent material provided by your instructor? Using the principles of refraction and Snell's law, experimentally determine the index of refraction of a transparent medium.

Procedural Overview

In the Structured version of this lab activity, students shine a single light ray through a transparent material with an unknown index of refraction at five oblique incident angles while measuring the refraction angle as the light ray crosses the air–material boundary. Students calculate the sine of each incident angle θ_1 and the sine of each refraction angle θ_2 and plot a graph of sin θ_1 versus sin θ_2. Students then apply a line of best fit to their graph and use the slope of this line to determine an experimental value for the index of refraction of the transparent material:

$$\text{slope} = n_2/n_1$$

where n_2 is the index of refraction of the transparent material and n_1 is the index of refraction of air. For the purpose of this activity, students assume that the index of refraction of air is effectively equal to 1.00.

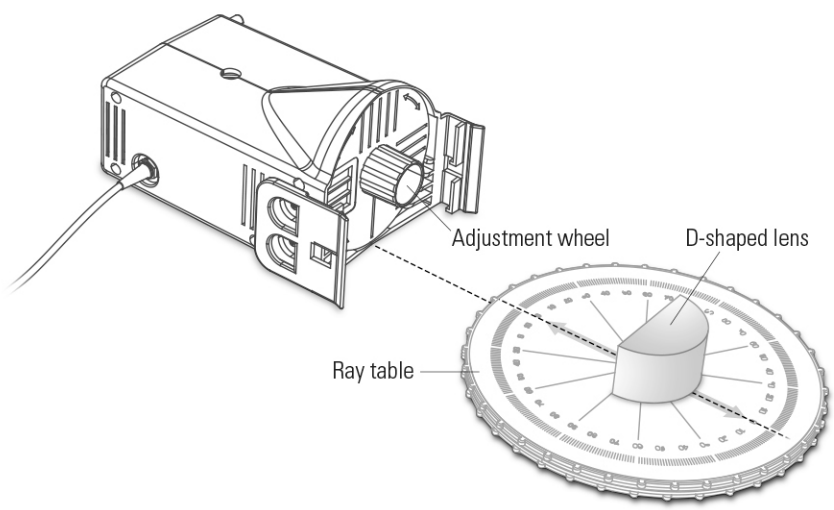
Pre-Lab Discussion and Activity

Because this lab activity is designed to be an application of Snell's law rather than a discovery-based investigation of refraction, it is recommended that the Guided Inquiry or Student Designed versions of this activity follow or conclude an in-class discussion introducing the topics of refraction and Snell's law. A sample pre-lab discussion and demonstration is described below.

The Structured version of this activity can be offered to students without a preceding in-class discussion as long as emphasis is placed on the background information outlined in the activity handout. The background section in the Structured version of this activity includes a brief explanation of refraction and Snell's law that will provide the student enough insight and information to successfully address the activity's objective and answer all analysis and synthesis questions in the lab activity.

Sample Discussion and Demonstration

Use the PASCO optics equipment listed in the Materials and Equipment section below to set up and perform the following discussion and demonstration. Assemble the light source, lens, and ray table as in the diagram. Set the D-shaped lens in the marked outline on the ray table with the frosted side of the lens down, against the ray table. Use a document camera or similar projection device above the setup to project an overhead view to the class.

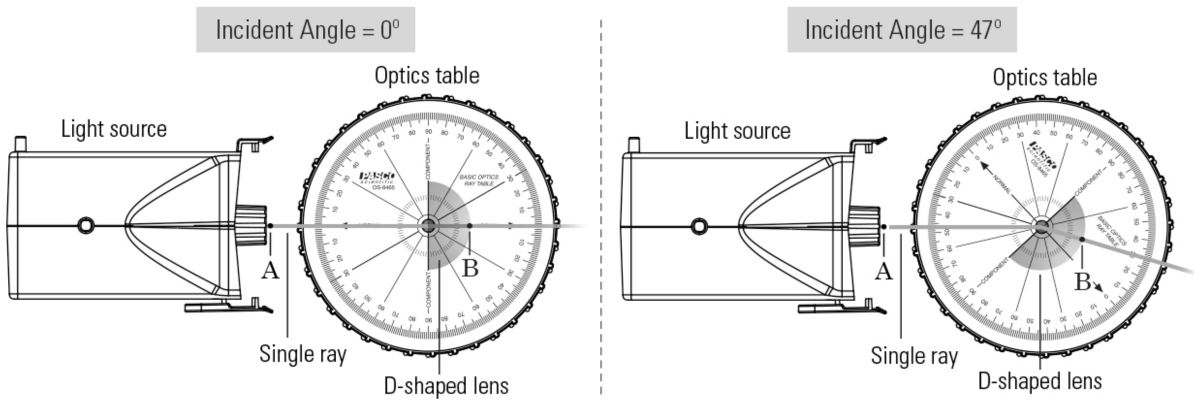


Plug in the light source to turn it on, and then turn the adjustment wheel on the front of the light source so that a single light ray is emitted. You may need to lower the room lights to make the light ray visible.

Rotate the ray table so the light ray enters the lens at the center of its flat surface with an incident angle of 0° (perpendicular). The path of the light ray should be a straight line through the lens.

Begin by explaining Fermat's principle to the class: Light traveling between two points always follows the path of least time. This implies that light traveling between two points in the same medium will always assume a straight line path. Students may correlate this to the common adage, "The shortest distance between two points is a straight line," which sounds similar, but can be dangerous as Fermat's Principle addresses time regardless of distance.

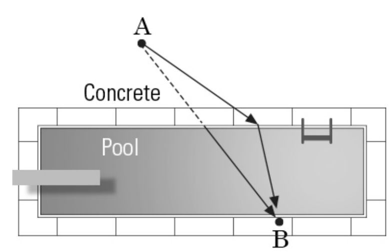
Have students observe your setup. Define two points between which the light ray travels: point A being the point at which the light ray is emitted from the light source, and point B being the point at which the light ray strikes the back (curved surface) of the lens.



Ask students to pay close attention to the light ray as you rotate the top of the ray table (rotating the top of the ray table also rotates the lens) approximately 47°. Use the degree scale on the ray table as your guide, and be sure that the light ray is incident on the center of the lens's flat surface at all times. Students should observe the light ray bending (refracting) as you rotate the ray table. At an incident angle of 46°, the light ray should align with the 30° mark on the opposite side of the lens.

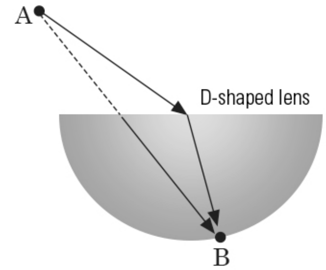
Have students observe your adjusted setup. Redefine point B as the new point at which the light ray strikes the back (curved surface) of the lens. Ask students, "Why do you think the ray no longer travels in a straight line to point B?" Guide students with Socratic questioning centered on the idea that the light ray will always follow the path of least time between points A and B, and that the velocity of the light ray must be different in the lens than it is in air to facilitate this.

To help solidify this idea, draw the (bird's eye view) diagram at right on the board without any of the lines. Ask students to imagine a large pool surrounded by concrete. At point A is a person who needs to get to point B on the other side of the pool. Assuming that the person can walk on the concrete about 50% faster than they can swim, what path will get them there in the least time? Draw the straight line segment between points A and B and ask if the path should be straight.

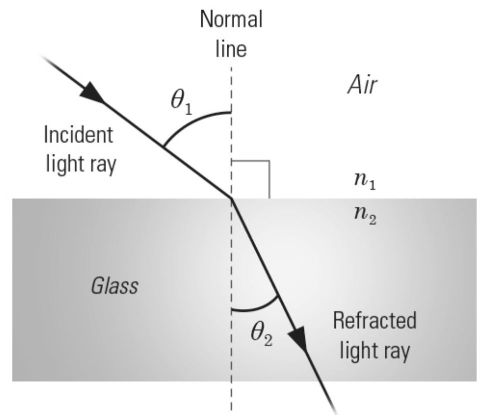


Again, guide students with Socratic questioning centered on the idea that the person is attempting to find the path of least time between points A and B. Because the person is faster on concrete than in water, they are trying to maximize their path on the concrete and minimize their path in the water so that the least amount of overall time is required. The result is the bent two-segment path shown in the diagram. Draw the two-segment path on the board and indicate the smaller in-water line segment in this path compared to the first.

Correlate this analogy to the lens demonstration by drawing the diagram at right showing the D-shaped lens and both light paths: a straight path, and the bent path observed in the demonstration. Explain that a straight line path between points A and B requires the light ray to travel a greater distance through the lens material than a bent path. If the speed of the light is slower in the lens than in air, the result is a longer travel time for the straight path. The light ray follows the bent path to minimize the travel time from point A to point B. This bending behavior of light is known as *refraction* and it occurs when light experiences a change in speed as it crosses a boundary between two transparent materials.



The amount of refraction experienced by light as it crosses a boundary between two transparent materials is dependent on the angle at which the light is incident upon the boundary and the speed of light in each material. The formula relating these quantities is known as Snell's law:



$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \qquad (1)$$

where θ_1 is the incident angle the incoming light ray makes relative to the normal line at the boundary, θ_2 is the angle the refracted light ray makes relative to the normal line at the boundary, and n_1 and n_2 are the indices of refraction for the two materials. The *index of refraction n* of a transparent material is a unitless quantity defined as the ratio of the speed of light in a vacuum c ($c = 3.00 \times 10^8$ m/s) to the speed of light in the material v:

$$n = \frac{c}{v} \qquad (2)$$

PRE-LAB QUESTIONS

❓ 1. In a few complete sentences, explain what refraction is as it applies to light.

Suitable responses must include:

- Refraction is the bending behavior of light as it travels across the boundary between transparent materials.
- Refraction is the result of the change in light speed as the light travels across the boundary between transparent materials.

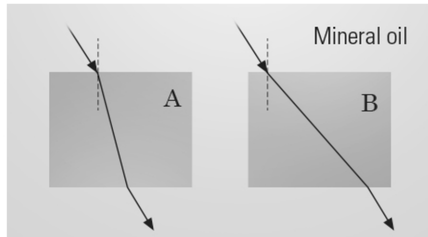
Sample response:

Refraction describes how light bends as it crosses a boundary between two transparent materials. The bending behavior is the result of the light traveling at a different speed in each material. According to Fermat, light will always follow the path of least time between two points. If the speed of light is different in two different transparent materials, light will follow a bent path to minimize travel time through the materials.

❓ 2. If the speed of light is 3.00×10^8 m/s in a vacuum, and 2.25×10^8 m/s in water, what is the index of refraction of water n_{water}?

$$n_{\text{water}} = \frac{c}{v_{\text{water}}} = \frac{3.00 \times 10^8 \text{ m/s}}{2.25 \times 10^8 \text{ m/s}} = 1.33$$

3. Two identically-shaped blocks of transparent material are submerged in mineral oil. The two materials, A and B, have different indices of refraction. Two identical light rays travel through the oil and through each block. Both light rays are incident on the surface of each material at the same angle. Based on the diagram below, in which material is the speed of light faster? Justify your answer.



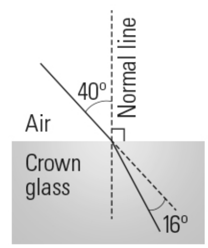
The speed of light in material B is faster than in material A. Students may choose to justify their answer using the arguments:

Fermat's principle: The ray traveling through material B travels farther within the material than the ray traveling through material A, even though both rays traverse the same vertical distance. According to Fermat's principle, this is only possible if the speed of light is greater in material B than material A.

Snell's law: Since $n_1 \sin\theta_1 = n_2 \sin\theta_2$ then $\frac{v_2}{v_1} = \frac{\sin\theta_2}{\sin\theta_1}$ which implies that if θ_2 is less than θ_1, then v_2 is less than v_1, and vice versa.

For material A, θ_2 is less than θ_1, therefore v_2 is less than v_1. For material B, θ_2 is greater than θ_1, therefore v_2 is greater than v_1. If the speed of light in material A is less than the speed of light in mineral oil, and the speed of light in material B is greater than the speed of light in mineral oil, then the speed of light in material A is less than the speed of light in material B.

4. Shown to the right is a ray of light traveling through air and incident on the surface of a flat piece of crown glass. The incident light ray forms an angle of 40° relative to the normal line at the surface. If the path of the light ray diverges by 16° toward the normal line when it enters the glass, what is the index of refraction of the glass? Assume that $n_{air} = 1.00$.



Rearranging Snell's law:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$n_2 = n_1 \frac{\sin\theta_1}{\sin\theta_2}$$

where n_1 is the index or refraction of air, θ_1 is the incident angle of the incoming light ray relative to the normal line, and θ_2 is the angle between the refracted ray and the normal line. Because the original path of the light ray bisects the line formed by the surface of the glass, we can infer that $\theta_2 = 40° - 16°$.

$$n_2 = 1.00 \frac{\sin 40°}{\sin 24°} = 1.58$$

Materials and Equipment

It is recommended that students use the D-shaped acrylic lens that comes with the PASCO Basic Optics Ray Table as their "transparent material with unknown index of refraction" in this activity. The lens is D-shaped to facilitate easy and accurate measurements of refraction angle. However, this lab activity can be done using any transparent material as long as that material has flat uniform sides. This includes any cubes, prisms, or rectangular prisms (cuboids). Refer to Teacher Tip 1 below for instructions on measuring incident and refraction angles using these shapes.

- PASCO Basic Optics Ray Table[1]
- Transparent material with unknown index of refraction (D-shaped acrylic lens)[1]
- PASCO Basic Optics Light Source[2]

Probeware Resources

Below are web-link and QR codes that will direct you to instructional video resources for individual pieces of PASCO probeware, sensors, and other hardware used in the lab activity. These same links and codes are provided to students in their activity handouts.

[1] www.pasco.com/ap27

PASCO Basic Optics Ray Table

[2] www.pasco.com/ap26

PASCO Basic Optics Light Source

Teacher Tips

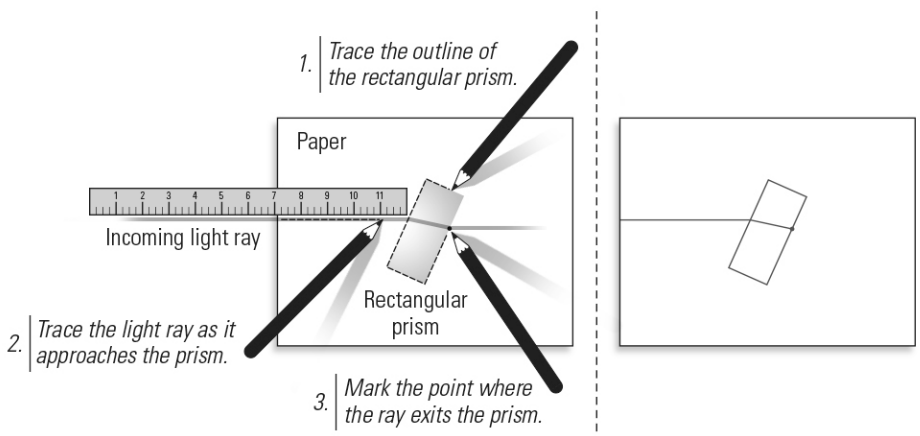
Tip 1 – Using Odd-Shaped Transparent Materials

- It is recommended that students use the D-shaped acrylic lens that comes with the PASCO Basic Optics Ray Table as their "transparent material with unknown index of refraction" in this activity. The lens is D-shaped to facilitate easy and accurate measurements of refraction angle. However, this lab activity can be done using any transparent material as long as that material has flat uniform sides. This includes any cubes, prisms, or rectangular prisms (cuboids). The following steps are recommended for measuring the incident and refraction angle using these shapes:

  1. Place the shape on a piece of white paper with one flat side down and another flat side (face) perpendicular to the paper, and then use a pencil to trace the outline of the shape onto the paper.

  2. Place the light source in front of the shape with a single light ray incident on the shape's face.

  3. Use a pencil and straight-edge to trace the light ray on the paper as it approaches and meets the shape, and then make a small mark on the paper indicating where the ray exits the shape.



  4. Remove the shape from the paper and use the straight-edge and pencil to connect the point at which the incoming light ray met the shape to the point where it exited the shape.

  5. Finally, use a protractor to measure the incident and refraction angle as the light ray entered the shape.

Tip 2 – Using the D-shaped Lens

- The lens that comes with the PASCO Basic Optics Ray Table is D-shaped to facilitate easy and accurate measurements of the refraction angle. It is imperative that the incident ray strike the center of the flat side of the D-lens; otherwise, the measured incident and refracted angles will be inaccurate. Students can use the markings on the ray table as a guide: place the lens in the center of the ray table within the D-shaped outline. A small mark in the outline indicates the center of the flat surface.

Sample Data

Below are sample data, acquired using the experimental setup and procedure outlined in the Structured version of the lab activity, and answers to questions in the Data Analysis section.

Data Analysis

Table 1: Incident and refraction angles of a light ray crossing from air into an unknown transparent medium

Trial	Incident Angle θ_1 (°)	Refraction Angle θ_2 (°)	sin θ_1	sin θ_2
1	10.0	7.0	0.174	0.12
2	25.0	16.5	0.423	0.284
3	40.0	25.5	0.643	0.431
4	55.0	33.5	0.819	0.552
5	70.0	39.5	0.940	0.636

1. Calculate the sine of the incident angle θ_1 and refraction angle θ_2 for each trial. Record your results into Table 1.

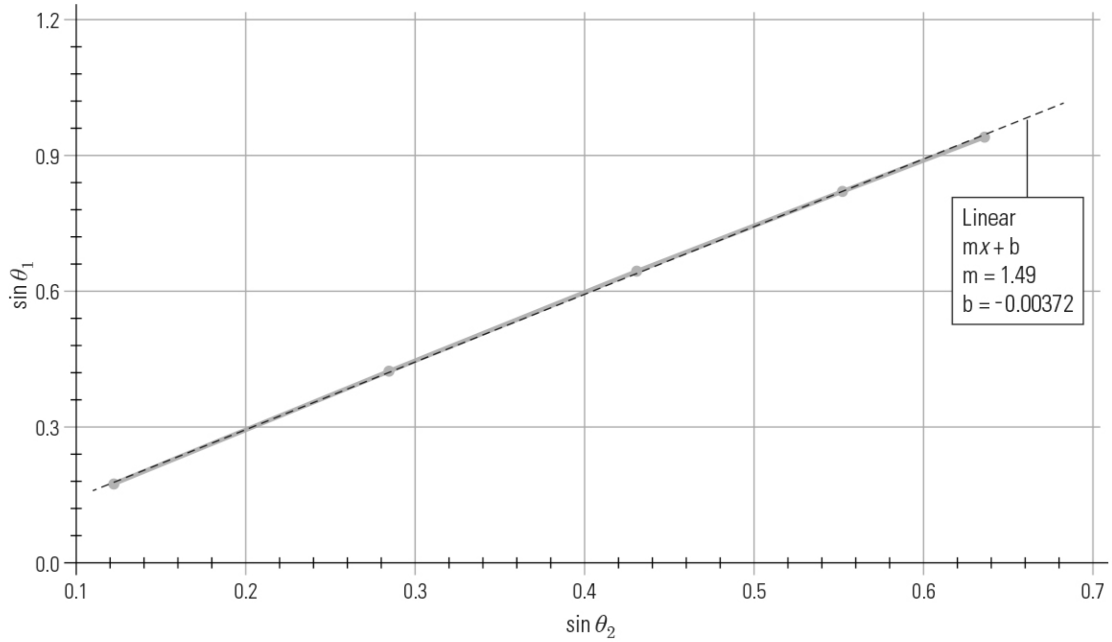
   Calculation using sample data for Trial 1:

   $\sin\theta_1 = \sin 10.0° = 0.174$

   $\sin\theta_2 = \sin 7.0° = 0.12$

2. Plot a graph of *sin* θ_1 versus *sin* θ_2 in the blank Graph 1 axes below. Be sure to label both axes with the correct scale and units (if any).

Graph 1: Sine of incident angle versus sine of refraction angle for a light ray crossing from air into an unknown transparent medium



3. Draw a line of best fit through your data in Graph 1. Determine and record the equation of the line here:

   Best fit line equation: $\sin\theta_1 = 1.49\sin\theta_2 - 0.00$

4. Use the slope from the best fit line to determine an experimental value for the index of refraction n_2 of the D-shaped transparent material:

   $\text{slope} = \frac{n_2}{n_1}$ where n_1 = 1.00 (index of refraction for air)

   Index of refraction n_2: 1.49

Calculation using sample data from Graph 1:

$$\text{slope} = \frac{n_2}{n_1}$$

$$n_2 = n_1 \times \text{slope} = (1.00)(1.49) = 1.49$$

Guided Inquiry Questions

Below are sample responses to the Guiding Questions found in the Guided Inquiry version of this lab activity.

❷ 1. Snell's law defines the behavior of light as it passes from one transparent material to a second transparent material:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad (1)$$

What does each variable in Snell's law represent?

Snell's law has four variables: n_1, n_2, θ_1, and θ_2.

n_1 is the index of refraction of the first transparent material.

n_2 is the index of refraction of the second transparent material.

θ_1 is the angle formed in the first material between the incoming light ray and the line normal to the surface of the boundary between the two materials.

θ_2 is the angle formed in the second material between the bent light ray and the line normal to the surface of the boundary between the two materials.

❷ 2. What variables in Snell's law can be measured directly using the measuring tools available to you? What variables will be part of your experiment and why?

Angles θ_1 and θ_2 in Snell's law can be measured directly using spatial measurement tools such as rulers and protractors. The index of refraction for either transparent material cannot be measured directly and requires some form of indirect measurement or measurement strategy.

Students should identify the angles θ_1 and θ_2 as the variables in their experiment, as these are the only variables in the expression that can be measured directly. Also, it can be assumed that both indices of refraction in the expression are constant.

❷ 3. What tools will you use to measure each variable listed in your previous answer and how would you use them to make measurements?

Students can use any spatial measurement tool (or combination of tools) to measure both θ_1 and θ_2 angles, such as a ruler, meter stick, protractor, compass, and PASCO Basic Optics Ray Table. Refer to the procedure outlined in the Structured version of this lab activity or Teacher Tip 1 for instructions on how to use these tools to make accurate measurements of each angle.

The exact use of these tools depends on the shape of the transparent material being used, as this may affect where and how students define the normal line at the boundary between transparent materials in their experiment.

❷ 4. How will you assemble the equipment in your experiment so that these variables can be measured accurately? What special setup requirements are necessary and why?

Assembly may differ between groups. Refer to the Set Up section outlined in the Structured version of this lab activity or Teacher Tip 1 for instructions on how to assemble equipment to make accurate measurements of either angle.

Students should assemble their equipment to take advantage of geometric simplicity. That will help make measurements as accurate as possible. Students should avoid using any oblique or rounded sides of the transparent material in the experiment.

Students using the D-shaped lens that is part of the PASCO Basic Optics Ray Table should make special note that the shape of the lens simplifies the measurement or refraction angle by having one flat and one cylindrical surface. Light incident at a non-zero angle on the center of the flat surface refracts as it enters the lens, but will not experience refraction as it exits the lens through its cylindrical surface. This simplification requires that the light strike the center of the flat surface.

❷ 5. How will you organize your data so that an experimental value for refractive index can be determined from it? Will you use a graphical method, tabular, or other? Explain your method and why you chose it.

It is recommended that students choose a graphical method that employs linearization techniques to analyze their data, as this method is prevalent in AP Physics exams. Assuming that students choose a setup and procedure similar to that outlined in the Structured version of this activity and plot sin θ_1 versus sin θ_2, they should put the dependent variable, sin θ_2, on the *x*-axis and independent variable, sin θ_1, on the *y*-axis of the graph so the slope equals n_2/n_1. From the slope values, students can solve for the index of refraction of their material.

6. Do you think it is better to make one measurement of each variable and then use that one value to determine the index of refraction of your material, or make several measurements using different values for your independent variable? Explain your reasoning.

   Students will likely conclude that if their experiment is based on one measurement, it is likely that there will be error that can cause their results to be inaccurate or less representative. Making several measurements of the dependent variable based on a good variance of the independent variable will provide an averaged and more accurate (representative) value for θ_1 and θ_2.

Assessment Questions: Sample Responses

Sample responses to the Analysis and Synthesis questions found in each version of the lab activity:

Analysis Questions

1. What is your experimental value for the index of refraction of your transparent material? How did you determine this value from your data?

   Based on the data shown in the Sample Data section above, the index of refraction of the unknown transparent material is 1.49.

   Students performing the Student Designed or Guided Inquiry version of this activity will acquire values that vary between groups based on the measurement tools used and the procedural strategies chosen. Students performing any of the three versions of this activity may also show variability in their values between groups due to variability in the transparent material(s) being used.

   The D-shaped lens that comes with the PASCO Basic Optics Ray Table is made from cast clear acrylic with a published index of refraction of 1.491. The sample data shown above was acquired using this lens.

   Students following the procedure outlined in the Structured version of this lab activity measure the refraction angle θ_2 formed by a single ray of light incident on the D-shaped lens at five different incident angles θ_1. This data is used to plot a graph of sin θ_1 (on the y-axis) versus sin θ_2 (on the x-axis). The shape of student graphs should be linear with a slope equal to n_2/n_1 where n_1 is equal to the index of refraction of air (n_{air} = 1.00) From this, students determine an experimental value for n_2.

2. Below is a list of refractive indices for common materials. Use this table and your experimental value for the index of refraction to determine a potential candidate for your transparent material. Calculate the percent error between your experimental value and the index of refraction value of your candidate.

Material	Index of Refraction
Quartz	1.41
Acrylic glass	1.49
Polycarbonate	1.58
Dense crown glass	1.67
Diamond	2.42

$$\text{Percent error} = \left|\frac{\text{Theoretical} - \text{Experimental}}{\text{Theoretical}}\right| \times 100$$

   Students should choose a candidate whose index of refraction value in the table most closely matches their experimental value. Based on the data shown in the Sample Data section above, the best candidate for the unknown transparent material used is acrylic glass.

   Sample calculation:

$$\text{Percent error} = \left|\frac{1.49 - 1.49}{1.49}\right| \times 100 = 0.00\%$$

3. Find another lab group that tested the same transparent material. Calculate the percent difference between your experimental value and their experimental value of the index of refraction.

$$\text{Percent difference} = \frac{\left|\text{Group}_2 - \text{Group}_1\right|}{\left|\left(\text{Group}_1 + \text{Group}_2\right)/2\right|} \times 100$$

   Sample calculation:

$$\text{Percent difference} = \frac{\left|1.53 - 1.49\right|}{\left|\left(1.49 + 1.53\right)/2\right|} \times 100 = 2.65\%$$

❓ 4. What are factors that might have caused error in your measured value of index of refraction?

Some factors may include, but are not limited to:

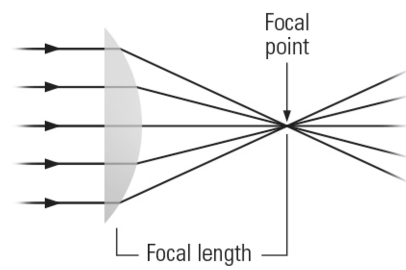
- Random error associated with the measurement tools used
- The index of refraction for air is not exactly equal to 1.00, which affects the calculated value
- The ray of light was too thick, which made it difficult to estimate the exact incident and refraction angle
- When using the D-shaped lens, the incident ray of light didn't strike the very center of the lens's flat side, which caused the refraction angle to be inaccurate

Synthesis Questions

❓ 1. A solid piece of clear transparent material has an index of refraction of 1.61. If you place it into a clear transparent solution and it seems to disappear, approximately what is the index of refraction of the solution? How do you know?

If the material has an index of refraction of 1.61, the transparent solution also has an index of refraction of 1.61. Refraction occurs as light travels from one transparent material to another with a different index of refraction. Refraction and reflection of light are the reasons we see objects. Opaque objects are seen because light is reflected off of them, and transparent objects are seen because light is refracted as it passes through them. If a transparent material seems to disappear when surrounded by another transparent material, it is because light is not refracted as it travels between them. This can only happen if the index of refraction of both materials is the same.

❓ 2. Some lenses are shaped with one flat side and one spherically-shaped side. This shape is designed to focus parallel light rays onto a single point. In a few sentences, explain how the spherical shape of the lens' surfaces causes parallel light rays to focus on a single point. (Assume the light is travelling through air into a lens with an index of refraction greater than that of air.)



Parallel light rays striking a spherical surface will have different incident angles dependent on the distance above and below the center of the surface: zero at the center of the surface, and increasing as the distance above and below the center increases. According to Snell's law, as the incident angle increases, so does the refraction angle (if $n_1 < n_2$). Therefore, light rays incident on the lens's center will travel through the lens with their path unaltered, and rays above and below the center will have increasing refraction angles, redirecting their paths to the lens's focal point.

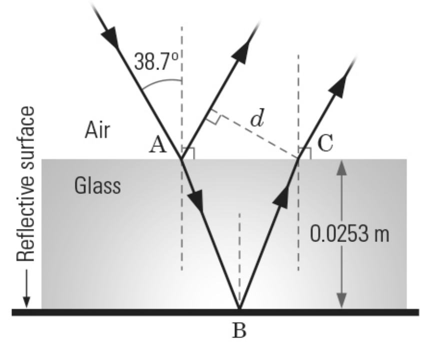
❓ 3. A ray of light travels from glass to air with an incident angle of 37° from the normal. What is the refraction angle? Assume $n_{glass} = 1.50$ and $n_{air} = 1.00$.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1$$

$$\theta_2 = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right) = \sin^{-1}\left(\frac{1.50}{1.00}\sin(37°)\right) = 65°$$

4. A laser beam is incident at an angle of 38.7° on a 0.0253 m thick piece of glass with a fully reflective coating on its bottom surface. Part of the laser beam is reflected off the top surface of the glass at point A, and part is transmitted through the glass to point B where it is reflected and sent out of the glass at point C. If both surfaces of the glass are flat and parallel, what is the perpendicular distance d between the two outgoing beams? Assume $n_{glass} = 1.50$ and $n_{air} = 1.00$.



Using Snell's law and the law of reflection, it can be shown that the angle between the line perpendicular from the first reflected ray to point C and the surface of the glass between points A and C is equal to the incident angle of the original incoming light ray.

For the purpose of this solution, the thickness of the glass is defined as h, and the distance between points A and C along the surface of the glass is equal to $2a$. Given those definitions, the following equations can be established:

$$\cos\theta_1 = \frac{d}{2a} \quad \text{and} \quad \tan\theta_2 = \frac{a}{h}$$

$$d = 2a\cos\theta_1 \qquad a = h\tan\theta_2$$

Combining the two equations yields:

$$d = 2h\tan\theta_2\cos\theta_1$$

Use Snell's law to solve for θ_2, and insert the result into the equation for d:

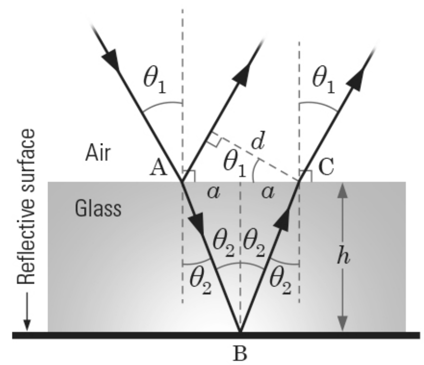
$$n_1\sin\theta_1 = n_2\sin\theta_2$$

$$\theta_2 = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right)$$

$$d = 2h\tan\left(\sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right)\right)\cos\theta_1$$

$$d = 2(0.0253\text{ m})\tan\left(\sin^{-1}\left(\frac{1.00}{1.50}\sin(38.7°)\right)\right)\cos(38.7°)$$

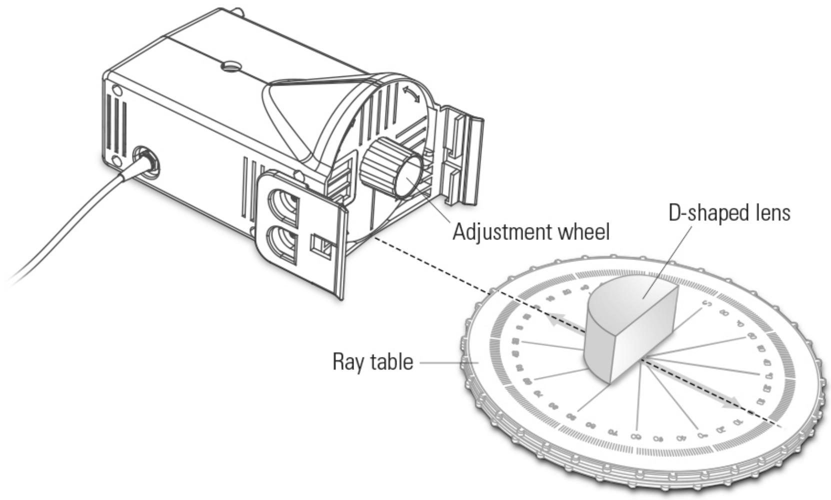
$$d = (0.0506\text{ m})\tan\left(\sin^{-1}(0.417)\right)(0.780) = 0.0181\text{ m}$$



Extended Inquiry Suggestions

A natural extension to this activity is a laboratory exploration and challenge involving total internal reflection. Begin with a short demonstration:

Assemble the PASCO Basic Optics Ray Table, PASCO Basic Optics Light Source, and D-shaped lens like the picture below. Set the D-shaped lens in the marked outline on the ray table with the frosted side of the lens down. Use a document camera or similar projection device above the setup to project an overhead view to the class.



Plug in the light source to turn it on, and then turn the adjustment wheel on the front of the light source so that a single light ray is emitted. You will need to lower the room lights to make the light ray more visible. Rotate the ray table so the light ray enters the lens at the center of its curved surface with an incident angle of 0° and passes through the center of the lens's flat surface. The path of the light ray should be a straight line through the lens.

Have students observe the light ray as it enters the lens through its curved surface and exits through its flat surface. Ask students to identify the incident angle of the light ray as it enters the lens's curved surface. Students should recognize that the incident angle is zero for two reasons: first, the light ray experiences zero refraction as it enters the lens, but more importantly, because the shape of the lens is cylindrical, the incident angle is always zero if the light ray enters through the lens's cylindrical surface and passes through the center of the lens's flat surface.

Rotate the ray table 20° and have students observe how the light ray still does not refract as it enters the lens, but now it experiences refraction as it leaves the lens due to the non-zero incident angle at the lens–air boundary. Students should note that the refraction angle as the light leaves the lens is greater than the incident angle, unlike what they observed in the lab activity where the incident angle was greater than the refraction angle.

Ask students to explain why this is different. Ideally students will realize that in their experiment, light travelled from air to their unknown transparent material, which assumedly had a higher index of refraction than air. In this demonstration, the light travels from the transparent material to air; from a material with high index of refraction to a material with lower index of refraction.

Have students also observe the faint light ray reflected from the lens's flat surface. Explain to students that although their focus in the Snell's law lab activity was on how light refracts at a boundary between two transparent materials, some amount of the incident light also reflects from the boundary. This is due to the wave nature of light.

While students pay close attention to the refracted light ray, continue to slowly rotate the ray table, increasing the incident angle. Keep rotating until the refracted light ray disappears and the entire ray is reflected back through the lens (at an incident angle approximately equal to 45°). Stop rotating when the incident angle is approximately 50°. Indicate to students that at some point, the incident angle reached a threshold where the light stopped crossing the lens–air boundary and was totally reflected back through the lens. Identify this behavior as *total internal reflection*.

Break the class into their lab groups. When students are in their groups, write Snell's law on the board:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \tag{1}$$

Challenge students to work in their lab groups and use Snell's law to derive a mathematical expression for the minimum incident angle above which total internal reflection occurs, also known as the *critical angle* θ_c. Groups should be encouraged to use simple algebraic manipulation and consider the physical constraints necessary for total internal reflection. Allow groups to use their equipment from the Snell's law lab activity to explore these physical constraints.

Derivation of Total Internal Reflection Expression

Students should observe that total internal reflection only occurs when the light travels from a material with a greater index of refraction to a material with a lesser index of refraction ($n_1 > n_2$). The key to deriving the correct expression is to observe that total internal reflection begins when the refraction angle equals 90°:

$$n_1 \sin\theta_c = n_2 \sin(90°)$$

$$\sin\theta_c = \frac{n_2}{n_1}$$

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

Once each group has established their expression, have them use the expression to calculate a theoretical value for the critical angle using the same transparent material from the lab activity. Students should use their experimental values of the index of refraction in their calculations. Have groups test their theoretical values using their lab equipment and compare as a class the accuracy and techniques used by each group.

NAME PERIOD DATE

6. SNELL'S LAW

STRUCTURED

Driving Question | Objective

What is the index of refraction of a transparent material provided by your instructor? Using the principles of refraction and Snell's law, experimentally determine the index of refraction of a transparent medium.

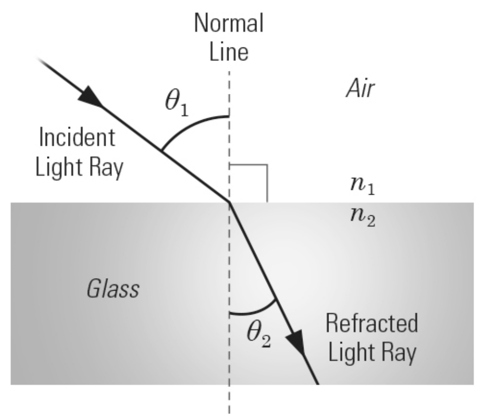
Materials and Equipment

- PASCO Basic Optics Ray Table[1]
- Transparent material with unknown index of refraction
- PASCO Basic Optics Light Source[2]

[1]www.pasco.com/ap27
PASCO Basic Optics Ray Table

[2]www.pasco.com/ap26
PASCO Basic Optics Light Source

Background



Light crossing a boundary between two transparent materials changes direction if the speed of light within those materials is different. This direction change is known as *refraction*.

Light traveling from a material in which it has high speed (like air) to a material in which it has slower speed (like glass) experiences refraction *toward* the normal line perpendicular to the boundary, and light traveling from a material in which it has slower speed to a material in which it has greater speed experiences refraction *away* from the normal line.

The amount of refraction experienced by light as it passes between two transparent materials is dependent on the angle at which the light is incident upon the boundary between the materials and the *index of refraction* of each material. The formula relating these quantities is known as Snell's law:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \tag{1}$$

where n_1 and n_2 are the indices of refraction of the first and second material, θ_1 is the incident angle the incoming light ray makes relative to the normal line, and θ_2 is the angle the refracted light ray makes relative to the normal line.

In this experiment you will employ Snell's law to determine the index of refraction of a D-shaped piece of transparent material. For the purpose of this experiment, assume that the index of refraction of air is effectively equal to 1.00.

RELEVANT EQUATIONS

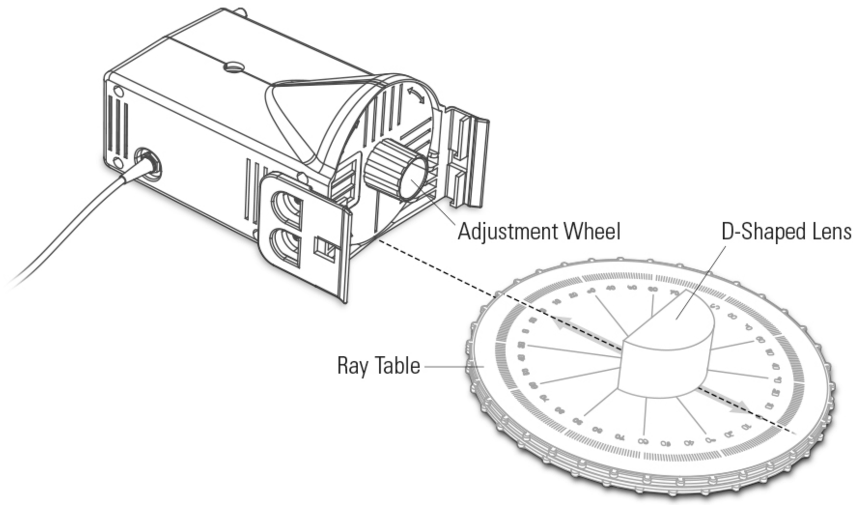
$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \tag{1}$$

Procedure

SET UP

1. Plug in the light source to turn it on, and then turn the wheel on the front of the light source so that a single light ray is emitted. Place the light source flat on the lab table.

2. Place the ray table in front of the light source so it is not more than 10 cm from the light source, and then adjust the position of the ray table so that the single light ray crosses the exact center of the ray table (along the "normal" line).



3. Set the D-shaped lens in the marked outline on the ray table, with the frosted side of the lens down, against the ray table.

4. Rotate the ray table so the light ray enters the lens at the center of its flat surface with an incident angle of 10°. Use the degree scale on the ray table to determine the incident angle.

   NOTE: The light ray refracts as it crosses the boundary from air to the lens material, but it does not *refract as it crosses the boundary from lens to air. This is because the circular shape of the lens causes the incident angle of the light ray at the lens–air boundary to be zero.*

COLLECT DATA

5. Use the degree scale on the ray table to measure the incident and refraction angles. Record both angles in the Trial 1 row of Table 1 in the Data Analysis section below.

6. Rotate the ray table to increase the incident angle by 15° so the light ray enters the lens at the center of its flat surface with an incident angle of 25°.

7. Measure and record the new incident and refraction angles in the Trial 2 row of Table 1.

8. Repeat the data collection steps three additional times, increasing the incident angle by 15° each time. Record the incident and corresponding refraction angles for each trial into Table 1.

Data Analysis

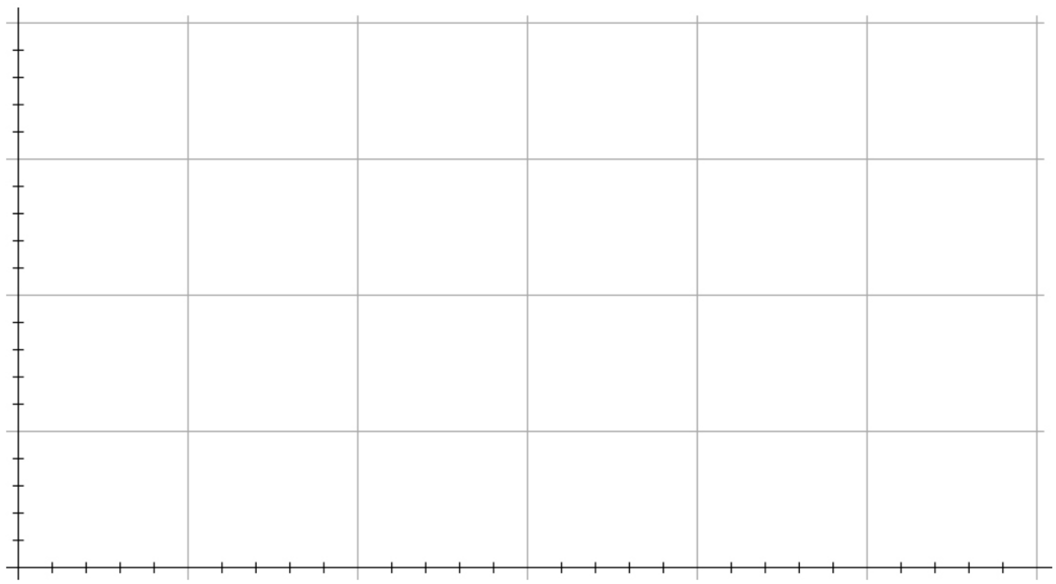
Table 1: Incident and refraction angles of a light ray crossing from air into an unknown transparent medium

Trial	Incident Angle θ_1 (°)	Refraction Angle θ_2 (°)	sin θ_1	sin θ_2
1				
2				
3				
4				
5				

1. Calculate the sine of the incident angle θ_1 and refraction angle θ_2 for each trial. Record your results into Table 1.

2. Plot a graph of *sin* θ_1 versus *sin* θ_2 in the blank Graph 1 axes below. Be sure to label both axes with the correct scale and units (if any).

Graph 1: Sine of incident angle versus sine of refraction angle for a light ray crossing from air into an unknown transparent medium



3. Draw a line of best fit through your data in Graph 1. Determine and record the equation of the line here:

   Best fit line equation: ____

4. Use the slope from the best fit line to determine an experimental value for the index of refraction n_2 of the D-shaped transparent material:

   slope = $\frac{n_2}{n_1}$ where n_1 = 1.00 (index of refraction for air)

   Index of Refraction n_2: ____

Analysis Questions

1. What is your experimental value for the index of refraction of your transparent material? How did you determine this value from your data?

2. Below is a list of refractive indices for common materials. Use this table and your experimental value for the index of refraction to determine a potential candidate for your transparent material. Calculate the percent error between your experimental value and the index of refraction value of your candidate.

Material	Index of Refraction
Quartz	1.41
Acrylic glass	1.49
Polycarbonate	1.58
Dense crown glass	1.67
Diamond	2.42

$$\text{Percent error} = \left|\frac{\text{Theoretical} - \text{Experimental}}{\text{Theoretical}}\right| \times 100$$

3. Find another lab group that tested the same transparent material. Calculate the percent difference between your experimental value and their experimental value of the index of refraction.

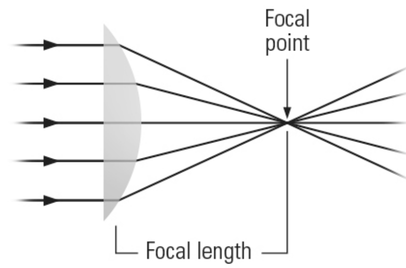
$$\text{Percent difference} = \frac{\left|\text{Group}_2 - \text{Group}_1\right|}{\left|\left(\text{Group}_1 + \text{Group}_2\right)/2\right|} \times 100$$

4. What are factors that might have caused error in your measured value of index of refraction?

Synthesis Questions

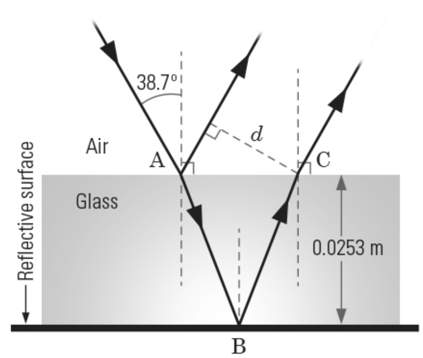
1. A solid piece of clear transparent material has an index of refraction of 1.61. If you place it into a clear transparent solution and it seems to disappear, approximately what is the index of refraction of the solution? How do you know?

2. Some lenses are shaped with one flat side and one spherically-shaped side. This shape is designed to focus parallel light rays onto a single point. In a few sentences, explain how the spherical shape of the lens' surfaces causes parallel light rays to focus on a single point. (Assume the light is travelling through air into a lens with an index of refraction greater than that of air.)



3. A ray of light travels from glass to air with an incident angle of 37° from the normal. What is the refraction angle? Assume $n_{glass} = 1.50$ and $n_{air} = 1.00$.

4. A laser beam is incident at an angle of 38.7° on a 0.0253 m thick piece of glass with a fully reflective coating on its bottom surface. Part of the laser beam is reflected off the top surface of the glass at point A, and part is transmitted through the glass to point B where it is reflected and sent out of the glass at point C. If both surfaces of the glass are flat and parallel, what is the perpendicular distance d between the two outgoing beams? Assume $n_{glass} = 1.50$ and $n_{air} = 1.00$.



7. FOCAL LENGTH OF A CONVERGING LENS

Connections to the AP® Physics 2 Curriculum*

The lab activity correlates to the following pieces of the AP® Physics 2 framework:

Big Idea	6	Enduring Understanding	E	Essential Knowledge	5

Learning Objective 1: The student is able to use quantitative and qualitative representations and models to analyze situations and solve problems about image formation occurring due to the refraction of light through thin lenses.
Science Practices: 1.4, 2.2

Learning Objective 2: The student is able to plan data collection strategies, perform data analysis and evaluation of evidence, and refine scientific questions about the formation of images due to refraction for thin lenses.
Science Practices: 3.2, 4.1, 5.1, 5.2, 5.3

Time Requirement

Preparation Time: 10 minutes

Lab Activity: 40 minutes

Prerequisites

Students should be familiar with the following concepts:

- *Refraction*: the path and speed of light change as the light crosses a boundary between two transparent media with dissimilar indices of refraction.
- *Real images* are formed by the convergence of light waves to produce a focused image visible on a screen.
- *Virtual images* are formed by the divergence of light rays and appear as if the image is spatially located behind the lens or mirror.
- The thin lens equation and its variables.

Driving Question | Objective

What is the focal length of the converging lens provided by your instructor? Use the principles of refraction and the thin lens equation to experimentally determine the focal length of a converging lens.

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Procedural Overview

In the Structured version of this lab activity, students observe the formation distance of a real image produced by a converging lens, and analyze its dependence on the distance between the lens and the object (image source). Students identify and measure the image distance using five different object distances and use their data to plot a graph of inverse object distance versus inverse image distance. The shape of student graphs should be linear with the *y*-intercept equal to $1/f$, where f is the focal length of their converging lens. From these relationships, students determine an experimental value for the focal length of their converging lenses.

Pre-Lab Discussion and Activity

This lab is designed to follow the "Spherical Mirror Reflection" activity in which students determine the radius of curvature of a concave spherical mirror using a graphical method. Because the procedures outlined in the Structured versions of that and this lab activity are so similar, it is recommended to follow an open-inquiry progression in which the Structured or Guided Inquiry version of the Spherical Mirror Reflection activity is first performed by students, followed by the Guided Inquiry or Student Designed version, respectively, of this activity. Doing this will help to assess students' abilities to design and conduct an activity based on their experience using linearization and other science practices outlined in the previous activity.

After performing the "Spherical Mirror Reflection" activity, students should have a good understanding of the two types of images formed in geometrical optics applications: virtual images and real images, and the differences and similarities between them. Students should also know how to identify the spatial location of real images in geometrical optics applications.

Below is a pre-lab discussion best suited for students who have not performed the Spherical Mirror Reflection activity, but is also suitable for those that have.

First explain to students the major difference between images formed by lenses versus images formed by mirrors: a lens is a piece of optics that uses refraction to bend light in such a way that it converges to form images, while a mirror redirects light using reflection so that it converges to form an image. Both form images, and both have the ability to form *real* and *virtual* images. Below are more formal definitions of each image form:

Real image – an image of an object physically formed on a screen or other viewing plane due to the convergence of light being emitted or reflected from an object.

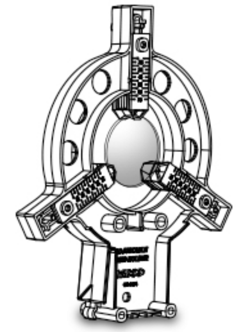
Virtual image – an image of an object formed by reflected or emitted light rays that are diverging. A virtual image appears to be in a position from which the actual light rays do not originate, and because of this a virtual image can never be formed on a screen.

An equation exists for both spherical mirrors and lenses that describes the spatial relationship between the optical component (mirror or thin lens), the distance from the component to an object, and distance from the component to the image of the object. For lenses, this is known as the thin lens equation:

$$\frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i} \tag{1}$$

where f is the focal length of the lens, s_o is the distance from the lens to an object, and s_i is the distance from the lens to the point at which the image of the object is in focus (image distance).

In the Structured version of this activity, the process of identifying the point at which the real image is formed from a converging lens is addressed step-by-step in the procedure section. However, students performing the Guided Inquiry and Student Designed versions of this activity may need a short demonstration showing this process, as this may be an important part of their experimental procedure.

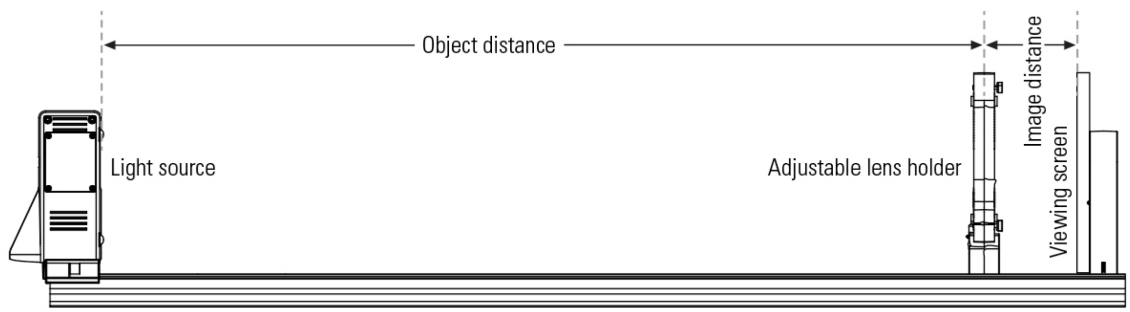


Use the PASCO optics equipment listed in the Materials and Equipment section below to set up and perform the following demonstration:

Begin by mounting a converging lens in the adjustable lens holder like the picture to the right. It is recommended to use a converging lens with a focal length equal to 20 cm.

Assemble the adjustable lens holder and the remainder of the PASCO optics equipment as in the picture below:

1. Lay the optics track flat on the lab table and mount the light source at one end of the track with the crossed-arrow image on the light source pointing down the length of the track.

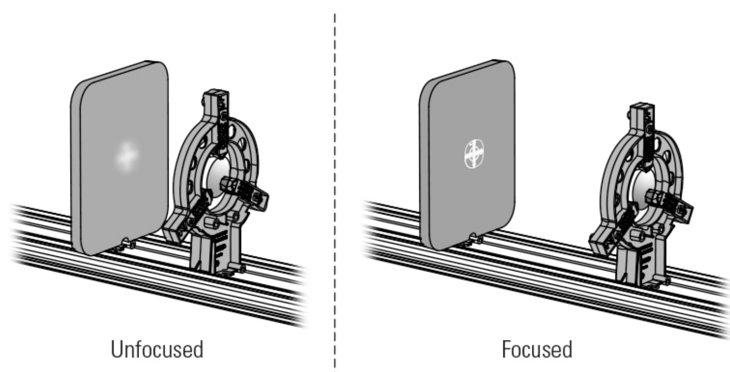


2. Mount the viewing screen at the opposite end of the track with the face of the viewing screen pointed at the light source.

3. Mount the lens holder to the track so that it and the viewing screen are separated by a distance equal to approximately half the focal length of the lens.

With the light source on, in this configuration the illuminated crossed-arrow target on the light source will act as the object and the image of the object will be formed on the viewing screen. Explain to students that the equipment is designed to be used in a straight path to best emulate the linear configurations depicted in ray diagrams, and to enable the accurate use of Equation 1.

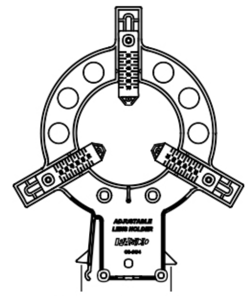
Show students how the converging light rays refracted by the lens appear on the viewing screen: the image should appear as a blur near the center of the screen. Have students observe the image on the screen as you slowly slide the lens holder away from the screen and closer to the light source. The image should condense and become more focused as the distance between the lens holder and screen increases. Stop sliding the lens holder at the point the image is most sharply focused on the screen (you may need to slide the lens holder back and forth until you can determine where the image is the most sharply focused). In this configuration, the screen is at the location of the real image formed by the converging lens.



Explain to students that with the screen at its current distance from the lens, measurements of the variables in Equation 1 (object distance s_o and image distance s_i, where object distance is measured from the front of the light source to the lens, and image distance is measured from the lens to the front of the screen), can now be accurately made.

Have students observe that there may be more than one real image that can be formed without moving the light source or screen; continue sliding the lens holder closer to the light source until a second real image forms on the screen, and note that in this configuration, measurements of the variables in Equation 1 can again be accurately made.

Another helpful tool for students may be the small indicators molded onto the bottom of each PASCO Basic Optics component holder. These indicators show the position of each component in its holder so can be used to make measurements of object and image distance using the metric scale on the optics track.


Position Indicators

Materials and Equipment

- PASCO Optics Track[1] or PASCO Dynamics Track with Optics Carriages[2]
- PASCO Basic Optics Light Source[3]
- PASCO Basic Optics Viewing Screen[4]
- PASCO Adjustable Lens Holder[5]
- Converging lens with unknown focal length

Probeware Resources

Below are web-link and QR codes that will direct you to instructional video resources for individual pieces of PASCO probeware, sensors, and other hardware used in the lab activity. These same links and codes are provided to students in their activity handouts.

[1]www.pasco.com/ap28
PASCO Optics Track

[2]www.pasco.com/ap29
PASCO Dynamics Track Optics Carriages

[3]www.pasco.com/ap26
PASCO Basic Optics Light Source

[4]www.pasco.com/ap31
PASCO Basic Optics Viewing Screen

[5]www.pasco.com/ap30
PASCO Adjustable Lens Holder

Lab Preparation

In this activity, students experimentally determine the focal length of a spherical converging lens, and then compare their experimental value to the actual value.

If your students are using a lens from the PASCO Basic Optics Geometric Lens set, note that each lens has its focal length shown on its component holder base: one with 0.100 m focal length, and one with 0.200 m focal length. Each published value has a tolerance of 10%. It is recommended that you cover these values with tape or other masking material so students do not have a preconceived idea of what their results should show.

If you are using lenses with unknown focal lengths, you will need to identify these values or determine them empirically before the lab activity commences. Some third-party manufacturers will publish the accepted component values in the literature that accompanies them. If not, it is recommended that you follow the procedure outlined in the Structured version of this lab activity to empirically determine each lens's focal length.

Teacher Tips

Tip 1 – Linearization Strategies for the Thin Lens Equation

- In the Structured version of this activity, students plot a graph of $1/s_o$ versus $1/s_i$, which will produce a linear relationship that applies to a converging thin lens. Students then apply a line of best fit to that data whose *y*-intercept will equal $1/f$, from which students can ascertain an experimental value for the focal length of their lens.

  However, when performing the Guided Inquiry or Student Designed versions of this activity, students may choose another common manipulation of the thin lens equation to help linearize their data. In this manipulation, students eliminate the fractions in the thin lens equation, producing a linear relationship between the product of the object and image distances, and the sum of the object and image distances:

  $$\frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i}$$

  $$\frac{1}{f} = \frac{s_i + s_o}{s_o s_i}$$

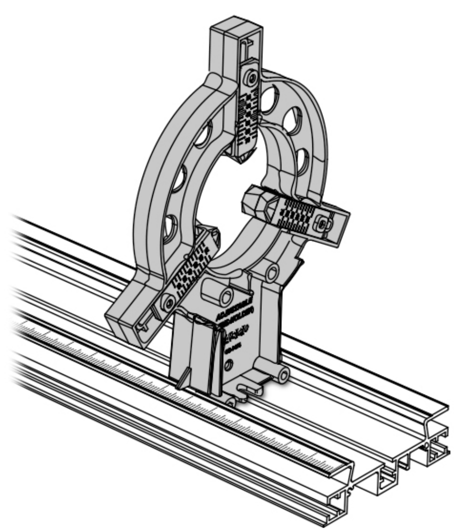
  $$s_o s_i = f\left(s_i + s_o\right)$$

  Students should then plot a graph of $(s_i s_o)$ versus $(s_i + s_o)$, which will produce a linear relationship using a converging thin lens, and then apply a line of best fit to that data. The *slope* of that line will equal their experimental value for the focal length.

Tip 2 – Using Lenses with Long Focal Lengths

- If the focal length of the lens being used in this activity is too long, students will find that the image formation distance will be immeasurable, as it will be longer than their workspace or optics track. Students using the PASCO Optics Track or Dynamics Track with Optics Carriages will need to use lenses with a focal length shorter than 250 mm to avoid running out of track when making measurements.

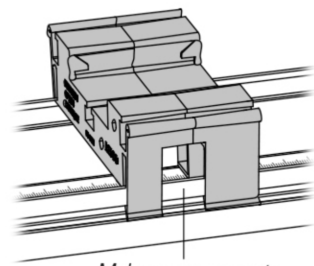
Tip 3 –Using PASCO Basic Optics System Components



- PASCO Basic Optics system component holders such as the Adjustable Lens Holder are designed to fit in the wide central channel of the PASCO Optics Track or the PASCO Dynamics Track Optics Carriages (see Teacher Tip 4). Place the base of the component holder on the bench and push down firmly to snap it in place. To move it, squeeze the tab on the base of the component holder and slide it along the track.
- Use the metric scale on the optics track to make spatial measurements with these components. Each component holder has a small molded tab extending from its base that indicates the position of the optical component in its holder.

Tip 4 –Using PASCO Dynamics Track Optics Carriages

- PASCO Dynamics Track Optics Carriages, in conjunction with a PASCO dynamics track, can be used in place of a PASCO Optics Track. The carriages are designed to snap onto the top of the dynamics track, and are molded with a central channel to receive PASCO Basic Optics system component holders, similar to the PASCO Optics Track.
- To mount the carriages and basic optics component holders, place the carriage on top of a dynamics track and press down to snap the outer tabs of the carriage into the side slots of the track. Place the base of the component holder on the carriage and push down firmly to snap it in place. The component holder should snap into the recesses in the center of the carriage, locking it in place. You can then slide the carriage and component combination along the track to the desired position. To detach the carriage, squeeze the arms on either side and pull up.
- To make spatial measurements using the carriage-component holder combination, use the metric scale on the dynamics track. Each carriage has a molded opening through which the metric scale on the dynamics track is visible. The front square edge indicates the position of the optical component in its holder.



Tip 5 – Appropriate Choice of Object Distances

- Students performing the Guided Inquiry or Student Designed version of this lab activity may choose an experimental procedure in which they measure image distance at varying object distances (similar to that outlined in the Structured version of this lab activity). It is important for these students to test a range of object distances that does not consist entirely of values much larger than the focal length of the lens, or values that are tightly grouped.

  At object distances much larger than the focal length of the lens (values > 6 × focal length), the resulting image distances have very small variance. This can make it difficult for students to measure differences in image distances accurately, especially if the difference between image distance values does not exceed the resolution of the measurement tool being used.

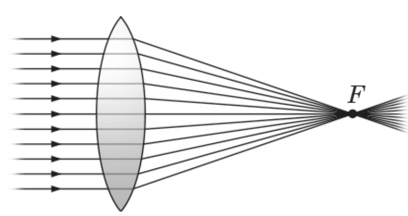
  Similarly, choosing a range of object distances that are too tightly grouped may also produce very small variance in image distance values, resulting in added difficulty in making accurate measurements.

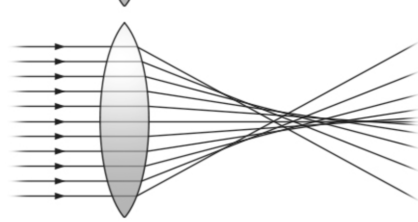
  Students should be encouraged and guided to choose a range of object distances that begins with a value as small as possible, but still permits the measurement of the image formation distance, and then extends to a value approximately six times the focal length of the lens.

Tip 6 – Correcting Spherical Aberration

- In most applications of spherical converging lenses, it is assumed that the shape and surface of the lens is such that all incoming light rays parallel to the optical axis will refract and converge at the lens's focal point *F*. However, this assumption fails for incident light rays not near the optical axis, which converge behind the focal point.

  As a result, images formed by most converging lenses appear focused at their center, but blurry along the outer edge. This is known as *spherical aberration* and can confuse students when attempting to determine where the actual image formation from a converging lens is, spatially.





Students can correct for spherical aberration by doing the following:

1. Cut a small piece of paper just larger than the size of the lens.
2. Cut a small round hole in the center of the paper about 1–2 cm in diameter.
3. Hold the paper over the lens surface with the center of the hole aligned with the center of the lens.

With the paper in place, those light rays not near the optical axis will be blocked, resulting in a more focused image.

Sample Data

Below are sample data, acquired using the experimental setup and procedure outlined in the Structured version of the lab activity, and answers to questions in the Data Analysis section.

Data Analysis

Table 1: Object distance and corresponding image distance using a converging lens

Trial	Object Distance s_o (m)	Image Distance s_i (m)	$1/s_o$ (m^{-1})	$1/s_i$ (m^{-1})
1	0.240	0.390	4.17	2.56
2	0.340	0.264	2.94	3.79
3	0.440	0.225	2.27	4.44
4	0.540	0.205	1.85	4.88
5	0.640	0.194	1.56	5.15

1. Calculate the inverse of each object distance and image distance in Table 1. Record your results in the $1/s_o$ and $1/s_i$ columns of the table.

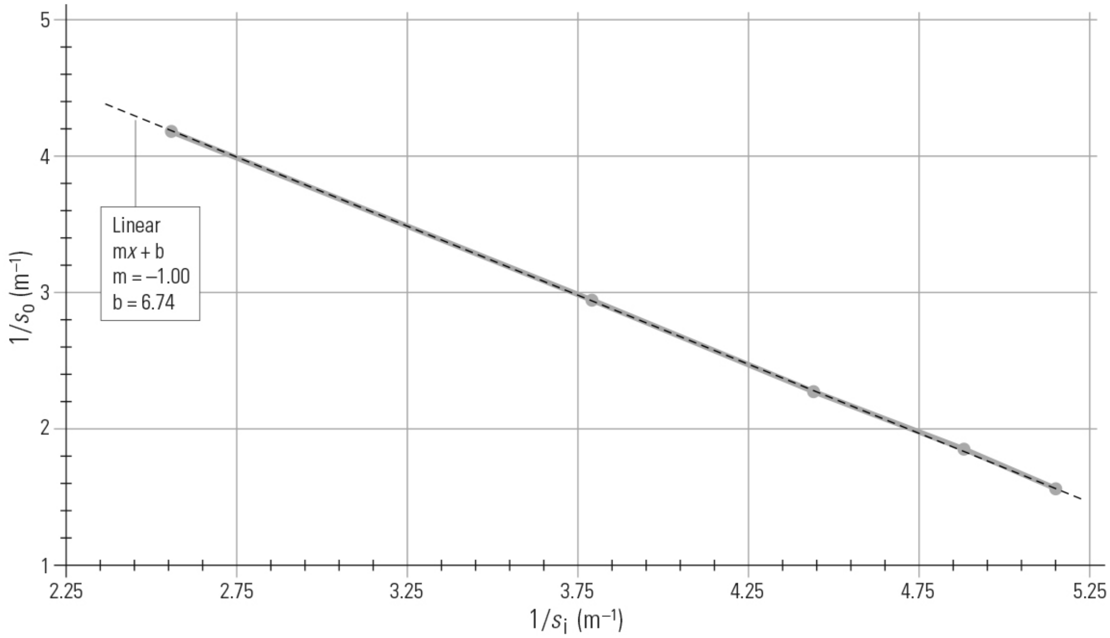
   Calculation using sample data for Trial 1

   $$\frac{1}{s_o} = \frac{1}{0.240\text{ m}} = 4.17\text{ m}^{-1}$$

   $$\frac{1}{s_i} = \frac{1}{0.390\text{ m}} = 2.56\text{ m}^{-1}$$

2. Plot a graph of $1/s_o$ versus $1/s_i$ in the blank Graph 1 axes below. Be sure to label both axes with the correct scale and units.

Graph 1: Inverse object distance versus inverse image distance using a converging lens



3. Draw a line of best fit through your data in Graph 1. Determine and record the equation of the line here:

Best fit line equation: $\frac{1}{s_o} = -1.00\frac{1}{s_i} + 6.74\ \text{m}^{-1}$

4. Use the y-intercept from the best fit line to determine an experimental value for the focal length f of your lens:

$$y\text{-intercept} = \frac{1}{f}$$

Focal length f (m): 0.148 m

$$y\text{-intercept} = \frac{1}{f}$$

$$f = \frac{1}{y\text{-intercept}} = \frac{1}{6.74\ \text{m}^{-1}} = 0.148\ \text{m}$$

Guided Inquiry Questions

Below are sample responses to the Guiding Questions found in the Guided Inquiry version of this lab activity.

1. The thin lens equation defines the position of the real image formed by a thin spherical lens:

$$\frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i} \quad (1)$$

Where f is the lens focal length, s_o is the distance from the lens to an object (*object distance*), and s_i is the distance from the lens to the focused image of the object (*image distance*).

a. Explain how Equation 1 helps inform which measurements you will make in your investigation?

The primary objective in this lab activity is for students to determine the focal length of their converging lens. Student responses should explain that Equation 1 identifies quantities (image distance and object distance) that can be measured and used to determine lens focal length. By focusing their experiment on these quantities, students can follow a straightforward path in their investigation.

b. Which variables in Equation 1 can be measured directly using measurement tools that are available to you?

Object distance s_o and image distance s_i can be measured directly using any spatial measurement tool: meter stick, ruler, tape measure, et cetera. Focal length f cannot be measured directly and requires some form of indirect measurement, or unique measurement strategy. Students should be encouraged not to attempt to measure focal length directly and concentrate their efforts on identifying a simple strategy to determine it indirectly using the relationship outlined in Equation 1.

c. What tools would you use to measure each variable listed in your previous answer and how would you use them to make measurements?

Students can use any spatial measurement tool to measure both object and image distance, but it should be recommended that they use a meter stick, metric ruler, or measuring tape. Object distance should be measured from the object or light source being used to the position of the lens. Image distance should be measured from the position of the lens to the front of the viewing screen on which a focused image of the object is formed.

2. When setting up your equipment, does it matter which side of the lens the object is on? Explain. Also, why is it important to keep the plane of the lens perpendicular to the optical axis and the object along the axis?

It should not matter which side of the lens the object is on because both surfaces of the lens are the same radius of curvature and light is refracted the same when entering the lens from either side.

It is important to keep the plane of the lens perpendicular to the optical axis when making measurements associated with the variables in Equation 1. If the lens was tilted slightly, the light entering one side of the lens would have a different incident angle than the complimentary ray entering the other end, thus producing a different angle of refraction.

3. What equipment will you use and how will you set it up so that each variable can be measured as accurately as possible?

Although the Structured version of this activity identifies specific PASCO optics components for students to use, students can use any brand of optics components as long as those components include a converging thin lens, an “object” of which an image will be formed, and a viewing screen on which the image of the object will be formed. Students should be creative in their experiment design, but should follow these general rules for equipment setup:

- Students can choose any object as long as it is sufficiently illuminated or emits light so that a reasonable image of the object can be formed. It is recommended that students use a light source (light bulb, lamp, candle, et cetera) as their object.
- The lens and object should be placed in such a way that the distance between the object and the lens can be measured in a straight line.
- A viewing screen should be used to project the image of the object, and should be placed in a straight line path with all of the other components.

4. If you place a converging lens in the center of a straight optics track and a light source (object source) at one end of the track facing the lens, how would you use a viewing screen to locate the image of the light source?

   Students should indicate that in this configuration, to identify the position of an image formed from the converging lens, the viewing screen should be placed past the lens (on the side opposite the light source) in-line with the lens and light source, and then slid up and down the track until a focused image of the light source is formed on the screen. This is assuming that the distance between the lens and light source is greater than the focal length of the lens.

5. Do you think it is better to make one measurement of each variable and then use that one value to determine *f* for your lens, or make several measurements using different values? Explain your reasoning.

   Students will likely conclude that if their experiment is based on one measurement, it is likely that there will be error that can cause their results to be inaccurate or less representative. Making several measurements of the dependent variable based on a good variance of the independent variable will provide an averaged and more accurate (representative) value for focal length.

6. According to Equation 1, what happens to the image distance when the object distance approaches the focal length of the lens? Should you measure the image distance when the object distance is near the focal length of the lens? Justify your answer.

   Students should recognize that the relationship outlined in Equation 1 indicates that the image distance approaches infinity as the object distance approaches the focal length of the lens.

   No. In this case, the image distance should not be measured, as it is greater than the length of the optics bench.

Assessment Questions: Sample Responses

Sample responses to the Analysis and Synthesis questions found in each version of the lab activity:

Analysis Questions

1. What is your experimental value for the focal length of your lens? How did you determine this value from your data?

   Based on the data shown in the Sample Data section above, $f = 0.148$ m

   Students performing the Student Designed or Guided Inquiry version of this activity will acquire values that vary between groups based on the measurement tools used and the procedural strategies chosen. Students performing any of the three versions of this activity may also show variability in their values between groups due to the variability in the published focal length of each converging lens. The PASCO Basic Optics Geometric Lens set has two converging lenses with published focal lengths of 0.100 m and 0.200 m and tolerance of 10% each. The sample data shown above was acquired using a 50-mm diameter double-convex glass lens with a published focal length of 0.150 mm and tolerance of 10%.

   Students following the procedure outlined in the Structured version of this lab activity identify and measure the image distance using five different object distances and use their data to plot a graph of inverse object distance versus inverse image distance. The shape of student graphs should be linear with the *y*-intercept equal to $1/f$. From this, students calculate an experimental value for *f*.

2. What are factors that might have caused error in your measured value of the focal length? Explain how each factor you list could have been avoided or minimized.

   The list of factors that could cause error may include, but is not limited to:

   - Difficulty identifying the point at which the image was located. This may be caused by spherical aberration. See Teacher Tip 6 above.
   - Components were not aligned in a straight line. Students can avoid this by using a fixed optics track or table to help accurately align components.
   - Measurements of component positions were made incorrectly: for example, students may measure object and image distances based on the position of the component holders and not the actual components in the holders. Students should make spatial measurements based on the position of the components in the holders and not the holders themselves. See Teacher Tips 3 and 4 above.

3. Ask your teacher for the actual value of the focal length of your lens. Calculate the percent error between your experimental value and the actual value.

$$\text{Percent error} = \left|\frac{\text{Actual} - \text{Experimental}}{\text{Actual}}\right| \times 100$$

Sample calculation:

$$\text{Percent error} = \left|\frac{0.150\text{ m} - 0.148\text{ m}}{0.150\text{ m}}\right| \times 100 = 1.33\%$$

4. What do you predict happens to the image distance from a converging lens as the object distance approaches the focal length of the lens? Justify your answer: use mathematical reasoning or data from your experiment, or both, to support your answer.

Based on the mathematical relationship in Equation 1, as the object distance approaches the focal length of the lens, the image distance approaches infinity.

Students may choose to justify their response using their data and a graph of object distance (on the *y*-axis) versus image distance (on the *x*-axis). This graph should show a horizontal asymptote where object distance is equal to focal length, thus indicating that as object distance approaches the lens focal length, image distance approaches infinity.

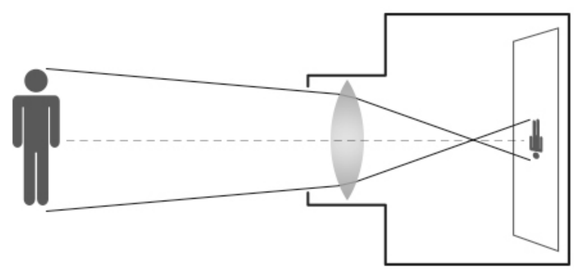
Synthesis Questions

1. An object sits in front of a converging lens 27.2 cm away from it. If the image of the object is formed 68.3 cm on the opposite side of the lens, what is the focal length of the lens?

$$\frac{1}{f} = \frac{1}{s_i} + \frac{1}{s_o} = \frac{s_o + s_i}{s_o s_i}$$

$$f = \frac{s_o s_i}{s_o + s_i} = \frac{(0.272\text{ m})(0.683\text{ m})}{0.272\text{ m} + 0.683\text{ m}} = 0.195\text{ m}$$

2. A camera uses a 10.5 cm focal-length converging lens to focus the image of a person onto a piece of camera exposure film. If the person is 2.15 m away from the lens, how far away must the film be from the lens to focus the image on the film? Explain how the focus on a camera might work.



$$\frac{1}{f} = \frac{1}{s_i} + \frac{1}{s_o}$$

$$\frac{1}{s_i} = \frac{1}{f} - \frac{1}{s_o} = \frac{s_o - f}{s_o f}$$

$$s_i = \frac{s_o f}{s_o - f} = \frac{(2.15\text{ m})(0.105\text{ m})}{2.15\text{ m} - 0.105\text{ m}} = 0.110\text{ m}$$

The focus on a camera mechanically adjusts the distance between the camera lens or lenses and the surface on which the image will be exposed or recorded. This allows the user to adjust the image distance so that the image is in focus regardless of the object distance.

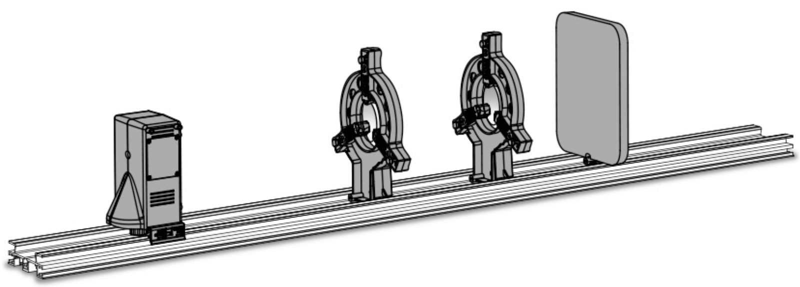
3. Imagine you were given a converging lens and a meter stick and sent outside on a sunny day. In a few sentences, describe a method to measure, as accurately as possible, the focal length of the lens using only the lens, a meter stick, and your outside surroundings. Explain your reasoning.

   Student responses should use clear and complete sentences and explain the reasoning behind their procedural choices. Responses may be similar to this sample:

   To determine the focal length of a converging lens using only the lens, a meter stick, and outside surroundings, hold the lens above the ground with the sun shining through it onto the ground. Move the lens up and down above the ground until the bright spot formed by the lens is at its brightest and smallest. In this position, measure the distance between the lens and the ground. This distance will equal the focal length of the lens. Based on the relationship shown in Equation 1, image distance and focal length of a converging lens are approximately equal when the object distance is very large. Because the sun is so far away, the object distance is large enough to assume that the image distance is equal to the focal length of the lens.

Extended Inquiry Suggestions

Conclude this lab activity with an exploration and discussion of optics systems that use more than one converging lens. Have each lab group join one other lab group, forming half as many lab groups as the class started with. Each new combined lab group should have a total of two converging lenses with known focal length (the focal lengths can be different), one from each constituent group. Have each new lab group take its equipment and construct an optics system that uses two converging lenses to form images. Students using PASCO optics equipment will have a setup similar to the diagram below:



Have students spend 10–15 minutes exploring the behavior of light in the two-lens system and answer the following questions in their groups:

1. How does the image in a single lens system change when a second lens is introduced?

2. What type of image is formed in the two-lens system, real or virtual?

3. Generally speaking, how does the behavior of the two-lens system differ from the single-lens system? What are some key differences?

After the groups have been given time to explore and answer the questions, address the groups one at a time, having each explain their responses to the entire class. Write some of the similar responses on the board.

Ask the class, "Based on the answers on the board, is it possible that the system of lenses behaves such that the image formed by the first lens acts as the object for the second lens? What evidence do we have that supports our answer to this question?" Students will hopefully understand that the answer is "yes." Systems of lenses behave such that the image formed by preceding lenses in a system acts as the object for succeeding lenses, both physically and mathematically. Anecdotal evidence includes:

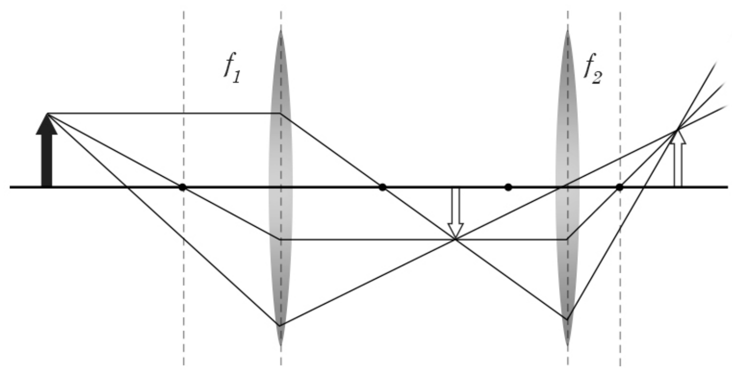
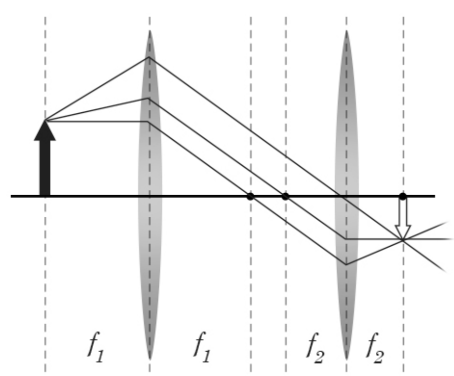
- The real image produced in a one-lens system is inverted, but not in a two-lens system. Reasoning: this could mean that a single lens inverts the image once, and a second lens inverts the image from the first lens a second time.

- When the object is placed at the focal length of the primary lens, regardless of the position and focal length of the secondary lens, the real image produced by the secondary lens appears at that lens's focal length. Reasoning: based on Equation 1, if object distance is equal to focal length for the primary lens, image distance is equal to infinity; if the image distance from the primary lens serves as the object distance for the secondary lens, the object distance for the secondary lens is equal to infinity, which, according to Equation 1, makes the image distance from the secondary lens equal to the focal length of the secondary lens.

Now challenge the groups to use their equipment to test the theory that the image formed by the first lens acts as the object for the second lens. Ask that the groups work independently to determine their own test procedure, and to take notes of the steps they chose and the data that supports or refutes their test so they can present it to the class.

After groups have been given 15–20 minutes to design and execute their tests, address the groups one at a time, having each explain its test design and the results of its test. Student tests should confirm that the image formed by preceding lenses in a system acts as the object for succeeding lenses, and that Equation 1 holds true for systems of lenses, given this relationship. Some student groups may also determine that this behavior is extensible to any system of lenses, regardless of the number of lenses.

For further supporting evidence, use the following ray diagrams to geometrically show how the image from a two-lens system is formed:

Two lens system: The real image formed by the primary lens acts as the object for the secondary lens.	
Two lens system (object at focal length): The parallel lines emerging from the primary lens indicate the image distance is equal to infinity.	

NAME PERIOD DATE

7. FOCAL LENGTH OF A CONVERGING LENS

STRUCTURED

Driving Question | Objective

What is the focal length of the converging lens provided by your instructor? Use the principles of refraction and the thin lens equation to experimentally determine the focal length of a converging lens.

Materials and Equipment

- PASCO Optics Track[1] or PASCO Dynamics Track with Optics Carriages[2]
- PASCO Basic Optics Light Source[3]
- PASCO Basic Optics Viewing Screen[4]
- PASCO Adjustable Lens Holder[5]
- Converging lens with unknown focal length

[1]www.pasco.com/ap28
PASCO Optics Track

[2]www.pasco.com/ap29
PASCO Dynamics Track Optics Carriages

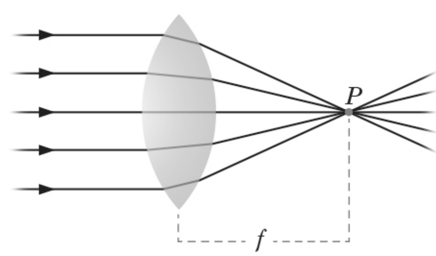
[3]www.pasco.com/ap26
PASCO Basic Optics Light Source

[4]www.pasco.com/ap31
PASCO Basic Optics Viewing Screen

[5]www.pasco.com/ap30
PASCO Adjustable Lens Holder

Background

Parallel light rays entering a converging (*double-convex*) lens change direction as the rays are refracted at both the front (*incident*) and back (*emergent*) surfaces of the lens. If we assume that the path of the light entering the lens is perpendicular to the plane of the lens, the final refracted angle of the ray as it leaves the lens will direct it to a *focal point* P along the optical axis. The distance from the lens to P is known as the *focal length* f of the lens.

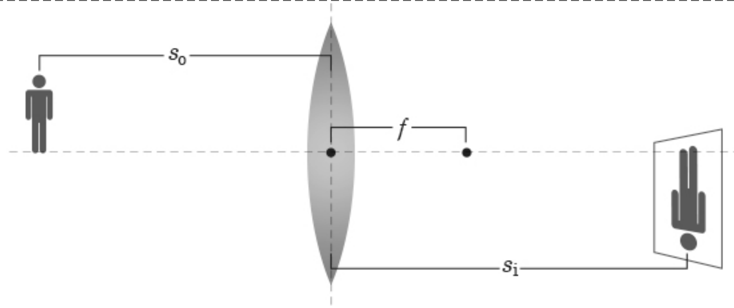


This behavior is a result of refraction, the geometric properties of the lens shape, and angles at which the incoming light rays strike the surface of the lens. Given the shape of the lens, its focal length can be quantified using a complex formula involving the curvature at each lens surface, the thickness of the lens, and the index of refraction for both air and the lens material. However, if we assume that the thickness of the lens is very small, the equation simplifies to a form involving only the distance between the lens and the object and the lens and the image produced by the lens.

This form is called the *thin lens* equation:

$$\frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i} \qquad (1)$$

Where f is the lens focal length, s_o is the distance from the lens to an object, and s_i is the distance from the lens to the point at which the image of the object is in focus.



RELEVANT EQUATIONS

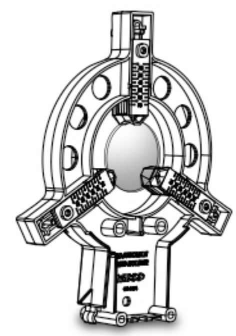
$$\frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i} \tag{1}$$

Procedure

SET UP

1. Lay the optics track flat on your lab table and mount the light source to it so that the "screen zero" (see the bottom of the light source for the screen zero indicator) is aligned with the 6-cm mark on the track. Make sure the crossed-arrow image on the light source points down the length of the track.

2. Mount the lens in the center of the holder using the three adjustable arms, and then mount the holder on the track so that the lens is aligned with the 30-cm mark.



3. Mount the viewing screen to the track on the opposite side of the lens as the light source, with the front of the screen facing the lens.

4. Plug in the light source to turn it on.

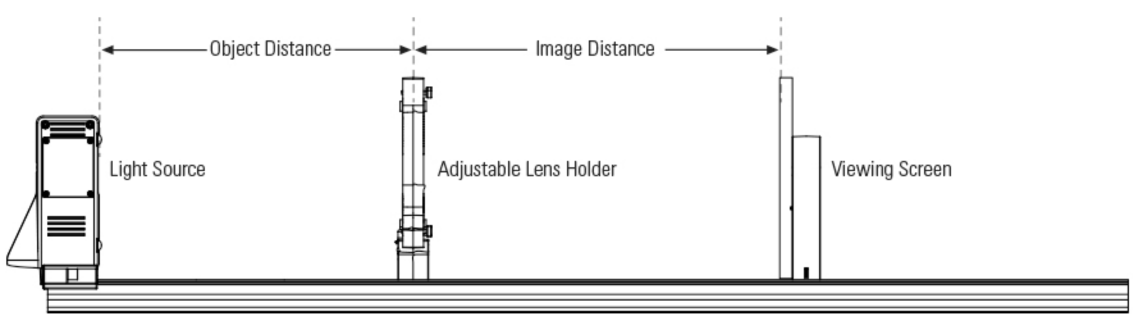
COLLECT DATA

5. Slide the viewing screen up and down the length of the optics track until the image of the crossed-arrow target is in focus on the screen.

   NOTE: The image may not be in perfect focus; however, the image location *is where the image is best focused. You may need to slide the screen back and forth through the image location until you can determine where the image is the most sharply focused.*

6. Using the graduated scale on the optics track, determine the object distance s_o and the image distance s_i. Record these values in Table 1 in the Data Analysis section below.

   NOTE: Object distance is measured from the position of the lens to the front of the light source, and image distance is measured from the position of the lens to the front of the viewing screen.



7. Slide the lens holder 10 cm farther down the track, away from the light source. (Do not change the light source position.)

8. Slide the viewing screen up and down the length of the optics track until the image of the crossed-arrow target is again in focus on the screen.

9. Record the new object distance and image distance next to Trial 2 in Table 1.

10. Repeat the data collection steps three more times, increasing the distance between the lens holder and the light source by 10 cm in each trial. Record the object distance and corresponding image distance for each trial into Table 1.

Data Analysis

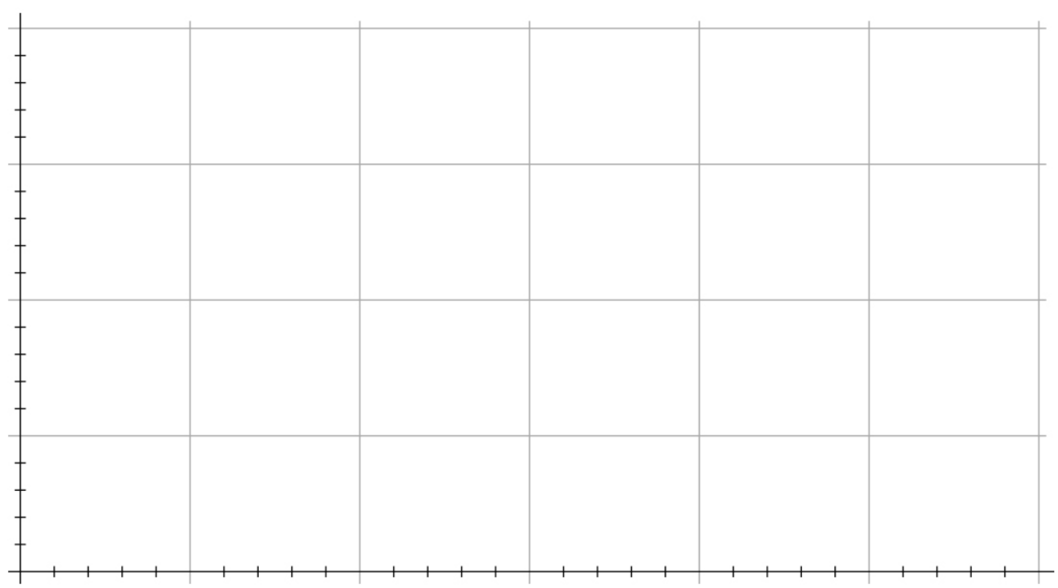
Table 1: Object distance and corresponding image distance using a converging lens

Trial	Object Distance s_o (m)	Image Distance s_i (m)	$1/s_o$ (m^{-1})	$1/s_i$ (m^{-1})
1				
2				
3				
4				
5				

1. Calculate the inverse of each object distance and image distance value in Table 1. Record your results in the $1/s_o$ and $1/s_i$ columns of the table.

2. Plot a graph of $1/s_o$ versus $1/s_i$ in the blank Graph 1 axes below. Be sure to label both axes with the correct scale and units.

Graph 1: Inverse object distance versus inverse image distance using a converging lens



3. Draw a line of best fit through your data in Graph 1. Determine and record the equation of the line here:

   Best fit line equation: ____

4. Use the y-intercept from the best fit line to determine an experimental value for the focal length f of your lens:

$$y\text{-intercept} = \frac{1}{f}$$

Focal Length f (m): ______________________________

Analysis Questions

1. What is your experimental value for the focal length of your lens? How did you determine this value from your data?

2. What are factors that might have caused error in your measured value of the focal length? Explain how each factor you list could have been avoided or minimized.

3. Ask your teacher for the actual value of the focal length of your lens. Calculate the percent error between your experimental value and the actual value.

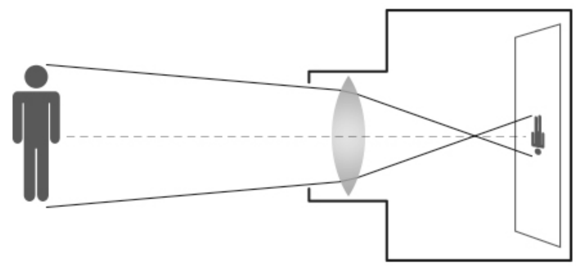
$$\text{Percent error} = \left|\frac{\text{Actual} - \text{Experimental}}{\text{Actual}}\right| \times 100$$

4. What do you predict happens to the image distance from a converging lens as the object distance approaches the focal length of the lens? Justify your answer: use mathematical reasoning or data from your experiment, or both, to support your answer.

Synthesis Questions

1. An object sits in front of a converging lens 27.2 cm away from it. If the image of the object is formed 68.3 cm on the opposite side of the lens, what is the focal length of the lens?

2. A camera uses a 10.5 cm focal-length converging lens to focus the image of a person onto a piece of camera exposure film. If the person is 2.15 m away from the lens, how far away must the film be from the lens to focus the image on the film? Explain how the focus on a camera might work.



3. Imagine you were given a converging lens and a meter stick and sent outside on a sunny day. In a few sentences, describe a method to measure, as accurately as possible, the focal length of the lens using only the lens, a meter stick, and your outside surroundings. Explain your reasoning.

8. INTERFERENCE AND DIFFRACTION

Connections to the AP® Physics 2 Curriculum*

The lab activity correlates to the following pieces of the AP® Physics 2 framework:

Big Idea	6	Enduring Understanding	C	Essential Knowledge	3

Learning Objective 1: The student is able to qualitatively apply the wave model to quantities that describe the generation of interference patterns to make predictions about interference patterns that form when waves pass through a set of openings whose spacing and slits are small, but larger than the wavelength.
Science Practices: 1.4, 6.4

Time Requirement

Preparation Time: 10 minutes

Lab Activity: 40 minutes

Prerequisites

Students should be familiar with the following concepts:

- Light demonstrates wave properties and can be quantified using characteristics such as wavelength and wave speed.
- A diffraction pattern can be observed as light waves pass through an opening whose dimensions are comparable to the wavelength.
- An interference pattern can be observed as light waves pass through a set of openings whose spacing is comparable to the wavelength.
- Both diffraction and interference are the result of constructive and destructive interference between coherent light waves as they occupy the same space at the same time.

Driving Question | Objective

How can the wave nature of light be used to determine the distance between two closely-spaced narrow parallel slits? Use a coherent light source and the principles associated with double-slit interference and diffraction to experimentally determine the spacing between the slits in a double-slit aperture.

Procedural Overview

In the Structured version of this lab activity, students shine laser light through a double-slit aperture provided by their instructor, producing a double-slit interference and diffraction pattern. Students measure the distance from the 0^{th} maxima to the 1^{st} through 5^{th} integer-order maxima in their interference pattern, and then plot a graph of those distance measurements versus their corresponding integer-order from 1 to 5. This graph displays a linear relationship. After students apply a line of best fit to the graph, they use the slope of the best-fit line to calculate the spacing between the narrow slits in the double-slit aperture.

Pre-Lab Discussion and Activity

Students performing the Structured version of this lab activity use a PASCO Diffraction Plate (PASCO Part Number OS-8850) that contains single and multiple slits that produce different diffraction and interference patterns. Students performing the Structured version of this lab activity should be instructed to only use one of the three double-slit apertures on the diffraction plate in their investigation. These double-slit apertures are labeled "D," "E," and "F" on the diffraction plate. Students performing the Guided Inquiry or Student Designed version of this activity should receive these instructions only if they are making use of the same PASCO Diffraction Plate.

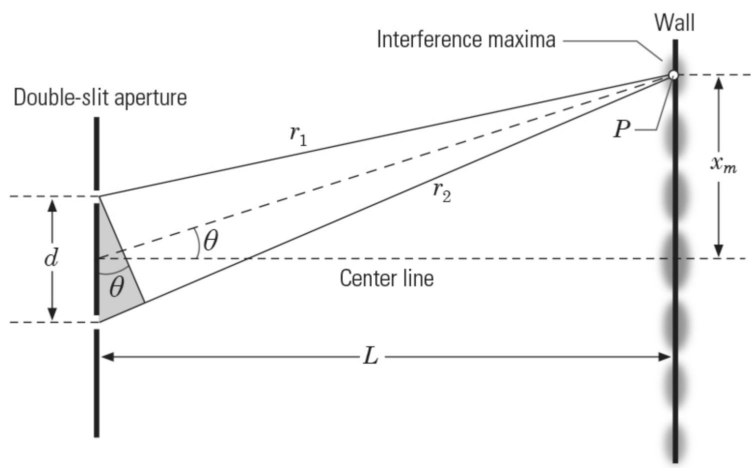
Before performing this lab activity, all students should have a basic understanding of the equation that describes the spatial conditions for constructive interference (interference maxima) in a double-slit interference pattern:

$$d \sin\theta = m\lambda \qquad (1)$$

where d is the spacing between the slits, θ is the angle subtended by the center line and the m^{th}-order maxima in the interference pattern, λ is the wavelength of the coherent light source, and m is an integer (m = 0, 1, 2,…).

The following question set can be used as the basis for a pre-lab classroom discussion to derive this equation, or it can be assigned as a question set that students can work on at home. In either case, preface it with a brief discussion of the diagram below of a physical setup used to generate a double-slit interference pattern on a wall.

PRE-LAB QUESTIONS



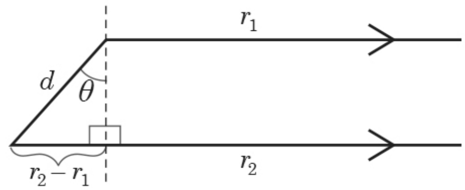
In the diagram above, coherent light is shown incident on the double slits where, according to Huygens's principle, the light is re-emitted as independent light waves from each slit, each with identical wavelength. When the two re-emitted waves are incident on the wall, constructive and destructive interference cause areas of bright and dark fringes known as an interference *maxima* and *minima*. For the following questions, consider the paths of the light waves from the slits to an interference pattern maxima at some distance x_m from the central maxima. Assume that the distance between the double-slit pattern and the wall L is extremely large compared to both the slit spacing d and the linear distance x_m from the center line: $L >> d$; $L >> x_m$.

1. On the diagram, r_1 and r_2 are the path lengths of the waves from each slit, where r_2 is longer than r_1 by ($r_2 - r_1$). Based on the diagram and information above, can it be assumed that r_1 and r_2 are effectively parallel? Explain your answer.

   Yes, it can be assumed that the paths of the two waves are effectively parallel because the slit spacing d is small compared to the distance L between the double-slit aperture and the wall L ($L >> d$).

2. If r_1 and r_2 are effectively parallel, can it be assumed that the shaded triangle in the diagram is a right triangle? Justify your answer.

   If the two lines r_1 and r_2 are parallel, the line that touches the end of r_1 as it emerges from the slit and bisects r_2 into the segments $r_2 - r_1$ and r_1 will be perpendicular to r_2. This forms a right triangle where $r_2 - r_1$ subtends the angle θ.



3. Based on your response to the previous question, write an equation that relates the slit spacing d, the path length difference $r_2 - r_1$, and the angle θ.

   If the shaded triangle in the diagram is a right triangle, the three variables can be associated using the trigonometric relationship:

   $$\sin\theta = \frac{r_2 - r_1}{d}$$

   $$d\sin\theta = r_2 - r_1$$

4. If the two light waves meet at the wall at point P in the diagram, and constructive interference occurs, what can be said about the phase difference between the two waves? Do the two waves need to be exactly in phase?

   Ideally, students will realize that constructive interference occurs when two waves meet with the peaks of each wave aligned; which, in the case of two waves with identical wavelength, occurs when the waves are exactly in phase, but also occurs when the waves are out of phase by any integer multiple of the wavelength $m\lambda$ (where m is an integer; m = 0, 1, 2, 3, …).

5. How does this phase relationship correlate to the path length difference between the two waves? How are the phase relationship and path length difference related mathematically?

   Spatially, the phase difference between the two waves should be equal to the difference in their path lengths, or:

   $$r_2 - r_1 = m\lambda \quad (m = 0, 1, 2, 3, \ldots.)$$

6. Combine your responses to the previous questions to construct a mathematical expression that relates the position θ of the m^{th} bright fringe in a double-slit interference pattern (θ being the angular position relative to the center line) to wavelength λ, integer-order m, and slit spacing d.

   Students should combine the mathematical expressions established in the answers to the questions above to obtain any form of the following expression:

   $$d\sin\theta = m\lambda$$

USING THE PARAXIAL APPROXIMATION OF EQUATION 1

In this activity, students should be encouraged to use the *paraxial approximation* in their experiment to simplify Equation 1. In practice, the angular position θ of each interference maxima in a double-slit interference pattern is often expressed in terms of linear position x, where x_m is the linear distance between the 0^{th}-order maxima and the m^{th}-order maxima (refer to the diagram above) as observed in the interference pattern on a screen some distance L from the double-slit aperture.

Going further, if the distance L between the slits and the screen on which you view the interference pattern is large compared to x_m, an approximation can be used to simplify Equation 1:

$$\text{If } L \gg x_m\text{, then } \sin\theta \approx \tan\theta \approx \frac{x_m}{L} \text{ or, } \frac{dx_m}{L} \approx m\lambda$$

$$\text{Rearranging yields: } x_m \approx m\frac{\lambda L}{d}$$

Students performing the Structured version of this lab activity use this simplification and are introduced to it in the background section of their activity handouts. However, students performing the Guided Inquiry or Student Designed versions may need a brief introduction to this approximation before designing their experiment.

Materials and Equipment

- PASCO Diffraction Plate[1]
- Table clamp or large base (2)
- Support rod, 45-cm (2)
- Three-finger clamp (2)
- Stainless steel calipers
- Laser pointer with known wavelength
- White paper
- Pencil
- Ruler
- Tape
- Measuring tape

[1]PASCO Part Number OS-8850

Probeware Resources

Below are web-link and QR codes that will direct you to instructional video resources for individual pieces of PASCO probeware, sensors, and other hardware used in the lab activity. These same links and codes are provided to students in their activity handouts.

[1]www.pasco.com/ap33

PASCO Diffraction Plate

Safety

Follow these important safety precautions in addition to your regular classroom procedures:

- A laser can cause serious damage to the human eye. Do not look directly at the beam and only turn the laser on when necessary for alignment or data collection.
- Avoid laser beam reflections that may cause eye damage. Be aware of the laser beam at all times; you might unknowingly be producing stray reflections.

Lab Preparation

In this lab activity students experimentally determine the spacing between the two slits in a double-slit aperture, and then compare their experimental values to the actual values. If the actual values are unknown, obtain these or determine them empirically before the lab activity commences. This can be done using a setup, procedure, and analysis similar to that outlined in the Structured version of this lab activity.

Slit spacing values for any of the three double-slit patterns on the PASCO Diffraction Plate (PASCO Part Number OS-8850) are as follows: The PASCO Diffraction Plate has three double-slit patterns: "D," "E," and "F," with slit spacings of 0.125 mm, 0.250 mm, and 0.250 mm respectively, each with a tolerance of ±0.0075 mm.

Students are required to know the wavelength of the coherent light source (laser) used in this lab activity. Most lasers and laser pointers have the output wavelength value printed on a sticker on the outside of the laser. If not, you will need to obtain these values or determine them empirically before the lab activity commences. It is recommended that this be done using an emission spectrometer or linear diffraction grating:

1. Using a Spectrometer: Follow the manufacturer's instruction provided with your spectrometer to determine each laser wavelength. If the lasers your students are using are more powerful than 0.25 mW, do not shine the laser light directly into the spectrometer (unless otherwise specified by the manufacturer's instructions). In this case, aim the laser onto a dark, diffuse surface and shine the reflected laser light into the spectrometer.

2. Using a Linear Diffraction Grating: Shine the laser light through the diffraction grating and observe the interference pattern projected onto a wall or nearby surface. The two brightest spots on either side of the center, the 0th-order maxima, are the 1st-order maxima in the interference pattern. Measure the distance between the two 1st-order maxima and divide that number by two to determine x_1. Then measure the distance from the diffraction grating to the wall to determine L.

   Most linear diffraction gratings have line spacing d published in units of lines per millimeter, N, (lines/mm). Convert from lines per millimeter N to line spacing d in meters using this conversion equation:

   $$d = \frac{1 \times 10^{-3}\ \text{m/mm}}{N}$$

   Finally, calculate the laser wavelength using your values for x_1, d, L, and the following equation:

   $$\lambda = d \sin\left(\tan^{-1}\left(\frac{x_1}{L}\right)\right)$$

Teacher Tips

Tip 1 – Using Third-Party Double-Slit Apertures

- Most third-party double-slit apertures can be used in place of the PASCO Diffraction Plate specified in the Materials and Equipment list above, as long as that diffraction aperture is symmetrical with a known slit spacing. If you choose to use a third-party diffraction aperture, the instructions found in the Structured version student handout specific to the PASCO Diffraction Plate (that is, the instructions in the Set Up section) will need to be adjusted.

Tip 2 – Student Misconception: Huygens's Principle

- A common misconception among students is that a double-slit interference pattern is affected by changes in the distance between the laser and the double-slit aperture. According to Huygens's principle, light incident on the double slits is re-emitted as independent light waves from each slit. This implies that the behavior of the light waves emitted from the slits has no relationship to the distance between the laser and the double-slit aperture.

  This can be demonstrated by producing a double-slit interference pattern on the wall, and then increasing the distance between the laser light source and the double-slit aperture while maintaining constant distance between the double-slit aperture and the wall. The intensity of the interference pattern may decrease as the distance between the laser light source and the double-slit aperture increases, but the interference pattern profile (fringe positions) won't change.

Sample Data

Below are sample data acquired using the experimental setup and procedure outlined in the Structured version of the lab activity, and answers to questions in the Data Analysis section.

Data Analysis

Double-slit pattern: E

$\lambda =$ 533 nm

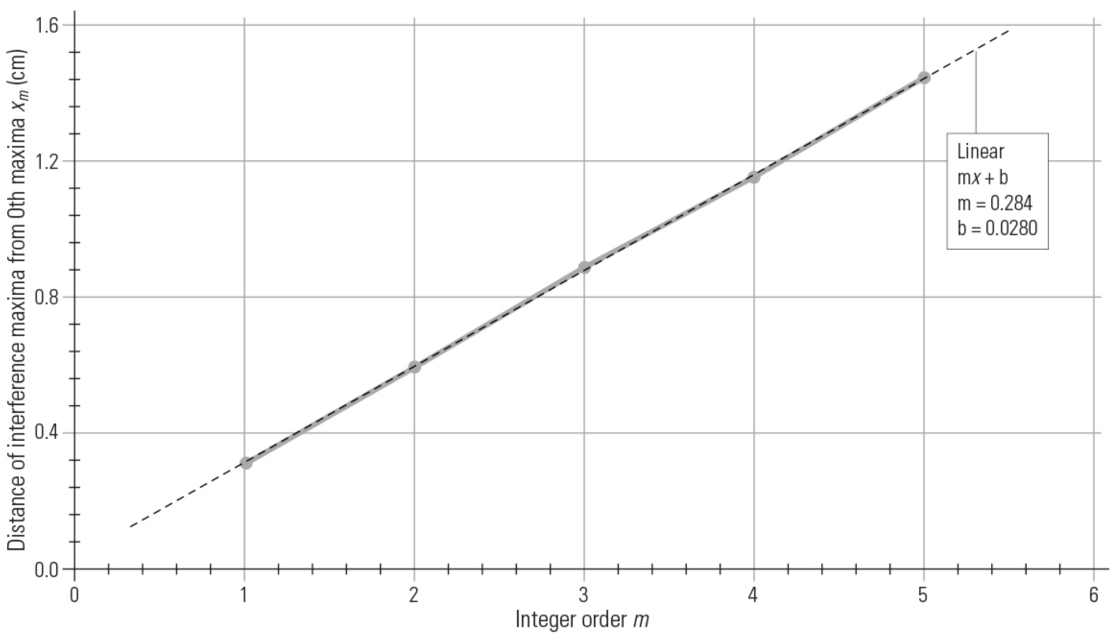
$L =$ 1.358 m

Table 1: Measurements for determining the slit spacing

Integer Order Maxima *m*	Distance from 0th Maxima x_m (cm)
1	0.310
2	0.595
3	0.890
4	1.155
5	1.450

1. Plot a graph of *distance of interference maxima from 0th maxima (x_m)* versus *integer order (m)* in the blank Graph 1 axes below. Be sure to label both axes with the correct scale and units.

Graph 1: Distance of interference maxima from 0th maxima versus integer order of double-slit interference maxima



2. Draw a line of best fit through your data in Graph 1. Record the equation of the line here:

Best fit line equation: $x_m = (0.284\ \text{cm})m + 0.0280\ \text{cm}$

3. Use the slope from the best fit line to determine the spacing d between the parallel slits on the diffraction plate:

$$\text{slope} = \frac{\lambda L}{d}$$

Slit spacing d (cm): 0.0255 cm

Calculation using sample data and results from Graph 1:

$$\text{slope} = \frac{\lambda L}{d}$$

$$d = \frac{\lambda L}{\text{slope}} = \frac{(5.33 \times 10^{-5}\ \text{cm})(135.8\ \text{cm})}{0.284\ \text{cm}} = 0.0255\ \text{cm}$$

Guided Inquiry Questions

Below are sample responses to the Guiding Questions found in the Guided Inquiry version of this lab activity.

1. What is the mathematical expression relating the position of bright fringes in a double-slit interference pattern to the spacing between the double slits producing the interference pattern? Identify each of the variables in this expression.

Students may identify this mathematical expression (or any alternate form):

$$d \sin\theta = m\lambda$$

where θ is the angular position (relative to the center line) of the m^{th} bright fringe in a double-slit interference pattern, λ is the wavelength of the coherent light shined onto the double slits, m is the integer-order bright fringe (maxima) in the interference pattern relative to the 0^{th}-order (center) bright fringe, and d is the slit spacing between the double slits.

Although the above expression is correct, students should be encouraged and guided to use this simplified version of the same equation:

$$x_m \approx m\frac{\lambda L}{d}$$

This equation has been simplified using the paraxial approximation where $\sin\theta \approx \tan\theta \approx x_m/L$. This simplified form will help students identify variables in their experimental design that can be measured directly. Students should note that this simplification is only possible if $L >> x_m$, where x_m is the linear distance from the 0^{th}-order (center) bright fringe to the m^{th}-order bright fringe, and L is the distance between the double-slit aperture and the surface on which the interference pattern is observed.

2. Which variables in the above expression can be measured directly, and which variable must be measured indirectly? Identify the tools and techniques you would use to measure each variable (other than the spacing between the slits)?

Equation 1: $d \sin\theta = m\lambda$ *Simplified expression*: $x_m \approx m\frac{\lambda L}{d}$

Technically, all of the variables in either of the above expressions can be measured directly; however, students would need high-resolution measurement tools that are likely not available to them.

Realistically, in Equation 1, integer order m and wavelength λ are the only two variables that can be measured directly. Integer order is measured by counting the number of bright fringes (maxima) on either side of the 0^{th}-order (center) bright fringe, and wavelength is generally a known quantity but can also be measured using classroom tools like a spectrometer. The angle θ will likely be too small to measure directly, but can be measured indirectly using the trigonometric relationship between x_m and L ($\theta = \tan^{-1}(x_m/L)$), *both of which can be measured directly using spatial measurement tools such as a meter stick, measuring tape, and high resolution calipers.*

In the simplified expression above for x_m, the variables m, x_m, L, and λ can all be measured directly using the same tools and techniques just mentioned .

❓ 3. Assuming you choose a graphical method to analyze your data, what will the dependent (measured) variable and independent (manipulated) variable on your graph be? Explain why you chose these variables.

Students may choose either of two dependent variable and independent variable combinations in their experimental design.

For one combination, students plot the distance from the 0^{th}-order maxima x_m (dependent variable) versus the integer order m (independent variable), similar to the procedure outlined in the Structured version of this lab activity. Choosing these variables will generate data that shows a linear relationship whose slope will equal $\lambda L/d$. Assuming that λ and L are held constant and known, students can apply a best fit line to their data and then calculate slit spacing d from the value for slope.

For the second combination, students plot the distance from the 0^{th}-order maxima x_m (dependent variable) versus the distance between the double-slit aperture and the wall L (independent variable). As long as students choose the 1^{st} integer maxima when measuring x_m, these variables will generate data that shows a linear relationship whose slope will equal λ/d. Assuming that λ is held constant and known, students can apply a best fit line to their data and then calculate slit spacing d from their value for slope.

This second combination is not recommended for students as the accuracy of x_m measurements is greatly affected by the measurement tool being used and the spacing between the double-slit aperture and the wall. See the sample response to the following question for more information.

❓ 4. The lab objective involves projecting a double-slit interference pattern onto a wall (or other surface) and using spatial measurements to experimentally determine the spacing between the slits. How does increasing the distance between the double-slit aperture and the wall affect the error in your measurements? Explain your answer.

Students will likely use spatial measuring tools with fixed resolution and associated uncertainty to measure x_m. If the values for x_m are small (comparable to the resolution of the measuring device), the relative error and overall accuracy in their measurements will be very large.

According to the equation:

$$x_m = m\frac{\lambda L}{d}$$

increasing the spacing L between the double-slit aperture and the wall will also increase the absolute size of x_m, which will decrease the relative error in their measurements, assuming students use the same measurement tool.

❓ 5. How is the error in your measurements affected if the distance between the double-slit aperture and wall is constant, but the distance between your coherent light source and the double-slit aperture increases? Explain your answer.

Students should recognize that, according to Huygens's principle, light incident on the double slits is re-emitted as independent light waves from each slit. This implies that the behavior of the light waves emitted from the double slits has no relationship to the distance between the coherent light source and the double-slit aperture. Going further, the double-slit interference pattern observed by students is unaffected by changes to the spacing between the light source and the double-slit aperture. If the interference pattern is unchanged, students will observe no change in their measurements, and thus, no difference in the error associated with those measurements.

Assessment Questions: Sample Responses

Sample responses to the Analysis and Synthesis questions found in each version of the lab activity:

Analysis Questions

❷ 1. What is your experimental value for the spacing between the double slits, and how did you determine this value from your data?

Based on the sample data shown in the Sample Data section above, d = 0.0255 cm

Students performing the Student Designed or Guided Inquiry version of this activity will acquire spacing values that vary between groups based on the measurement tools used and the procedural strategies chosen. Students performing any of the three versions of this activity may also show variability in their values between groups due to the variability in the actual slit spacing. The PASCO Diffraction Plate has three double-slit patterns: "D," "E," and "F," with slit spacings of 0.125 mm, 0.250 mm, and 0.250 mm, respectively, each with a tolerance of ±0.0075 mm. The sample data shown above was acquired using the "E" pattern.

Students following the procedure outlined in the Structured version of this lab activity measure the distance from the 0th maxima to the 1st through 5th integer-order maxima in their interference pattern, and then plot a graph of distance versus the corresponding integer-order m from 1 to 5. The slope of this graph, determined by the line of best fit, is then used to calculate the spacing between the narrow slits in the double-slit pattern.

❷ 2. What are factors that may have caused error in your experimental value of slit spacing? Explain how each factor you list could have been avoided or minimized.

The list of factors that could cause error may include, but is not limited to:

- Measurement error could result when students determine the center of each fringe in their interference pattern by observation. Students can minimize this error by increasing the distance between the double-slit aperture and the wall, which will in turn increase the size of each fringe in the interference pattern. Once the fringes are large enough, students can then use their spatial measurement tool to more accurately locate the center of each fringe before marking it.
- Measurement error may result when making spatial measurements using tools such as a meter stick, measuring tape, or calipers. Students can minimize this error by being precise in their measurement technique, regardless of the measuring tool's accuracy. The accuracy of the graphical analysis method used in the Structured version of this lab activity relies on the relative spacing between the interference fringes, not the absolute distances.
- The diffraction plate may not have been parallel to the wall, or perpendicular to the path of the coherent light source. This would effectively make the slit spacing smaller, causing the interference fringes to appear further apart. Students can minimize this by aligning the light source and double-slit aperture so that the light reflected from the double-slit aperture is incident on (or near) the point where the light is emitted from the light source.

❷ 3. Ask your teacher for the actual slit spacing value, and then calculate the percent error between your experimental value and the actual value.

$$\text{Percent error} = \left|\frac{\text{Actual} - \text{Experimental}}{\text{Actual}}\right| \times 100$$

Sample calculation:

$$\text{Percent error} = \left|\frac{0.0250\text{ cm} - 0.0255\text{ cm}}{0.0250\text{ cm}}\right| \times 100 = 2.00\%$$

❷ 4. Explain how your data would differ if you had used slits spaced twice as far apart and half as far apart.

According to the equation:

$$x_m = m\frac{\lambda L}{d}$$

slit spacing d and fringe spacing x_m are inversely proportional. Doubling slit spacing d would result in halving x_m (smaller fringe spacing), while cutting slit spacing d in half would double x_m (greater fringe spacing).

❷ 5. Would the data in your experiment differ if the distance between your laser and the double-slit aperture had been much greater? Justify your answer.

Changing the distance between the laser light source and the double-slit aperture will have no effect on student data. Students should indicate one of these two justifications:

- Huygens's principle: Light incident on the double slits is re-emitted as independent light waves from each slit, implying that the behavior of the light waves emitted from the double slits has no relationship to the distance between the coherent light source and the double-slit aperture. Going further, the double-slit interference pattern observed by students is unaffected by changes to the spacing between the light source and the double-slit aperture, indicating that data would not differ
- Students following the procedure outlined in the Structured version of this lab activity can use the equation:

$$x_m = m\frac{\lambda L}{d}$$

to justify their response. Students should indicate that the expression shows no dependence on the spacing between laser light source and the double slits. The measurable values in the expression remain unchanged, mathematically, if the spacing between the light source and the double-slit aperture changes.

Synthesis Questions

❷ 1. A CO_2 gas laser ($\lambda = 1.06 \times 10^{-5}$ m) emits light incident on two very narrow, closely spaced slits that produce a diffraction and interference pattern on a screen 1.00 m away. If the slits are spaced 0.150 mm apart, how far from the central maxima is the 10th-order maxima?

Students must use Equation 1 to derive their solutions to this problem. Students using the paraxial approximation in their solutions will find the 10th-order fringe to be located 70.7 cm from the central maxima, which violates the paraxial approximation requirement that $L >> x_m$.

$$d\sin\theta = m\lambda \quad \theta = \tan^{-1}\left(\frac{x_m}{L}\right)$$

$$d\sin\left(\tan^{-1}\left(\frac{x_m}{L}\right)\right) = m\lambda$$

$$\tan^{-1}\left(\frac{x_m}{L}\right) = \sin^{-1}\left(\frac{m\lambda}{d}\right)$$

$$x_m = L\tan\left(\sin^{-1}\left(\frac{m\lambda}{d}\right)\right)$$

$$x_{10} = (1.00\text{ m})\tan\left(\sin^{-1}\left(\frac{(10)(1.06\times10^{-5}\text{ m})}{0.150\times10^{-3}\text{ m}}\right)\right) = (1.00\text{ m})\tan\left(\sin^{-1}(0.707)\right) = 1.00\text{ m}$$

❷ 2. Imagine you had a rigid sheet of metal foil with two extremely narrow, closely-spaced slits in it used as a particle shield in some advanced experiment. Before the shield can be used, you need to determine the spacing between the slits. Using a gamma ray gun ($\lambda = 1.00 \times 10^{-11}$ m) and a detector array 40.13 m away, you find that the diffraction pattern has a distance between the central maxima and the 1st-order maxima of 0.01332 m. What is the spacing between the slits?

Because the spacing between the shield and the array is much larger than the distance between the 0th-order and 1st-order maxima ($L >> x_m$), the paraxial approximation can be used to simplify Equation 1:

$$x_m = m\frac{\lambda L}{d}$$

$$d = m\frac{\lambda L}{x_m} = \frac{\lambda L}{x_1} = \frac{(1.00\times10^{-11}\text{ m})(40.13\text{ m})}{0.01332\text{ m}} = 3.01\times10^{-8}\text{ m}$$

❷ 3. When you observe a double-slit interference and diffraction pattern, the pattern is always symmetrical about a central bright point. In a few sentences, explain why a double-slit interference and diffraction pattern mirrors itself about this central bright point?

Student responses should indicate that the symmetric appearance of a double-slit interference pattern is the result of the overlap and interference of two identical spherical waves. The waves are in phase and have origins equidistant from the central bright point. Responses may be similar to this sample:

The interference pattern observed after coherent light waves travel through a double-slit aperture is symmetric due to the wave nature of light and the geometry associated with each wave's path. According to Huygens's principle, coherent light incident on the double slits is re-emitted as independent spherical waves from each slit. As the waves propagate from the slits, they overlap and cause interference. Because the waves have the same wavelength and phase, wherever the path length difference between the waves is equal to an integer multiple of the wavelength, there will be an area of constructive interference (bright fringe). This path length requirement is geometrically symmetric about the center line between the two slits.

Extended Inquiry Suggestions

This lab primarily covers the topic of interference, but it may be useful to go over the principles of diffraction and how they apply to this double-slit experiment. Convey that the width of the slits governs the diffraction component of this experiment and not the interference, and that changing the width of the slits does not affect the number of fringes or their locations. Show the students the primary diffraction envelope and how the fringes are "contained" within it, and that the size of this envelope changes as the width of the slit changes, but the interference fringes do not move.

8. INTERFERENCE AND DIFFRACTION

STRUCTURED

Driving Question | Objective

How can the wave nature of light be used to determine the distance between two closely-spaced narrow parallel slits? Use a coherent light source and the principles associated with double-slit interference and diffraction to experimentally determine the spacing between the slits in a double-slit pattern provided by your instructor.

Materials and Equipment

- PASCO Diffraction Plate[1]
- Table clamp or large base (2)
- Support rod, 45-cm (2)
- Three-finger clamp (2)
- Stainless steel calipers
- Laser pointer with known wavelength
- White paper
- Pencil
- Ruler
- Tape
- Measuring tape

[1]www.pasco.com/ap33

PASCO Diffraction Plate

Background

Single-slit diffraction pattern

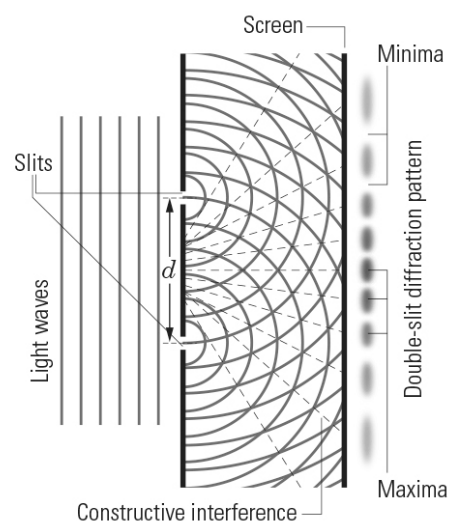
Double-slit interference pattern

Coherent light passing through a very narrow slit forms a distinct *diffraction* pattern, with clear areas of higher and lower intensity, on a screen. These areas are the result of conditional constructive and destructive interference between light waves at the viewing screen.

When coherent light passes through two very narrow, very closely spaced slits, the diffraction pattern includes many additional areas of bright and dark fringes compared to those from a single slit diffraction pattern.

The additional bright and dark fringes are the result of the interference of light emitted from each of the two slits. According to Huygens's Principle, when coherent light reaches two slits, each slit becomes an emission source for spherical light waves with the same wavelength as the original coherent light.

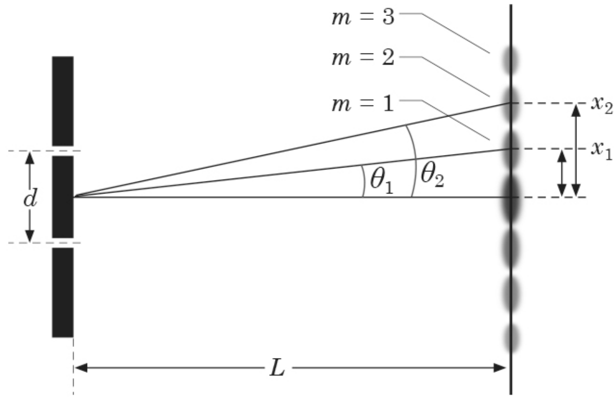
These light waves cross and interfere as they propagate, eventually reaching a screen where areas of constructive interference are seen as bright spots (maxima), and areas of destructive interference are seen as dark spots (minima).



The mathematical expression describing the position (angle) of the maxima within this double-slit interference pattern is:

$$d \sin\theta = m\lambda \qquad (1)$$

Where m is the order number, an integer ±1, ±2, ±3…, d is the distance between the slits, θ is the angle between each m order maxima in the interference pattern and the 0th-order (central brightest) maxima, and λ is the wavelength of the light source.



In practice, the angular position θ of each interference maxima is often expressed in terms of linear position x, where x_m is the linear distance between the 0th-order maxima and the mth-order maxima as observed in the interference pattern on a screen some distance L from the double-slit aperture. Going further, if the distance L between the slits and the screen on which you view the interference pattern is large compared to x_m, an approximation can be used to simplify Equation 1:

If $L \gg x_m$, then $\sin\theta \approx \tan\theta \approx \frac{x_m}{L}$ or, $\frac{dx_m}{L} \approx m\lambda$

Rearranging yields: $$x_m \approx m\frac{\lambda L}{d} \qquad (2)$$

RELEVANT EQUATIONS

$$d \sin\theta = m\lambda \qquad (1)$$

$$x_m \approx m\frac{\lambda L}{d} \qquad (2)$$

Safety

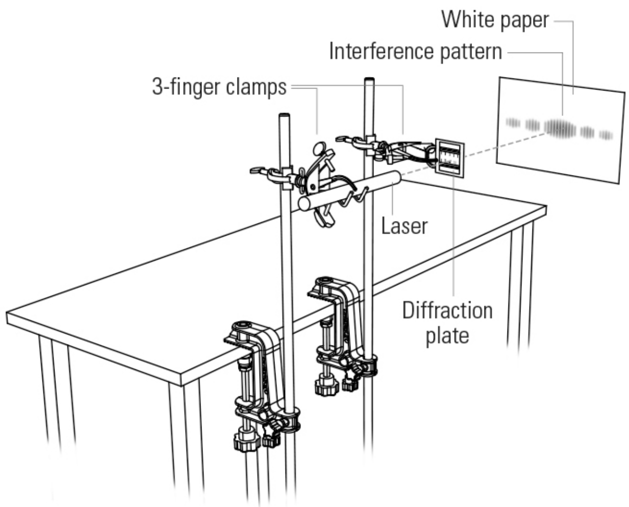
Follow these important safety precautions in addition to your regular classroom procedures:

- A laser can cause serious damage to the human eye. Do not look directly at the beam and only turn the laser on when necessary for alignment or data collection.
- Avoid laser beam reflections that may cause eye damage. Be aware of the laser beam at all times; you might be producing unknown stray reflections.

Procedure

SET UP

1. Assemble your equipment and align the laser as shown:



   a. Find an area on your lab table with enough space to align the components as in the illustration, with the laser aimed away from other lab groups, and toward a wall or other flat rigid surface on which you can attach the white paper.

   b. Use tape to hold the white paper in place against the wall or other surface.

   c. Lightly clamp the edge of the diffraction plate (the edge parallel to the slits on the plate) in the three-finger clamp. Be certain the double-slit patterns on the plate are vertical and the fingers of the clamp touch only the edge of the plate and not the plate film.

      NOTE: Over-tightening the clamp will damage the plate.

   d. Adjust the setup so that the laser beam is perpendicular to the diffraction plate, and the white paper and diffraction plate are parallel to each other.

   e. Have at least one meter of space between the diffraction plate and the paper. Use as large a distance as possible while ensuring visibility of the light on the paper.

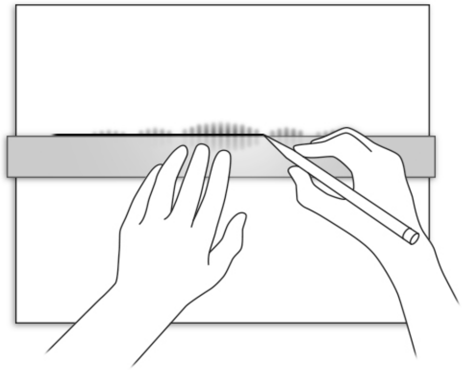
2. Choose one of the three double-slit patterns on the diffraction plate, “D,” “E,” or “F,” and adjust the laser so the beam shines on that pattern. A clear image of the interference pattern should appear on the white paper. Record which double-slit pattern you chose: “D,” “E,” or “F,” in the space indicated at the top of the Data Analysis section.

   NOTE: Aim the laser slightly downward at the diffraction plate to prevent stray reflection of the laser beam traveling upward into a classmate's eyes. Also, you might want to place an object behind your setup (behind the laser) to catch any reflection of the laser beam from the diffraction plate.

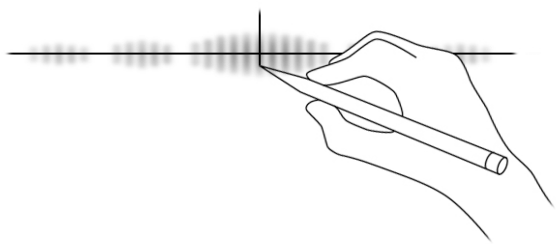
3. Once aligned, tighten the setup components so the laser and diffraction plate do not move during data collection.

COLLECT DATA

4. With the interference pattern clearly shown, use the ruler and pencil to draw a horizontal line through the center of the interference pattern on the paper.



5. Make a small mark on the line you just drew at the center of the 0^{th}-order (central brightest) maxima.



6. Continue to make small marks at the centers of the neighboring five maxima to the right or left of the 0^{th} maxima.

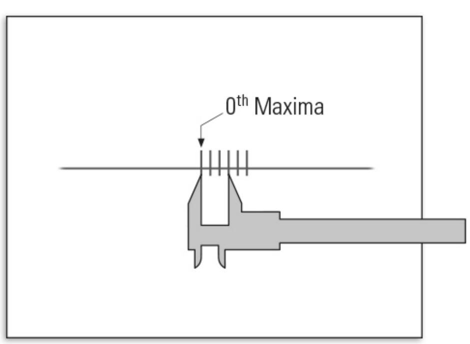
7. Turn off the laser, and then record its wavelength λ in the Data Analysis section below.

   NOTE: The wavelength is usually printed on a sticker on the outside of the laser. If it is not printed on the laser, ask your instructor for this value.

8. Use the measuring tape to measure the distance L between the diffraction plate and the white paper. Record this value in the Data Analysis section.

9. Remove the white paper and place it on your lab table.

10. Use the calipers to accurately measure the distance from the 0^{th} maxima mark (m = 0) to each higher order maxima (m = 1, 2, 3, 4, 5). Record each distance (x_m) in Table 1 in the Data Analysis section below.



Data Analysis

Double-slit pattern: ______________________

λ = ________________________________

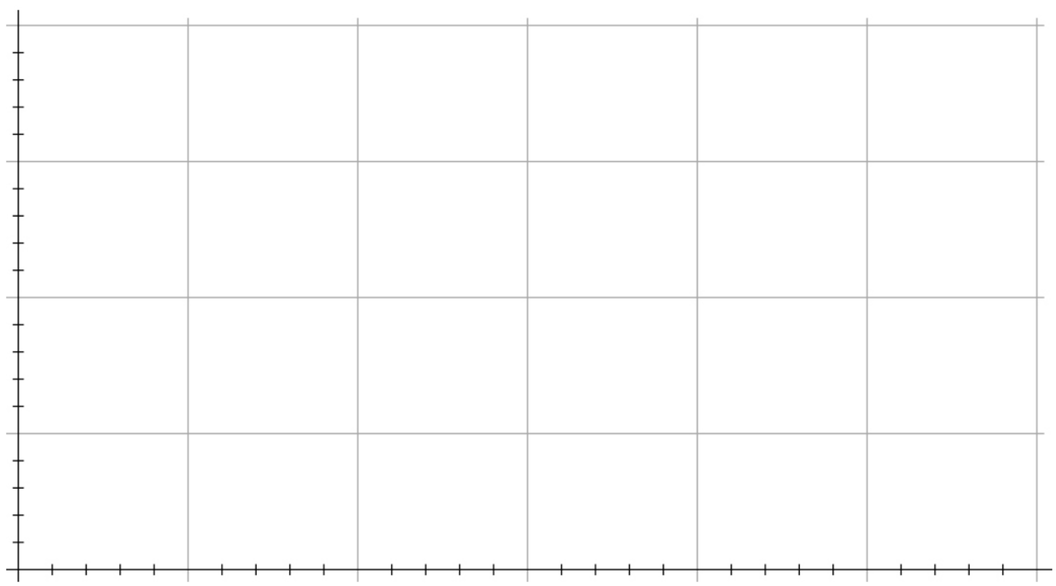
L = ________________________________

Table 1: Measurements for determining the slit spacing

Integer Order Maxima *m*	**Distance from 0th Maxima x_m (cm)**
1	
2	
3	
4	
5	

1. Plot a graph of *distance of interference maxima from 0th maxima (x_m)* versus *integer order (m)* in the blank Graph 1 axes below. Be sure to label both axes with the correct scale and units.

Graph 1: Distance of interference maxima from 0th maxima versus integer order of double-slit interference maxima



2. Draw a line of best fit through your data in Graph 1. Record the equation of the line here:

   Best fit line equation: ________________________________

3. Use the slope from the best fit line to determine the spacing d between the parallel slits on the diffraction plate:

   $$\text{slope} = \frac{\lambda L}{d}$$

   Slit spacing d (cm): ________________________________

Analysis Questions

1. What is your experimental value for the spacing between the double slits, and how did you determine this value from your data?

2. What are factors that may have caused error in your experimental value of slit spacing? Explain how each factor you list could have been avoided or minimized.

3. Ask your teacher for the actual slit spacing value, and then calculate the percent error between your experimental value and the actual value.

$$\text{Percent error} = \left|\frac{\text{Actual} - \text{Experimental}}{\text{Actual}}\right| \times 100$$

4. Explain how your data would differ if you had used slits spaced twice as far apart and half as far apart.

5. Would the data in your experiment differ if the distance between your laser and the double-slit aperture had been much greater? Justify your answer.

Synthesis Questions

1. A CO_2 gas laser ($\lambda = 1.06 \times 10^{-5}$ m) emits light incident on two very narrow, closely spaced slits that produce a diffraction and interference pattern on a screen 1.00 m away. If the slits are spaced 0.150 mm apart, how far from the central maxima is the 10^{th}-order maxima?

2. Imagine you had a rigid sheet of metal foil with two extremely narrow, closely-spaced slits in it used as a particle shield in some advanced experiment. Before the shield can be used, you need to determine the spacing between the slits. Using a gamma ray gun ($\lambda = 1.00 \times 10^{-11}$ m) and a detector array 40.13 m away, you find that the diffraction pattern has a distance between the central maxima and the 1^{st}-order maxima of 0.01332 m. What is the spacing between the slits?

3. When you observe a double-slit interference and diffraction pattern, the pattern is always symmetrical about a central bright point. In a few sentences, explain why a double-slit interference and diffraction pattern mirrors itself about this central bright point?

9. ELECTRIC FIELD MAPPING

Connections to the AP® Physics 2 Curriculum*

The lab activity correlates to the following pieces of the AP® Physics 2 framework:

Big Idea	2	Enduring Understanding	E	Essential Knowledge	2

Learning Objective 1: The student is able to determine the structure of isolines of electric potential by constructing them in a given electric field.
Science Practices: 6.4, 7.2

Time Requirement

Preparation Time: 40 minutes

Lab Activity: 50 minutes

Prerequisites

Students should be familiar with the following concepts:

- Electric fields are a useful way to model fundamental forces that can be exerted between charged objects at a distance without direct physical contact between the objects.
- Electric fields can be modeled using an array of vectors that indicate the direction and magnitude of the field at a given point in space.
- The *electric potential difference* (voltage) between two points in an electric field indicates the difference in electric potential energy per unit charge between the two points; the amount of energy (work) per unit charge required to move a charge from one point to the other.

Driving Question | Objective

How can the characteristics of the electric field surrounding oppositely-charged electrodes in two configurations, dipole and parallel plates, be determined experimentally? Use the principles of electric fields and electric potential energy to experimentally determine the lines of equal electric potential (isolines) surrounding oppositely-charged electrodes and the shape and direction of the electric field lines in each configuration.

Procedural Overview

In the Structured version of this lab activity, students draw 10 continuous electric field lines on each of two electrode configurations: two dipole electrodes, and two parallel-plate electrodes.

Students use a DC power supply to generate a 10-volt potential difference between the two electrodes drawn on semi-conductive paper. Using a volt meter, students probe the paper in the space surrounding the electrodes and mark points where the electric potential difference between the electrode and probe is constant. When enough marks are made, students connect them as a continuous line of equal electric potential (isoline).

After identifying and drawing several isolines surrounding both electrodes, students use them to draw continuous electric field lines. These originate from the positive electrode and terminate at the negative electrode. Each field line leaves the surface of the positive electrode at a right angle, crosses each isoline at a right angle, and terminates at the surface of the negative electrode at a right angle.

Pre-Lab Discussion and Activity

Before performing this lab activity, students must understand the strategies and rules associated with drawing two-dimensional electric field lines using isolines of electric potential. Students using the Structured version of this lab activity will find that information in their student handouts. Use the same background information from the Structured student handout as the basis for a classroom discussion for those students using the Guided Inquiry or Student Designed versions.

The core rules for drawing electric field line in a two-dimensional representation are:

- Electric field lines begin on a positive charge or charged object and terminate on a negative charge or charged object.
- The number and spacing of field lines leaving or entering a charged surface is proportional to the magnitude of charge on the surface. The density of electric field lines in a given area represents the relative strength of the electric field.
- Field lines always leave or enter perpendicular to the surface.
- Electric field lines cross isolines of electric potential at right angles.
- Electric field lines in the same electric field do not cross.

Each student group will use a PASCO Field Mapper Kit to construct their experimental setup, regardless of the activity version. As understanding the construction and operation of the semi-conductive paper and conductive ink in the PASCO Field Mapper Kit may play an important role in their experiment design, it is recommended that you integrate this information into the pre-lab discussion. A complete explanation can be found in the user's guide that accompanies the kit. A cursory explanation is provided here:

Each field mapper kit comes with metal push pins, wires, a corkboard work surface, and the two most critical elements: semi-conductive carbon-impregnated paper and a conductive ink pen. Each sheet of semi-conductive paper is designed to have conductive electrodes drawn on it using the ink from the conductive ink pen.

Once electrodes have been drawn on the paper, an electrical potential difference can be established between the electrodes by electrically connecting them to the positive and negative terminals on a power supply. Because the paper is semi-conductive, a standard voltmeter will measure the electrical potential difference between the electrodes on the paper, as well as the electrical potential difference between any two points on the paper by connecting the voltmeter probes directly to the paper.

Using measurements of electric potential difference on the paper, and the basic relationships between electric potential difference and electric field lines, students can establish a two-dimensional representation of the electric field surrounding the conductive electrodes.

Materials and Equipment

- PASCO Conductive Paper with dipole electrodes drawn in conductive ink[1]
- PASCO Conductive Paper with parallel-plate electrodes drawn on it in conductive ink[1]
- Pushpin, metal[1] (6)
- Cork board[1]
- Power supply, 18-V, 3-A
- Digital multimeter
- 4-mm banana plug patch cord (4)
- 4-mm banana plug patch cord alligator clip (4)
- T-pin, metal
- Felt-tip marker, silver
- Pencil

Probeware Resources

Below are web-link and QR codes that will direct you to instructional video resources for individual pieces of PASCO probeware, sensors, and other hardware used in the lab activity. These same links and codes are provided to students in their activity handouts.

[1]www.pasco.com/ap34

PASCO Field Mapper Kit

Safety

Follow these important safety precautions in addition to your regular classroom procedures:

- Do not connect the terminals of a power supply without a load; this will cause a short circuit.

Lab Preparation

For this activity, each student group will need two sheets of semi-conductive paper, one with a dipole electrode configuration drawn on it in conductive ink, the second with a parallel-plate electrode configuration drawn on it. It is recommended that these sheets be prepared by the instructor before the lab activity, but you may choose to have each group draw its own as long as you have a sufficient number of conductive ink pens—one for each group. Instructions on how to draw each electrode configuration are provided in the Appendix at the end of the Structured student handout.

Unless otherwise specified by the instructor, students carrying out the procedure in the Structured version of this lab activity expect to use sheets with electrode configurations prepared by the instructor.

Teacher Tips

Tip 1 – Voltmeter Input Impedance

- Students should use voltmeters with high input impedance (10 MΩ or higher). High input impedance will prevent distortion of the electric field due to current draw from the voltmeter.

Tip 2 – Using Voltmeter Probes

- Either PASCO voltage sensors and a data collection system or the PASCO electrometer can be used in place of the digital multimeter specified in the activity Materials and Equipment list. Students should measure electric potential difference by touching the voltmeter probes to the surface of the semi-conductive paper. Although any conductive contact between the probes and the paper will work, a probe with a pointed end will increase accuracy when determining the position of isolines on the paper. Students can attach an alligator clip to the end of the probes and then clamp a metal T-pin in the alligator clip to create a probe with a pointed end.
- Students should apply the same pressure between the voltmeter probes and the semi-conductive paper for every potential difference measurement. Inconsistently applied pressure will affect the precision in their measurements.
- While probing the paper as they make potential difference measurements, students should avoid contacting the grid marks on the semi-conductive paper. Touch the voltmeter probes only to the solid black areas of the paper or the conductive ink in the electrodes.

Tip 3 – Using Metal Pushpins

- Each PASCO Field Mapper Kit comes with several metal pushpins and a corkboard work surface. Students should mount their semi-conductive paper, with electrodes, to the corkboard using the pushpins. Use one pushpin in each corner of the paper to hold it in place on the corkboard, and then place one pushpin in each electrode (the position of the pushpin in the electrode is not important).

  Push each pushpin through the paper and into the corkboard far enough to hold the pushpin firmly in the cork. The pushpins in each electrode are now electrically connected to the electrodes and may be connected to potential difference sources, such as the positive and negative terminals of a power supply.

Tip 4 – Conductive Ink Pen

- The conductive ink pen in the PASCO Field Mapper Kit has a limited shelf life, as the ink may dry. To prevent drying, have the cap on the pen at all times except when it is used to draw electrodes onto the semi-conductive paper. Shake the pen thoroughly before drawing electrodes, and lightly squeeze the pen as you draw them. If your pens are dry, please visit the PASCO website for information regarding replacements.

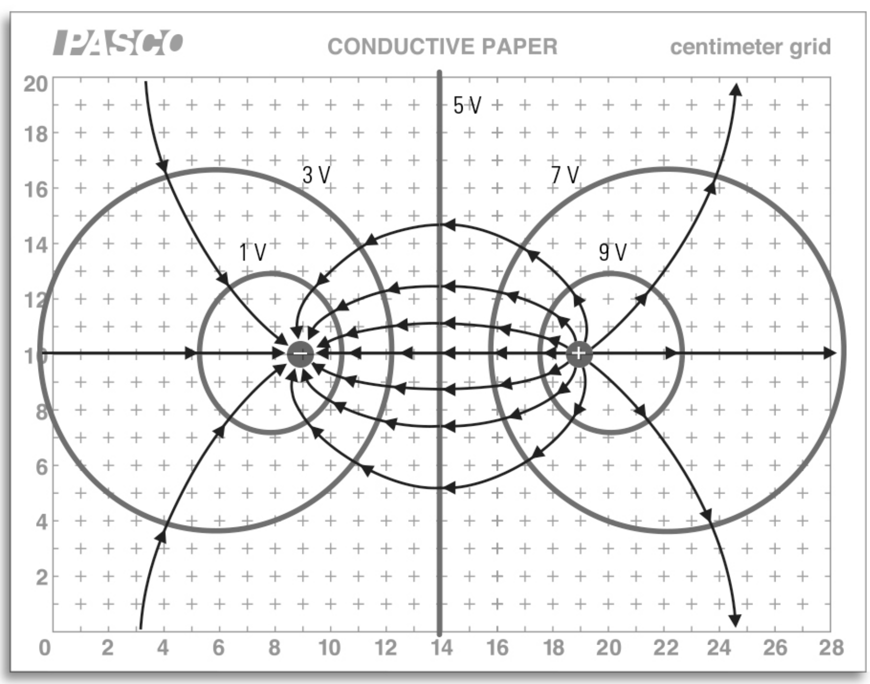
Sample Data

Below are sample data, acquired using the experimental setup and procedure outlined in the Structured version of the lab activity, and answers to questions in the Data Analysis section.

Data Analysis

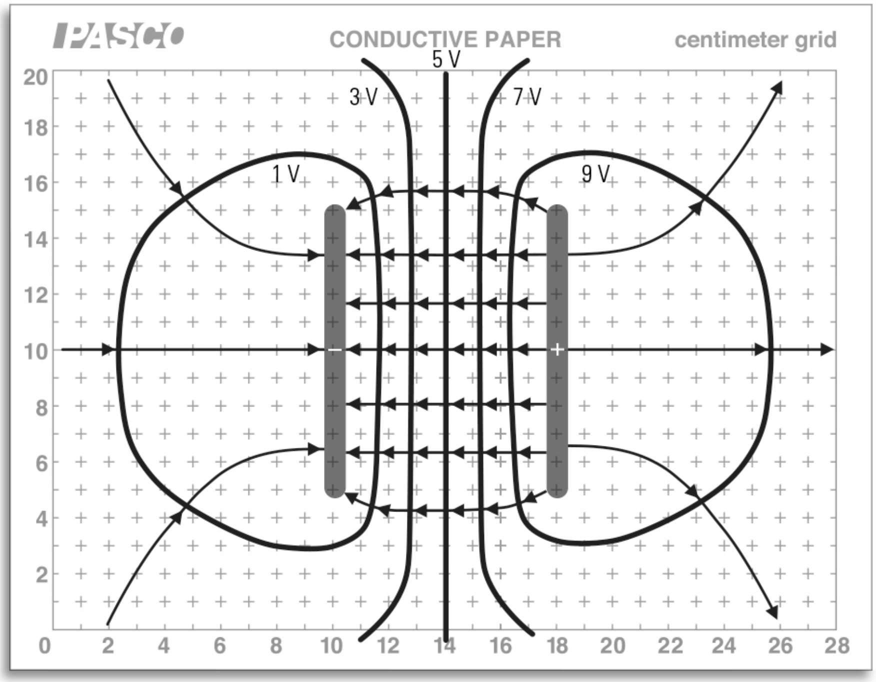
PART 1 – DIPOLE ELECTRODES

Sample student drawing of the isolines and electric field lines surrounding a dipole electrode configuration:



PART 2 – PARALLEL-PLATE ELECTRODES

Sample student drawing of isolines and electric field lines surrounding a parallel-plate electrode configuration:



Guided Inquiry Questions

Below are sample responses to the Guiding Questions found in the Guided Inquiry version of this lab activity.

❓ 1. What are electric field lines and how are they useful?

Student responses will vary. Correct responses should describe electric field lines as lines or vectors that represent the direction and magnitude of an electric field surrounding a charge or charged object.

These field lines can be used to determine the direction and magnitude of force experienced by a charge or charged object within the field.

❓ 2. Each group has two sheets of semi-conductive paper, each with an electrode configuration drawn on it in conductive ink. To create a two-dimensional electric field on the surface of the paper between the electrodes, you must first use a power supply to establish an electric potential difference between them. How will you connect a power supply to achieve this?

Students should electrically connect the positive terminal from their power supply to one of the electrodes, and the negative terminal to the other electrode. (The terminals are created by pushing a metal pushpin into each electrode on the semi-conductive paper.) These connections will produce a constant and continuous electric potential difference between the electrodes, equal to the voltage displayed on the power supply.

❷ 3. In an electric field line drawing, a greater density of field lines leaving or entering a surface (more lines per surface area) is representative of the charge density on the surface. Consider the parallel-plate electrode configuration. When a potential difference is established between the plates, each plate has an equal but opposite charge. Will the density of field lines leaving the inner surface of the positive electrode (the surface facing the negative electrode) be the same as the density of its outer surface? Explain your answer.

Charges move freely along the surface of a conductor. A single positively-charged plate will have a uniform charge density on its surface. However, the charge density of a positively charged plate placed near and parallel to an equally, but negatively, charged plate will not have a uniform charge density on either plate. As the opposite charges are attracted to each other, charges migrate to the inner surfaces of the plates. This creates an induced non-uniform charge distribution in which the inner surfaces of the plates have a higher charge density than the outer surfaces. Therefore, there will be a greater density of field lines on the inner surfaces of the parallel-plate electrodes compared to the outer surfaces.

❷ 4. Once an electric field has been established between the two conducting electrodes on the semi-conductive paper, how is a voltmeter used to measure the electric potential difference between two points in the field?

A standard voltmeter or electrometer, with high input impedance, measures the difference in electric potential between its probes (also known as electrodes), or between the two points or objects connected to its probes. This electric potential difference is measured in units of volts.

To measure the electric potential difference between two points on the paper, students connect the probes to different points on the surface of the paper. The value reported by the voltmeter represents the electric potential difference between those two points.

❷ 5. What are *isolines* of electric potential and how can the shape and values of isolines of electric potential be used to help draw electric field lines?

Students should indicate that isolines of electric potential are continuous lines in an electric field that represent a path of equal electric potential.

Electric field lines are always perpendicular to isolines of electric potential and in the direction of increasing electric potential. Isolines of electric potential can be used as a guide to drawing electric field lines as long as each electric field line crosses an isoline at a right angle, and is in the direction of increasing electric potential between isolines.

Assessment Questions: Sample Responses

Sample responses to the Analysis and Synthesis questions found in each version of the lab activity:

Analysis Questions

❷ 1. What are the primary similarities and differences between the electric fields surrounding each electrode configuration?

Some of the primary similarities between the electric field drawings:

- Both electric fields are symmetrical.
- The greatest field line density is directly between the electrodes.
- The electric fields are most uniform (constant field line density) directly between the electrodes.
- There exists a totally flat isoline of electric potential between the midpoints of the electrodes in both configurations.

Some of the primary differences between the electric field drawings:

- The parallel-plate electrodes have a uniform field line density over a greater distance between the electrodes.
- The isolines of electric potential are flat directly between the parallel-plate electrodes, and curved directly between the dipole electrodes (except at the midpoint).

❷ 2. Where was the electric field strongest in each electrode configuration? Justify your answer.

For both electrode configurations, the electric field strength is greatest in the space directly between the electrodes. Students can justify this using the magnitude of field line density in their drawings: the area where the field line density is greatest represents the area where the field strength is greatest.

❷ 3. What can be said about the voltage at each point of an isoline of electric potential?

The voltage measured using a voltmeter or electrometer is the same at any point on the same isoline of electric potential.

❷ 4. What do the electric field lines represent?

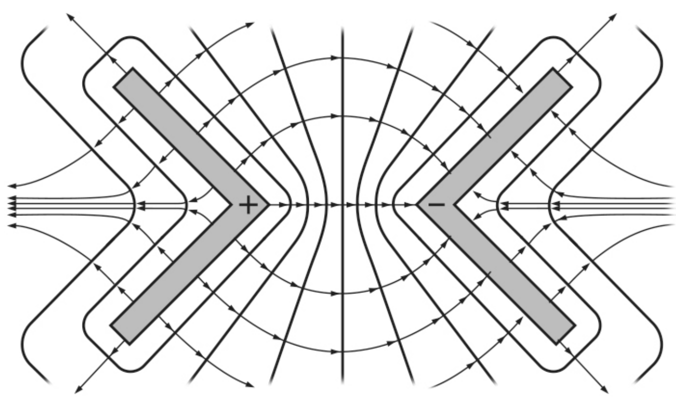
Electric field lines are lines or vectors that represent the direction and magnitude of an electric field surrounding a charge or charged object.

❷ 5. How would the shape of the electric field lines in each configuration change if you increased the voltage across the electrodes? Justify your answer.

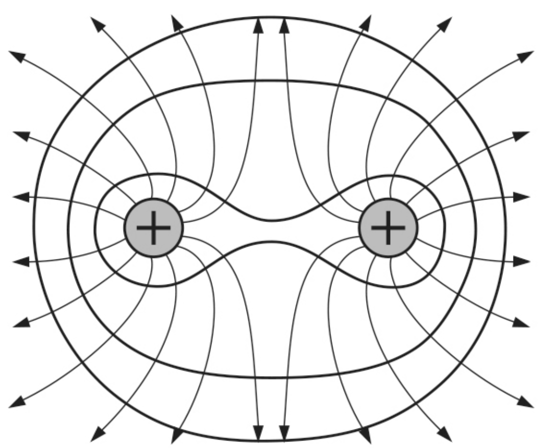
Increasing the voltage (potential difference) across the electrodes will not change the shape of the electric field in either configuration, and thus, will not change the shape of the electric field lines. However, an increased potential difference across the electrodes results in an increased charge density on each electrode, which would cause the electric field line density to increase.

Synthesis Questions

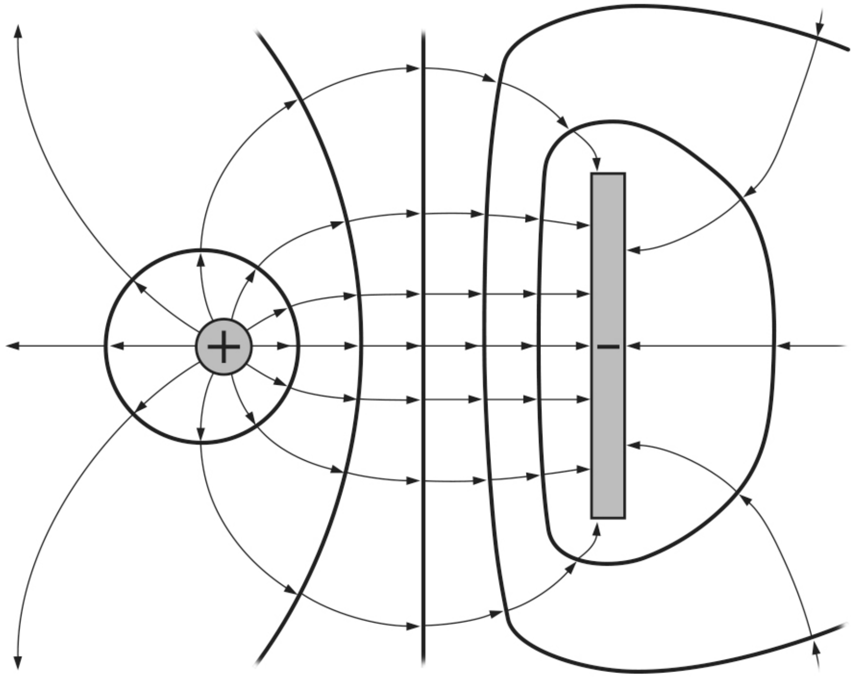
❷ 1. The diagram below shows an electrode configuration with corresponding isolines of electric potential. Indicate the shape of the electric field by drawing 15 evenly-spaced electric field lines from the positive electrode to the negative electrode.



❷ 2. The diagram below shows an electrode configuration with corresponding electric field lines. Draw three isolines of electric potential surrounding the electrodes. Space the isolines as evenly as possible.



❷ 3. Based on your experience in this lab activity, estimate the shape of the electric field surrounding the electrodes shown below: draw isolines of electric potential and enough electric field lines surrounding the electrodes to distinguish the shape of the field corresponding to your prediction. Assume that the amount of charge on both electrodes is equal and opposite.



Extended Inquiry Suggestions

Despite the qualitative nature of this lab, there are quantitative applications if teachers wish to pursue them.

Once students have drawn electric field lines, have them measure the voltage drop per centimeter and then calculate how the electric field strength varies along an electric field line. One way to do this is to tape the positive and negative probes of a voltmeter together so they are exactly 1 cm apart (banana plugs often have plastic housings that allow for this), and students can work their way along an electric field line, measuring the voltage drop at consistent intervals from the positive electrode to the negative electrode.

If time permits, students can repeat this lab with different electrode shapes. The lab is simplest when two point charges of opposite sign make up the electrodes, and the parallel plates used in this activity are especially important for the study of capacitors, but two circles, or a plate and a circle, can be alternative electrode patterns to illuminate common electric fields.

A circular electrode can be tested to show students that while the electric field surrounds the outside of the circle, it is zero inside. This is a great example of a Faraday shield, and there are many videos and photos online of people sitting safely inside a metal cage while huge bolts of lightning strike the cage.

As another application of a Faraday shield, put a student's cell phone inside an anti-static bag (computer boards and RAM often come in these), call the phone and see if it rings. Because of the shielding around the phone, resulting in an absence of an electric field as in the inside of a conductive sphere, the phone may not ring.

NAME PERIOD DATE

9. ELECTRIC FIELD MAPPING

STRUCTURED

Driving Question | Objective

How can the characteristics of the electric field surrounding oppositely-charged electrodes in two configurations, dipole and parallel plates, be determined experimentally? Use the principles of electric fields and electric potential energy to experimentally determine the lines of equal electric potential (isolines) surrounding oppositely-charged electrodes and the shape and direction of the electric field lines in each configuration.

Materials and Equipment

- PASCO Conductive Paper with dipole electrodes drawn in conductive ink[1]
- PASCO Conductive Paper with parallel-plate electrodes drawn on it in conductive ink[1]
- Pushpin, metal[1] (6)
- Cork board[1]
- Power supply, 18-V, 3-A
- Digital multimeter
- 4-mm banana plug patch cord (4)
- 4-mm banana plug patch cord alligator clip (4)
- T-pin, metal
- Felt-tip marker, silver
- Pencil

[1]www.pasco.com/ap34

PASCO Field Mapper Kit

Background

All charged objects produce electric fields in the space surrounding them. Knowing the shape, direction, and magnitude of an electric field is necessary to determine how a charged particle will interact with the field. Visualization is often helpful when analyzing fields and field forces; however, visualizing a 3-dimensional field can be difficult. A convenient way of representing an electric field is through the use of *electric field lines*.

Electric field lines are drawn lines that follow the path of the electric field, originating from a positive charge (or charged object) and terminating at a negative charge (or charged object). The lines never cross, and the density of lines (the number of lines in a given area) represents the magnitude of the field strength. These rules are written more formally as:

- Electric field lines must begin on a positive charge and terminate on a negative charge.
- The number of electric field lines drawn leaving a positive charge or approaching a negative charge is proportional to the magnitude of the charge.
- No two electric field lines originating from the same source can cross.

Another characteristic of electric field lines is that they always travel perpendicularly across lines of equal electric potential known as *isolines*. If a charged particle were to follow one of these lines, its electric energy (or voltage) would not change. In contrast, if a charged particle were to move in the direction of the electric field, across the isolines of electric potential, work must be done on the particle by the force from the electric field.

In this activity, you will identify different isolines of electric potential surrounding a pair of charged electrodes, and then use those isolines as guides to draw electric field lines and identify the magnitude and direction of the electric field.

Safety

Follow these important safety precautions in addition to your regular classroom procedures:

- Do not connect the terminals of a power supply without a load; this will cause a short circuit.

Procedure

Part 1 – Dipole Electrodes

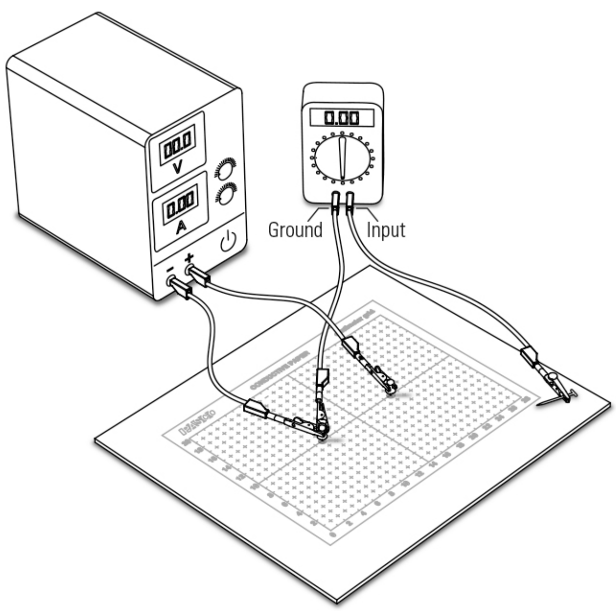
SET UP

1. Place the sheet of semi-conductive paper with the dipole electrodes drawn on it onto the cork board. Pin each corner of the paper to the cork board using metal pushpins.

   NOTE: If your instructor does not provide the semi-conductive paper with electrodes already drawn on them, follow the instructions in the Appendix at the end of this activity handout to draw the electrodes in conductive ink.

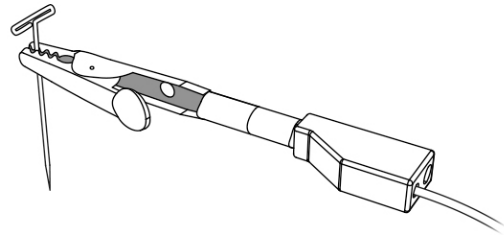
2. Press a metal pushpin into the center of each electrode, making sure that the pushpins are pressed firmly through the paper and into the corkboard.

3. With the power supply off, connect the electrode pushpins to the terminals on the power supply using two patch cords and alligator clips as shown: positive terminal to one electrode (this is the positive electrode), negative terminal to the other (this is the negative electrode).



4. Connect the two remaining patch cords to the DC voltage ports on the digital multimeter (DMM), and then adjust the DMM to measure DC voltage up to 10 VDC.

5. Attach alligator clips to the ends of the DMM patch cords. Clip the DMM “ground” or “COM” patch cord to the pushpin in the negative electrode (this is now the reference electrode). Connect the alligator clip on the other patch cord to the metal T-pin.



Collect Data

6. Turn on the power supply and adjust the voltage to 10 VDC.

7. Touch the tip of the T-pin to any black space surrounding either electrode on the semi-conductive paper and observe the voltage measurement on the DMM. The DMM should measure different voltages at different points on the paper. If not, make sure all of the alligator clips and pushpins are making good connections and retest.

   NOTE: Touch the tip of the T-pin only to the solid black areas of the semi-conductive paper. Do not touch the tip of the pin to the paper's grid marks.

8. Use the tip of the T-pin to probe different positions surrounding the electrodes until you find a location on the paper where the DMM reads 1.0 VDC. Use the felt-tip marker to make a small mark at this position.

9. Continue moving the probe to different points on the paper, identifying several positions surrounding the electrodes where the voltage is at 1.0 VDC. Mark each new point with the felt-tip marker until there are enough marks to accurately draw a smooth line that connects them. Label the line “1.0 V”.

   NOTE: Be sure to probe all areas surrounding the electrodes. Isolines of electric potential may connect in a closed path on the paper or extend off the edge of the paper and then re-enter the paper in an unexpected location.

10. Repeat the previous data collection steps, identifying and drawing the isolines of electric potential for each of these voltage values: 1.0 V, 3.0 V, 5.0 V, 7.0 V, and 9.0 V. Label each line with its corresponding voltage.

11. Turn the power supply off when finished.

Part 2 – Parallel-Plate Electrodes

Set Up

12. Using the sheet of semi-conductive paper with the two parallel-plate electrodes drawn on it, repeat the setup procedure outlined in Part 1. Clip the DMM ground patch cord to the pushpin in the negative plate electrode (this is now the reference electrode).

    NOTE: If your instructor does not provide the semi-conductive paper with electrodes already drawn on them, follow the instructions in the Appendix at the end of this activity handout to draw the electrodes in conductive ink.

Collect Data

13. Turn on the power supply and adjust the voltage to 10 VDC.

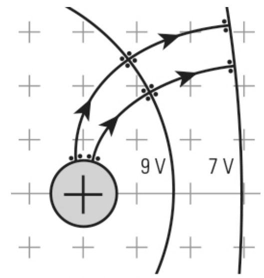
14. Following the Part 1 data collection steps, identify and draw the isolines of electric potential surrounding the parallel-plate electrodes for each of these DC voltage values: 1.0 V, 3.0 V, 5.0 V, 7.0 V, and 9.0 V.

15. Turn the power supply off when finished.

Data Analysis

Use the following steps to draw the electric field lines for each electrode configuration:

Part 1 – Dipole Electrodes



1. Using the pencil, draw an arrow starting from any point on the surface of the positive electrode and extending to the nearest isoline. Draw the arrow so it leaves the surface of the electrode at a right angle and intersects the first isoline at a right angle. You will find that to meet these requirements, the arrow must curve smoothly.

2. When you are satisfied with the shape of the arrow, draw over it using the felt-tip pen, and then draw a head on the arrow indicating the direction of the electric field: in the direction of decreasing electric potential from the positive electrode to the negative electrode.

3. From the tip of the arrow you just drew, draw another arrow like the previous one that extends from that isoline line to the next, and then another arrow to the next isoline, and so on until you reach the other electrode or the edge of the paper. Each arrow must obey the requirements outlined in the previous steps. Combined, the connecting arrows form one electric field line.

   NOTE: If your field line extends off the edge of the paper, it may also re-enter the paper in another location. Use the isolines of electric potential as your guide, drawing arrows from higher potential to lower potential.

4. Repeat these steps to draw a total of 10 electric field lines originating from the positive electrode, spacing the start of each line evenly along its surface.

Part 2 – Parallel-Plate Electrodes

5. Follow the Part 1 data analysis steps to draw 10 electric field lines from the positive parallel-plate electrode to the negative parallel-plate electrode as follows:

   a. Draw 7 of the 10 field lines leaving the inner surface of the positive electrode, and the remaining 3 field lines leaving the outer surface of the electrode.

   b. Space the 7 lines evenly along the inner surface, and the 3 lines evenly along the outer surface of the positive electrode. Draw each arrow with a smooth curve when necessary, and include arrow heads indicating the direction of the electric field.

   NOTE: This distribution of field lines is representative of the actual charge densities on the inner and outer surfaces of the plates. Because the plates are oppositely charged, there will be a non-uniform charge distribution due to the induction from the opposite plate.

Analysis Questions

❷ 1. What are the primary similarities and differences between the electric fields surrounding each electrode configuration?

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❷ 2. Where was the electric field strongest in each electrode configuration? Justify your answer.

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❷ 3. What can be said about the voltage at each point of an isoline of electric potential?

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❷ 4. What do the electric field lines represent?

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❷ 5. How would the shape of the electric field lines in each configuration change if you increased the potential difference between the electrodes? Justify your answer.

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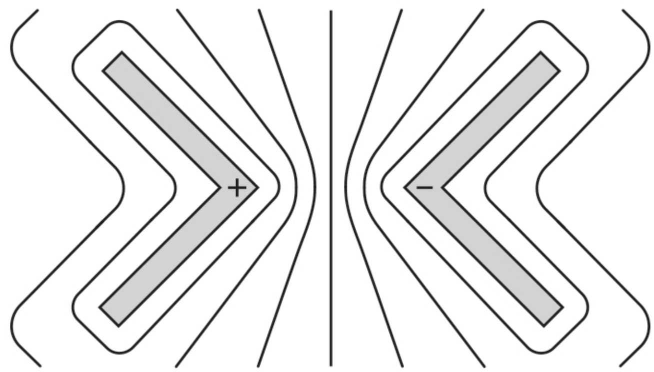
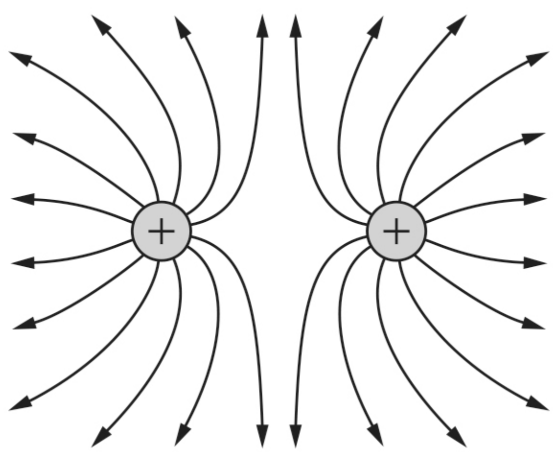
Synthesis Questions

1. The diagram below shows an electrode configuration with corresponding isolines of electric potential. Indicate the shape of the electric field by drawing 15 evenly-spaced electric field lines between the electrodes.



2. The diagram below shows an electrode configuration with corresponding electric field lines. Draw 3 isolines of electric potential surrounding the electrodes. Space the isolines as evenly as possible.



❓ 3. Based on your experience in this lab activity, estimate the shape of the electric field surrounding the electrodes shown below: draw isolines of electric potential and enough electric field lines surrounding the electrodes to distinguish the shape of the field corresponding to your prediction. Assume that the amount of charge on both electrodes is equal and opposite.

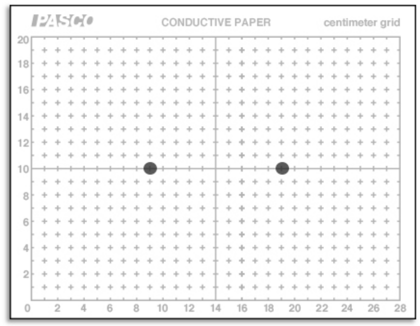
Appendix

Instructions: Drawing Electrodes Using Conductive Ink

- Conductive Paper, gridded (1 sheet/configuration) [1]
- Conductive ink pen[1]
- Ruler
- Pencil

[1]This equipment is from the PASCO Field Mapper Kit

PART 1 – DIPOLE ELECTRODES



1. Place a sheet of semi-conductive paper flat, printed side up, on your lab bench.

2. Use the pencil to sketch two dots, 10 cm apart, along the horizontal center-line on the gridded paper. Sketch each dot with a diameter of about 1 cm.

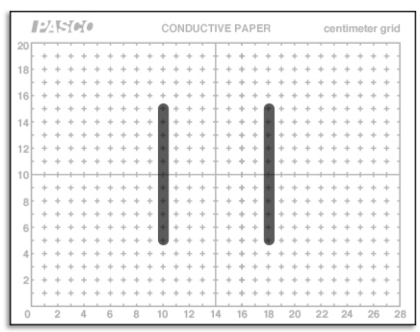
3. Shake the conductive ink pen (with the cap on) for 10-20 seconds. Then remove the cap and press the spring-loaded tip down on the semi-conductive paper, in the center of either dot.

4. Lightly squeeze the barrel on the pen until ink starts to slowly flow onto the paper, and then move the tip of the pen in circles inside the dot until the dot is completely filled with the ink.

5. If the dot is not completely filled in, draw over it again with the pen. A solid uniform dot is essential for good measurements.

6. Repeat the same process for the second dot. Allow the conductive ink 15–20 minutes to dry fully.

PART 2 – PARALLEL-PLATE ELECTRODES



7. Place the sheet of semi-conductive paper flat, printed side up, on your lab bench.

8. Use the ruler and pencil to sketch two straight lines, each 10 cm long, 8 cm apart, and perpendicular to the horizontal center-line on the gridded paper. Sketch each line with a width of about 0.5 cm.

9. Shake the conductive ink pen (with the cap on) for 10-20 seconds. Then remove the cap and press the spring-loaded tip down on the semi-conductive paper, in the center of either line.

10. Lightly squeeze the barrel on the pen until ink starts to slowly flow onto the paper, and then move the tip of the pen back and forth inside the sketched line until it is completely filled with the ink.

11. Use precision when drawing the electrodes. If the line is not of uniform thickness, it may not conduct, so go back and deposit more silver conductive material if necessary. If the line is not completely filled in, draw over it again. A solid uniform line is essential for good measurements.

12. Repeat the same process for the second line. Allow the conductive ink 15–20 minutes to dry fully.

10. MAGNETIC FIELDS

Connections to the AP® Physics 2 Curriculum*

The lab activity correlates to the following pieces of the AP® Physics 2 framework:

Big Idea	2	Enduring Understanding	D	Essential Knowledge	2

Learning Objective 1: The student is able to create a verbal or visual representation of a magnetic field around a long straight wire or a pair of parallel wires.
Science Practices: 1.1

Big Idea	2	Enduring Understanding	D	Essential Knowledge	3

Learning Objective 1: The student is able to describe the orientation of a magnetic dipole placed in a magnetic field in general and the particular cases of a compass in the magnetic field of the Earth and iron filings surrounding a bar magnet.
Science Practices: 1.2

Big Idea	2	Enduring Understanding	D	Essential Knowledge	4

Learning Objective 1: The student is able to use the representation of magnetic domains to qualitatively analyze the magnetic behavior of a bar magnet composed of ferromagnetic material.
Science Practices: 1.4

Time Requirement

Preparation Time: 15 minutes

Lab Activity: 30 minutes

Prerequisites

Students should be familiar with the following concepts:

- Magnetic fields are created by permanent magnets and current-carrying wires.
- A small magnet (such as the magnetized pointer in a compass) can be used to determine the direction of a magnetic field.

Driving Question | Objective

How do the characteristics of the magnetic field created by a bar magnet and a current-carrying coil differ? Determine the shape, including direction, of the magnetic fields created by a bar magnet and a current-carrying coil, and then compare the two, outlining the distinct differences and similarities between them.

Procedural Overview

In the Structured version of this lab activity, students compare the magnetic field pattern surrounding a bar magnet and a current-carrying coil. A Magnaprobe™ wand is used to determine the magnetic field pattern by placing the probe at multiple positions around the magnet and the coil, and recording the results by drawing an arrow in the direction the probe is pointing at each location.

Pre-Lab Discussion and Activity

To help students get familiar with the Magnaprobe wand, demonstrate how it works by using it to observe the Earth's magnetic field. Show the students that the magnet in the wand always points in the same direction no matter how you rotate the probe.

Since the Magnaprobe wand magnet is able to align itself with the magnetic field in all dimensions, students will notice that the magnet is not parallel to the surface of Earth. This is not due to the magnet being unbalanced. The angle at which the magnet dips depends on your location on Earth. In the northern hemisphere, the north end of the magnet points downward and in the southern hemisphere, the north end of the magnet points upward.

Materials and Equipment

- PASCO AC/DC Electronics Laboratory[1]
- Magnaprobe™ wand
- Power supply, 18-VDC, 3-A
- Bar magnet
- Magnet wire or enameled wire (4 m), 22-gauge
- Appendix Parts 1 and 2
- Sandpaper
- Scissors or wire cutters
- Beaker, 400-mL
- Wire lead
- Banana plug patch cord (2), 4-mm

Probeware Resources

Below are web-link and QR codes that will direct you to instructional video resources for individual pieces of PASCO probeware, sensors, and other hardware used in the lab activity. These same links and codes are provided to students in their activity handouts.

[1]www.pasco.com/ap04

PASCO AC/DC
Electronics Laboratory

Safety

Follow these important safety precautions in addition to your regular classroom procedures:

- Do not allow current to flow through the wires any longer than a few seconds. The wires (and possibly the power supply) can become hot and cause burns and damage equipment.
- Handle the magnets carefully. When held near each other, the magnets can suddenly snap together and pinch fingers.
- Keep the magnets away from electronic devices.

Teacher Tips

Tip 1 – Preparing the Coils before the Class Period

- If class time or materials are limited, you may want to prepare the coils prior to the class, following the beginning Set Up steps in Part 2 in the Structured version of the experiment.

Tip 2 – Using a Low Current Power Supply or the 550 Universal Interface

- If a high-current power supply is not available, you can use a lower-power supply (less than 1 amp) by constructing a coil with additional wraps. Good data can be obtained at 400 mA with a coil of 50 wraps. It is recommended that a smaller gauge wire be used, such as 26 gauge.
- A single D-cell battery can also be used if no power supply is available. A fresh D-cell battery can draw enough current to be used with a 10-turn or 50-turn coil. However, the battery will drain much faster when used with a 10-turn coil and will get hot very quickly due to the higher current draw.

Tip 3 – Alternative Materials

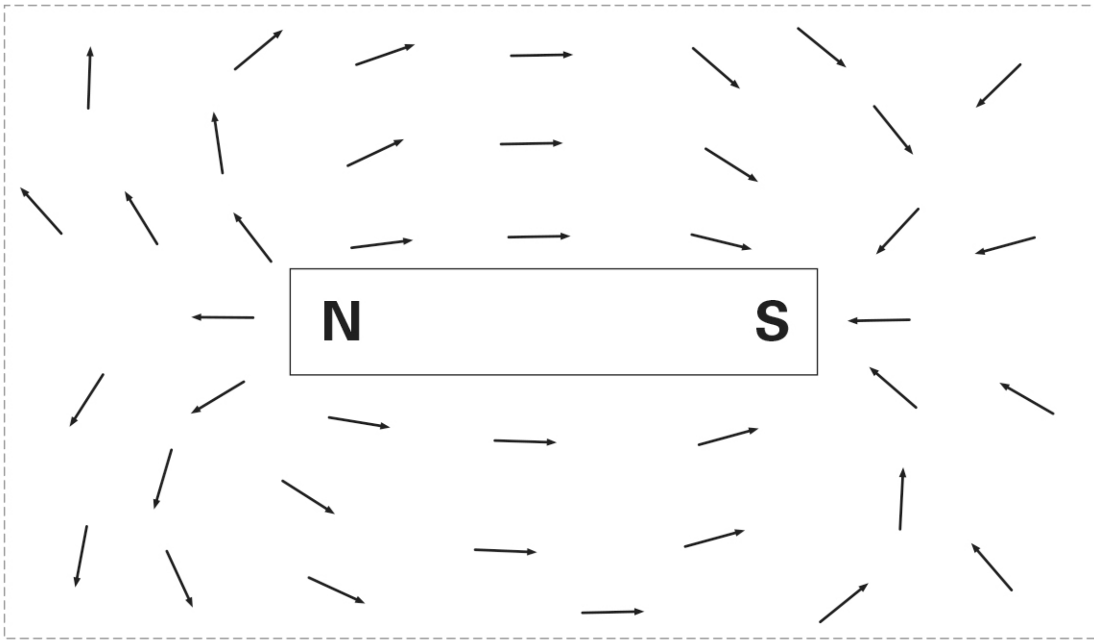
- PASCO AC/DC Electronics Laboratory alternative: The coil can be connected to the power supply using alligator wires and a pushbutton switch. Though a pushbutton switch is not required, it is highly recommended since it helps prevent current from flowing through the wires for an extended time.
- Bar magnet alternative: You can use stacks of rectangular ceramic magnets to create an alternate bar magnet.
- Magnaprobe wand alternative: A standard magnetic compass can be used in lieu of the Magnaprobe wand.

Sample Data

Below are sample data, acquired using the experimental setup and procedure outlined in the Structured version of the lab activity, and answers to questions in the Data Analysis section.

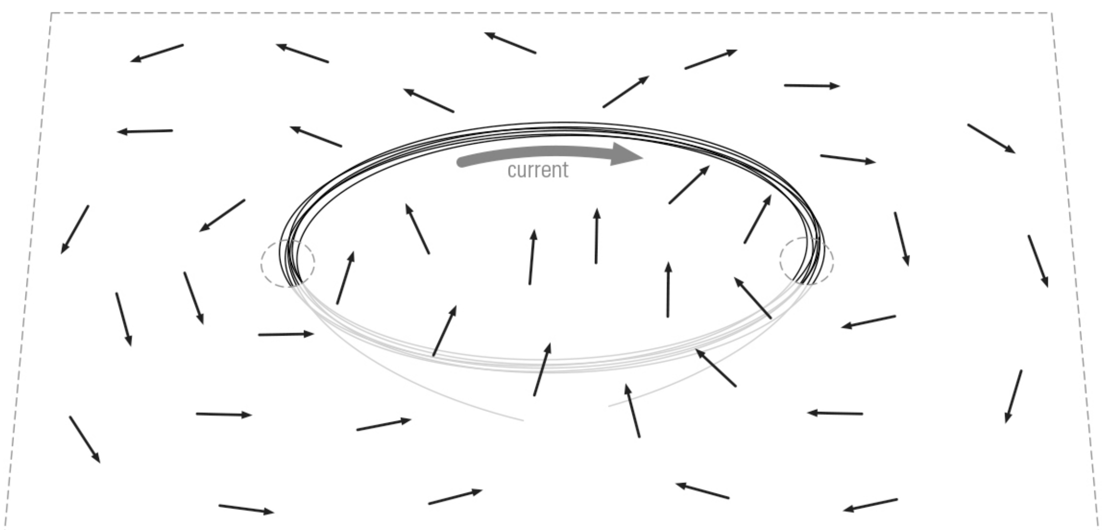
Data Analysis

Part 1 – Magnetic Field around a Bar Magnet



Magnetic field around a bar magnet

Part 2 – Magnetic Field around a Current-Carrying Coil



Magnetic field around a current-carrying coil

Guided Inquiry Questions

Below are sample responses to the Guiding Questions found in the Guided Inquiry version of this lab activity.

1. The magnet in the Magnaprobe wand points in the direction of a magnetic field. When documenting the magnetic field, which direction should you draw your arrow based on the indicators on the probe? Explain your reasoning.

   The north pole of the probe (colored red) indicates the direction of the field. The arrows need to be pointed in the direction of the field. Students may determine this by either reading the manual of the Magnaprobe wand or using a bar magnet marked with north.

2. How many arrows are needed in order to gain a good representation of the magnetic field? How should these arrows be arranged on the page? Explain your reasoning.

   No specific number of arrows is necessary, but a range of 25–50 arrows would be considered typical. The arrows should be spread around the page and not clumped together.

3. What is the benefit of creating a coil with multiple loops?

   A coil with multiple loops will create a stronger magnetic field that will be easier to detect using the Magnaprobe wand.

Assessment Questions: Sample Responses

Sample responses to the Analysis and Synthesis questions found in each version of the lab activity:

Analysis Questions

1. How does the magnetic field pattern created by a bar magnet compare to the magnetic field pattern created by a current-carrying coil? What are some of the significant differences and similarities between them?

   Both fields show curvature. However, the field surrounding a bar magnet shows the arrows pointing away from the north pole and curving toward the south pole of the magnet while the arrows circle around and through the current-carrying coil.

2. Describe how the coil used in your experiment could be rearranged or bent to create a magnetic field more closely resembling a field created by a bar magnet. Explain why you chose the bends and rearrangement that you did.

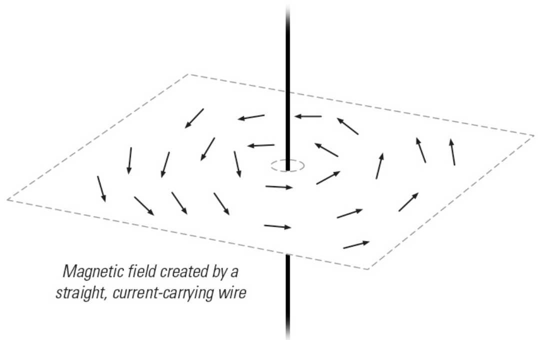
   The coils could be separated—stretched apart like a spring—until it's similar in length to the bar magnet. It is not bent in any way.

   The ends of the coil are analogous to the north and south poles of a bar magnet. Stretching out the coil forces the poles to be further separated, similar to the bar magnet.

3. Based on your exploration of the magnetic fields using the Magnaprobe wand, where did each magnetic field seem to be strongest? How did you come to these conclusions?

   The Magnaprobe wand shows that the field is stronger closer to the bar magnet or coil. This can be observed by the response of the Magnaprobe wand magnet. Students may notice that the magnet aligns itself with the magnetic field much more quickly and easily as the probe is brought closer to the bar magnet or coil.

4. If the coil in your experiment was unwound to create a very long, straight wire, what would the magnetic field look like in a plane perpendicular to the wire? Draw arrows in the diagram below to represent the magnetic field surrounding the wire when the current is directed upward.

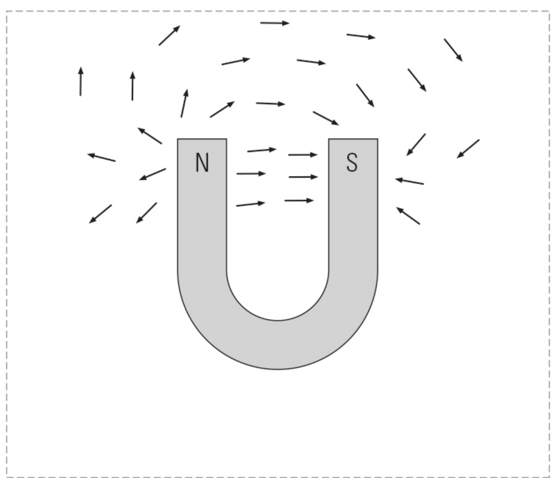


Magnetic field created by a straight, current-carrying wire

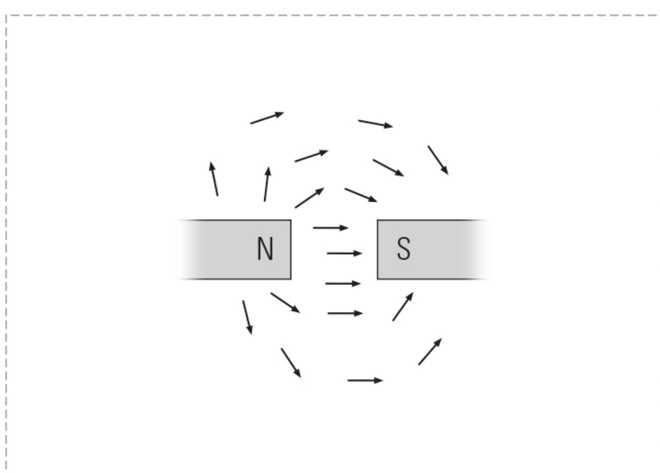
Synthesis Questions

1. Three arrangements of bar magnets are shown below. Draw the resultant magnetic field around the poles for each case.

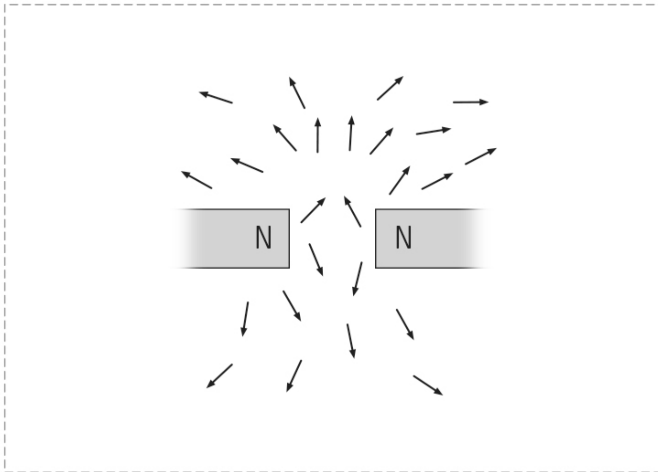
   a. Horseshoe magnet



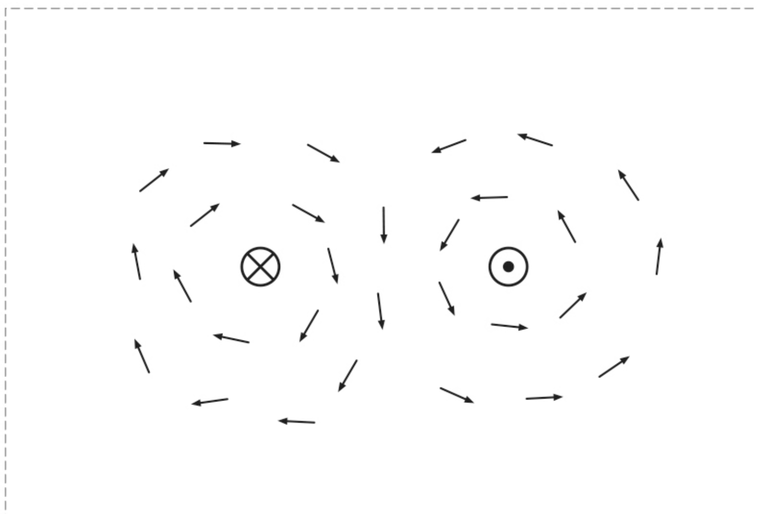
   b. North pole facing a south pole



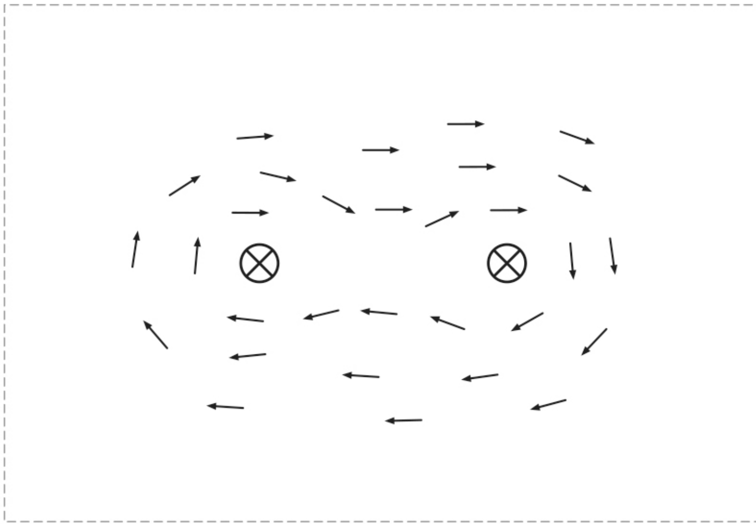
c. North pole facing a north pole



❷ 2. The symbols below represent two current-carrying wires extending vertically through the page (perpendicular to the surface of this page). ⊗ represents current flowing *into the page*, and ⊙ represents current flowing *out of the page*. Assuming that the magnitude of the current is the same in both wires, draw the magnetic field surrounding and between them.



❷ 3. The symbols below represent two current-carrying wires extending vertically through the page (perpendicular to the surface of this page), except now the current in both wires is flowing into the page. Assuming that the magnitude of the current is the same in both wires, draw the magnetic field surrounding (and between) them.



Extended Inquiry Suggestions

- Create an electromagnet by wrapping a wire around a long bolt or nail. Attach a power supply to the wire, similar to the coil in the experiment, and determine the magnetic field using the Magnaprobe wand.
- Use the Magnaprobe wand to determine the magnetic field around a very long, stiff wire, such as a coat hanger wire.
- Use iron filings to observe the magnetic field around the bar magnet and the coil. Place a sheet of paper over the bar magnet and sprinkle the iron filings on top of the paper. For the coil, obtain a piece of cardboard with two holes for the coil and sprinkle the iron filings on the cardboard while current is flowing through the coil.

NAME PERIOD DATE

10. MAGNETIC FIELDS

STRUCTURED

Driving Question | Objective

How do the characteristics of the magnetic field created by a bar magnet and a current-carrying coil differ? Determine the shape (including direction) of the magnetic fields created by a bar magnet and a current-carrying coil, and then compare the two, outlining the distinct differences and similarities between them.

Materials and Equipment

- PASCO AC/DC Electronics Laboratory[1]
- Magnaprobe™ wand
- Power supply, 18-VDC, 3-A
- Bar magnet
- Magnet wire or enameled wire (4 m), 22-gauge
- Appendix Parts 1 and 2
- Sandpaper
- Scissors or wire cutters
- Beaker, 400-mL
- Wire lead
- Banana plug patch cord (2), 4-mm

[1]www.pasco.com/ap04

PASCO AC/DC
Electronics Laboratory

Background

Sources of magnetic fields include permanent magnets and moving electrically charged objects, such as a current-carrying wire. Like electric fields, magnetic fields have direction. While electric field direction is determined by the distribution of positive and negative charges, magnetic field direction is determined by magnetic domains, which we label as *north* and *south*.

The direction and shape of a magnetic field can be determined by observing the direction in which a smaller magnet aligns itself in the field, similar to the pointer in a compass aligning to the earth's magnetic field. Use this technique with a Magnaprobe™ wand to determine the shapes and directions of the magnetic fields created by a bar magnet and a current-carrying coil, and then compare the two fields.

Safety

Follow these important safety precautions in addition to your regular classroom procedures:

- Do not allow current to flow through the wires any longer than a few seconds. The wires (and possibly the power supply) can become hot and cause burns and damage equipment.
- Handle the magnets carefully. When held near each other, the magnets can suddenly snap together and pinch fingers.
- Keep the magnets away from electronic devices.

Procedure

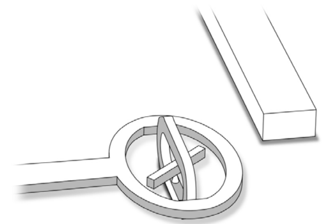
Part 1 – Magnetic Field around a Bar Magnet

SET UP

1. Remove the two Appendix pages from the end of this activity handout and set the second page (Part 2) aside.

2. Place the bar magnet in the center of the dashed area on the Part 1 appendix page, making sure that the polarity (N or S) of each end of the magnet is aligned with the markings provided on the paper.

COLLECT DATA



3. Hold the probe end of the Magnaprobe wand at some point near the magnet, close to the paper, and observe how the small magnet probe aligns itself in the field of the bar magnet. The north pole of the probe (colored red) indicates the direction of the field.

4. Beneath the probe, draw a small arrow (about the length of the magnet on the wand) with the arrow pointing in the direction of the field.

5. Repeat this for other points around the magnet until the area bordered by the dashed lines is filled with arrows. Keep the arrows evenly spaced (do not clump arrows) and include a sufficient number to identify the patterns of the magnetic field.

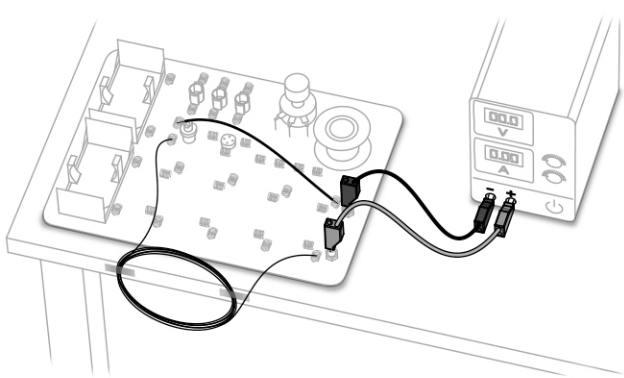
Part 2 – Magnetic Field around a Current-Carrying Coil

SET UP

6. Wrap the magnet (or enameled) wire 10 times around the beaker, forming a coil. Use scissors or wire cutters to leave about 20 cm of straight wire on each end, and then use sandpaper to sand off about 1 cm of insulation from the tips of each end of the wire.

7. Carefully slip the coil off the beaker and tape it to the edge of a table so half of the coil sits above the edge of the table.

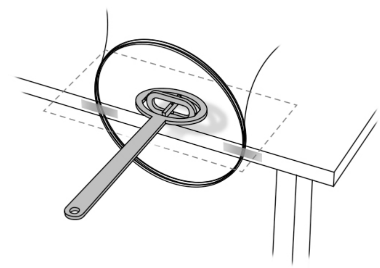
8. Connect the power supply, coil, and switch on the AC/DC Electronics Board as shown.



9. Turn the power supply on, press and hold the switch on the circuit board, and adjust the power supply so that 3.0 A of current is output to the coil. Release the switch after the adjustment is complete.

   CAUTION: Do not hold the switch for more than a few seconds. Repeatedly press the switch to adjust the power supply if needed, waiting 1–2 seconds in between adjustments.

COLLECT DATA



10. Observe the figure on the Part 2 appendix page. For your measurements in this part, hold the probe only in the plane perpendicular to the coil, as shown in the figure.

11. Hold the probe near the coil (in the plane perpendicular to it), and then press and hold the switch. Observe the orientation of the probe as it aligns itself in the magnetic field of the current-carrying coil. Release the switch.

    CAUTION: Do not hold the switch for more than a few seconds.

12. Draw an arrow in the blank plane on the Part 2 appendix page to represent the position and direction of the field while current was flowing through the coil. Also indicate the direction of current through the coil, using the convention of current flowing from positive to negative.

13. Repeat this for other points around and within the current-carrying coil until the dashed area is filled with arrows. Keep the arrows evenly spaced (do not clump arrows) and include a sufficient number to identify the patterns of the magnetic field.

Data Analysis

Part 1 – Magnetic Field around a Bar Magnet

Submit Appendix Part 1 showing the magnetic field lines of the bar magnet with your lab report.

Part 2 – Magnetic Field around a Current-Carrying Coil

Submit Appendix Part 2 showing the magnetic field lines of the current-carrying coil with your lab report.

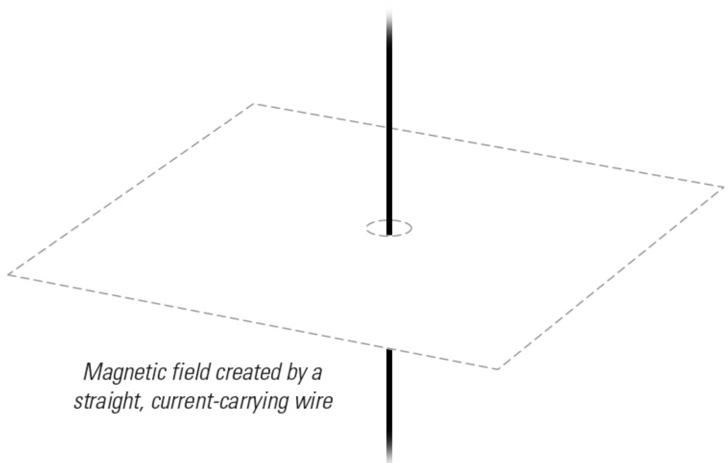
Analysis Questions

1. How does the magnetic field pattern created by a bar magnet compare to the magnetic field pattern created by a current-carrying coil? What are some of the significant differences and similarities between them?

❓ 2. Describe how the coil used in your experiment could be rearranged or bent to create a magnetic field more closely resembling a field created by a bar magnet. Explain why you chose the bends and rearrangement that you did.

❓ 3. Based on your exploration of the magnetic fields using the Magnaprobe wand, where did each magnetic field seem to be strongest? How did you come to these conclusions?

❓ 4. If the coil in your experiment was unwound to create a very long, straight wire, what would the magnetic field look like in a plane perpendicular to the wire? Draw arrows in the diagram below to represent the magnetic field surrounding the wire when the current is directed upward.

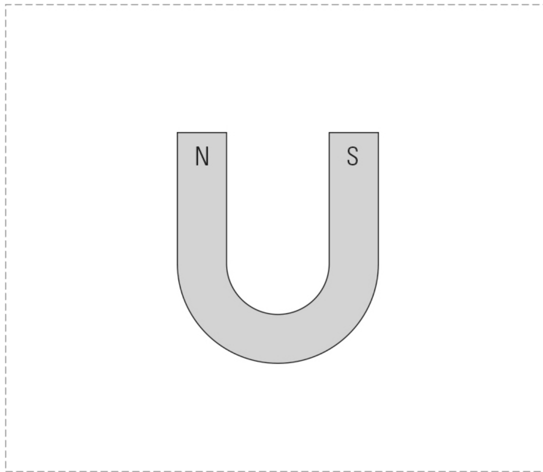


Magnetic field created by a straight, current-carrying wire

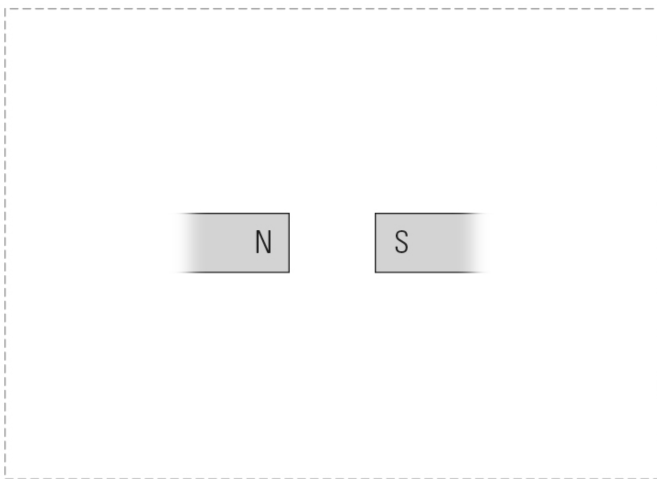
Synthesis Questions

1. Three arrangements of bar magnets are shown below. Draw the resultant magnetic field around the poles for each case.

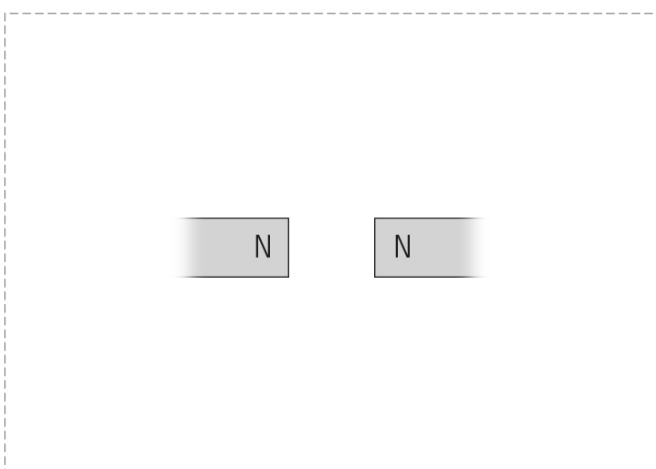
   a. Horseshoe magnet



   b. North pole facing a south pole



   c. North pole facing a north pole

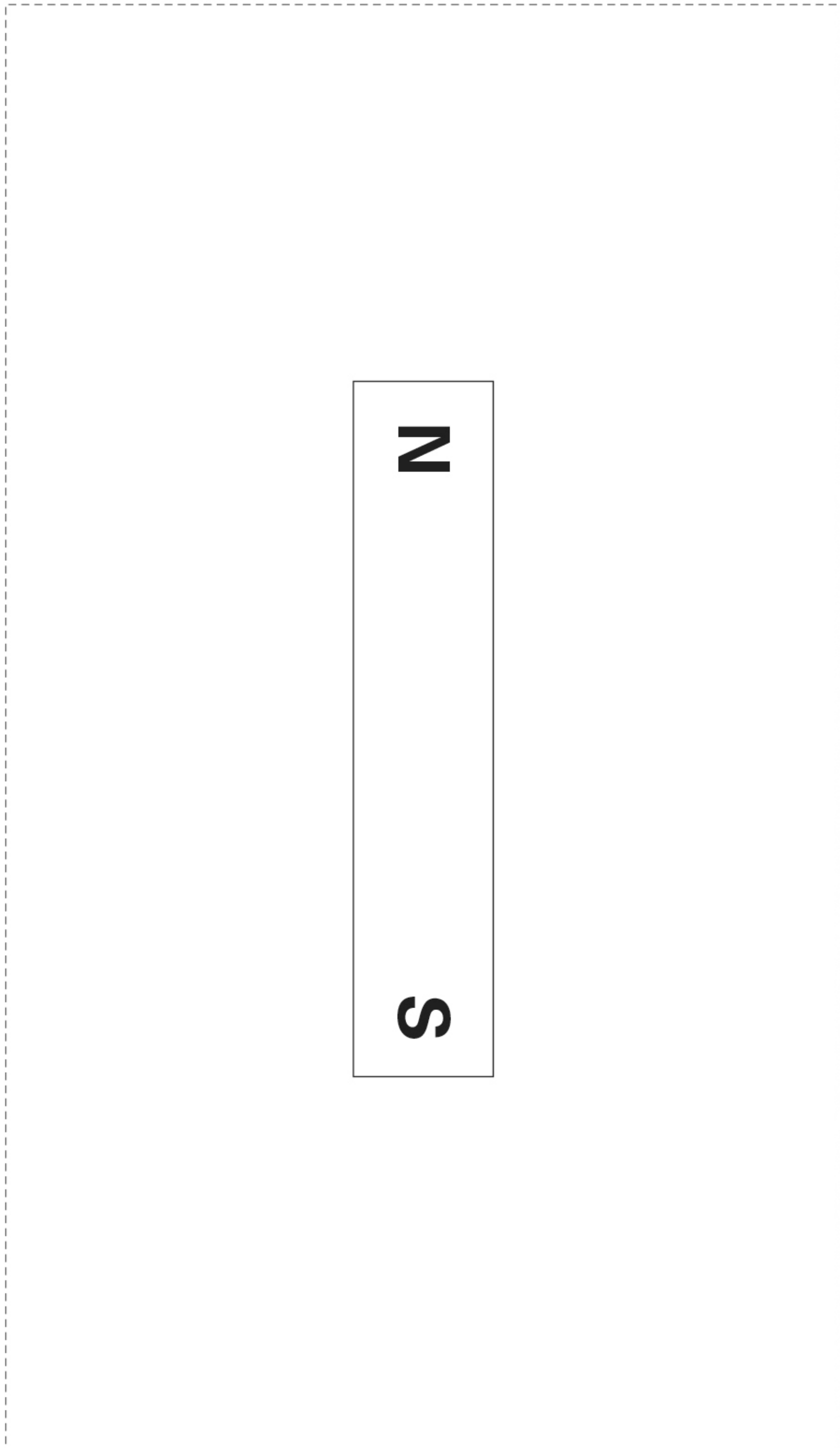


2. The symbols below represent two current-carrying wires extending vertically through the page (perpendicular to the surface of this page). ⊗ represents current flowing *into the page*, and ⊙ represents current flowing *out of the page*. Assuming that the magnitude of the current is the same in both wires, draw the magnetic field surrounding and between them.

3. The symbols below represent two current-carrying wires extending vertically through the page (perpendicular to the surface of this page), except now the current in both wires is flowing into the page. Assuming that the magnitude of the current is the same in both wires, draw the magnetic field surrounding (and between) them.

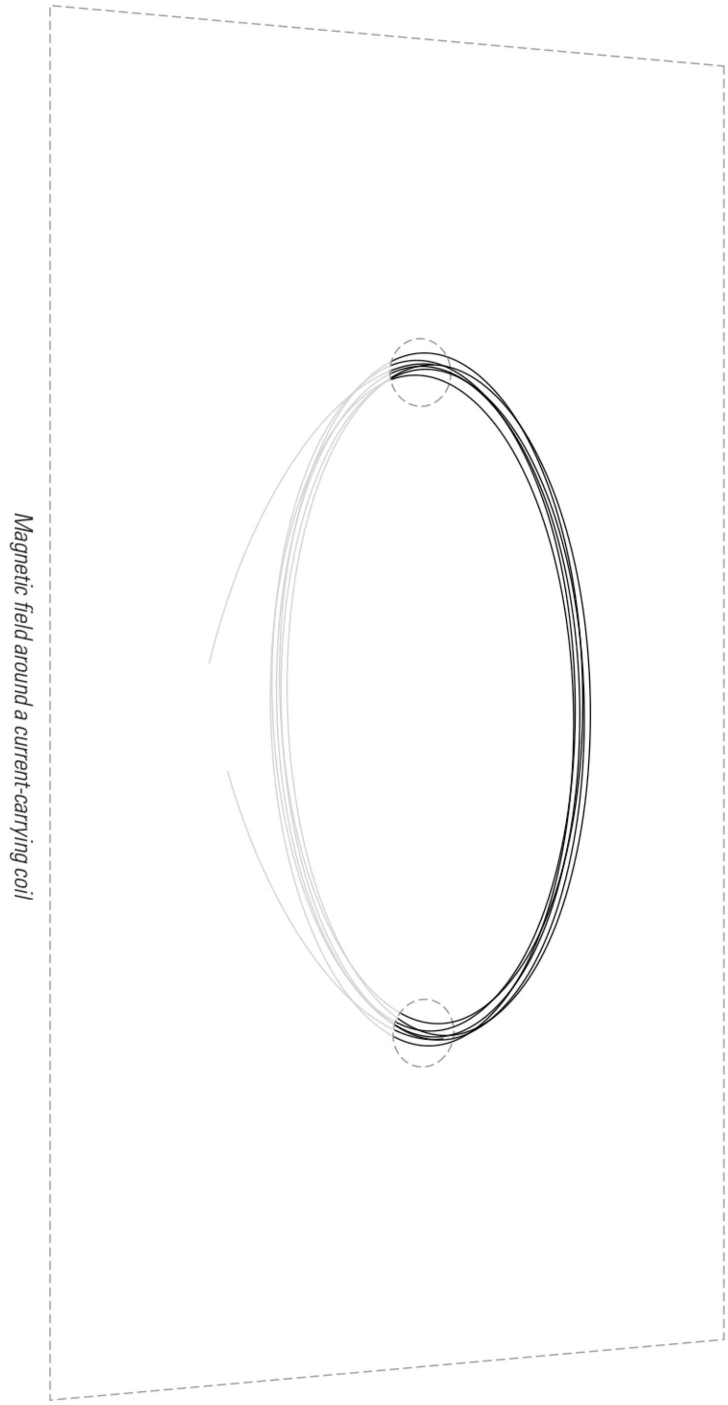
Appendix

Part 1 – Magnetic Field around a Bar Magnet



Magnetic field around a bar magnet

Part 2 – Magnetic Field around a Current-Carrying Coil



11. MAGNETIC FIELD STRENGTH

Connections to the AP® Physics 2 Curriculum*

The lab activity correlates to the following pieces of the AP® Physics 2 framework:

Big Idea	2	Enduring Understanding	D	Essential Knowledge	2

Learning Objective 1: The student is able to create a verbal or visual representation of a magnetic field around a long straight wire or a pair of parallel wires.
Science Practices: 1.1

Time Requirement

Preparation Time: 15 minutes

Lab Activity: 50 minutes

Prerequisites

Students should be familiar with the following concepts:

- Magnetic fields are created by permanent magnets and by current-carrying wires.
- Magnetic fields can be represented by magnetic field lines to visualize the strength and direction of the magnetic field.
- The magnetic field at the center of a current-carrying coil can be increased by adding additional loops to the coil.

Driving Question | Objective

How is the strength of the magnetic field at the center of a current-carrying coil dependent on the coil current and radius? Experimentally determine a mathematical relationship between the current, radius, and the magnetic field at the center of the current-carrying coil.

Procedural Overview

In the Structured version of this lab activity, students create five coils of different radii, all with the same number of turns of wire. Using five beakers of different radii, each coil is constructed by wrapping ten turns of wire around one of the beakers.

The procedure is divided into two parts. In the first part of the lab, the students explore the relationship between magnetic field strength and electric current. They use a magnetic field sensor to measure the magnetic field at the center of one of the coils at five different current levels. Students plot magnetic field strength versus current and analyze the relationship between them after applying a best fit line to the data.

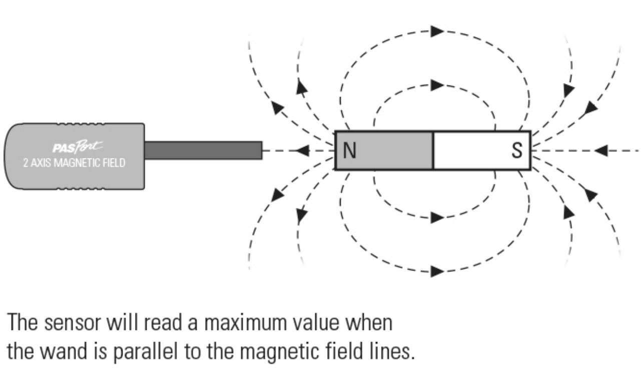
In the second part of the experiment, students explore the relationship between magnetic field strength and coil radius. They use the five coils of different radii and the same number of wire loops. The current is set at 3.0 amps for each coil and the magnetic field strength is measured at the center of each coil. Students plot magnetic field strength versus coil radius, which produces an inverse relationship. To obtain a linear relationship, students calculate inverse radii values and then plot magnetic field strength versus inverse radius. After applying a best fit line to the data, students use the slope to determine the vacuum permeability constant μ_0.

Pre-Lab Discussion and Activity

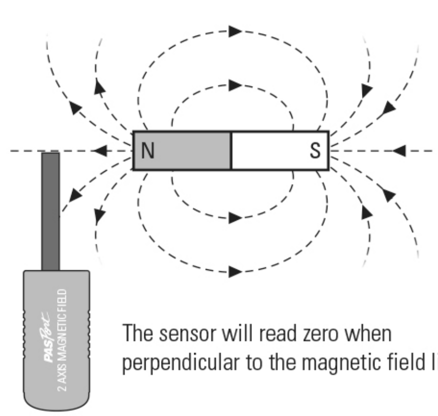
The magnetic field sensor can be a little confusing for students to use at first, since the sensor must be aligned parallel to magnetic field lines in order to make magnetic field strength measurements. The PASCO 2-Axis Magnetic Field Sensor has the ability to measure magnetic field strength in two directions. To keep things simple, only focus on using the axial direction and instruct your students to use this measurement during the lab.

After performing the activities in the Magnetic Fields lab, students should be familiar with the magnetic field surrounding a bar magnet and a coil. You may want to remind students of the magnetic field around a bar magnet by providing an image or drawing the magnetic field on the board. To help your students understand how to use the magnetic field sensor, below are some demonstrations and questions to discuss with your students.

1. Hold up a bar magnet and point the magnetic field sensor wand directly toward the north pole of the bar magnet, as shown, while monitoring the magnetic field measurement in your data collection system. Turn the wand 90 degrees so that the magnetic field sensor is now perpendicular to the magnetic field lines at the same point. Students should note that the magnetic field sensor reading decreases towards zero as the wand rotates toward 90 degrees to the magnetic field lines. Based on this test, ask the students if the sensor needs to be pointed parallel or perpendicular to the magnetic field in order to take a measurement.



The sensor will read a maximum value when the wand is parallel to the magnetic field lines.



The sensor will read zero when perpendicular to the magnetic field lines.

2. The magnetic field sensor can measure magnitude and direction. To demonstrate this, point the magnetic field sensor toward the north pole of the magnet (in parallel with the magnet) and then toward its south pole. The students should observe that the sensor reads a positive number when pointing towards the north pole and negative when pointing towards the south pole.

3. Use a compass to determine magnetic north and have the students also note the direction. Display a graph of magnetic field versus time using your data collection system. Point the wand of the sensor toward north and start recording data. Slowly rotate the sensor in a circle parallel to the floor as the students view the measurement change on the graph. Students should notice that the sensor reads a maximum positive value when the sensor is pointed south and a maximum negative value when pointed north.

There are two points to take away from the demonstration:

- The magnetic field sensor reads a maximum value while pointed in the north or south direction since the sensor is then parallel to earth's magnetic field. If the sensor was zeroed while in a zero-gauss chamber, the sensor would read a value of zero when pointed in the east or west direction since these directions would be perpendicular to earth's magnetic field.
- The magnetic field sensor reads a negative value when pointed toward the north because the earth's north pole is actually the magnetic south pole.

Materials and Equipment

- Data collection system
- PASCO 2-Axis Magnetic Field Sensor[1]
- Sensor rod[1]
- PASCO Sensor Extension Cable
- PASCO AC/DC Electronics Laboratory[2]
- Magnet wire or enameled wire, fine gauge (~10 m)
- Power supply, 18-VDC, 3-A
- 4-mm banana plug patch cord (2)
- Wire lead[2]
- Support rod, 45-cm
- Table clamp
- Right angle clamp
- Beakers of different diameter (5)
- Sandpaper
- Scissors or wire cutters
- Ruler

Probeware Resources

Below are web-link and QR codes that will direct you to instructional video resources for individual pieces of PASCO probeware, sensors, and other hardware used in the lab activity. These same links and codes are provided to students in their activity handouts.

[1]www.pasco.com/ap35

PASCO 2-Axis Magnetic Field Sensor

[2]www.pasco.com/ap04

PASCO AC/DC Electronics Laboratory

Safety

Follow these important safety precautions in addition to your regular classroom procedures:

- Do not allow current to flow through the wires any longer than a few seconds. The wires (and possibly the power supply) can become hot and cause burns and damage equipment.

Teacher Tips

Tip 1 – Preparing the Coils prior to the Class Period

- If class time or materials are limited, you may want to prepare the coils before the class. Prepare the coils for each lab group by wrapping ten loops of wire around each beaker, leaving 20 cm of straight wire on each end. Use wire strippers (if the insulation is plastic) or sandpaper to remove about 1 cm of insulation from the tips of each end of the wire.

Tip 2 – Using a Low Current Power Supply or the 550 Interface Signal Generator

- If a high-current power supply is not available, you can use a lower-current power supply (1 amp or less) and then construct the coils with additional loops. Good data can be obtained at 400 mA with a coil of at least 20 loops of a smaller gauge wire, such as 26 gauge.

Tip 3 – Using a PASCO Magnetic Field Sensor instead of a PASCO 2-Axis Magnetic Field Sensor

- It is not possible to zero the PASCO Magnetic Field Sensor, which means students will need to subtract the value the sensor reads when no current flows in the coils from the value that the sensor reads when current is flowing in the coil.
- The PASCO Magnetic Field Sensor has a lower resolution than the PASCO 2-Axis Magnetic Field Sensor. As a result, students may get noisy data that can lead toward unreliable measurements. Noise can be reduced by using current values other than those specified in the structured version of the experiment. For example, instead of using a 0.5 A current, students may try using a current of 0.4 A or 0.6 A. If students are getting particularly noisy data when no current is flowing through the coil, try repositioning the sensor so it is aligned parallel to earth's magnetic field.

Tip 4 –Other Alternative Equipment

- PASCO AC/DC Electronics Laboratory alternative: In place of the AC/DC Electronics Laboratory, connect the coil to the power supply using alligator wires and a push-button switch. Although a push-button switch is not required, it is highly recommended since it helps prevent current from flowing through the wires for an extended time.
- Power supply, 18-VDC, 3-A: If you are using a power supply which does not have a built in ammeter, you will need to use a separate multimeter or sensor to measure the current.

Sample Data

Below are sample data, acquired using the experimental setup and procedure outlined in the Structured version of the lab activity, and answers to questions in the Data Analysis section.

Data Analysis

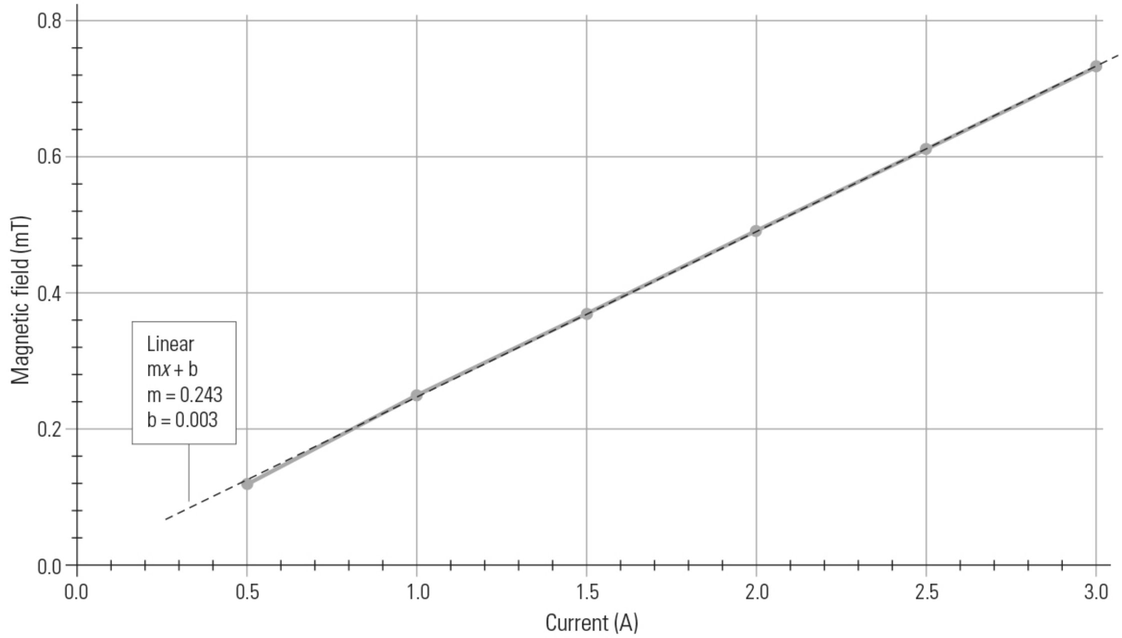
Part 1 – Magnetic Field and Current

Table 1: Change in magnetic field as the current changes through a coil, keeping coil radius constant

Current (A)	Magnetic Field (mT)
0.50	0.12
1.0	0.25
1.5	0.37
2.0	0.49
2.5	0.61
3.0	0.73

1. Plot a graph of *magnetic field* versus *current* in the blank Graph 1 axes. Be sure to label both axes with the correct scale and units.

Graph 1: Magnetic field versus current of a current-carrying coil with constant radius



2. Draw a line of best fit through your data in Graph 1. Determine and record the equation of the line here:

Best fit line equation: $B = (0.243\ \text{mT/A})I + 0.003\ \text{mT}$

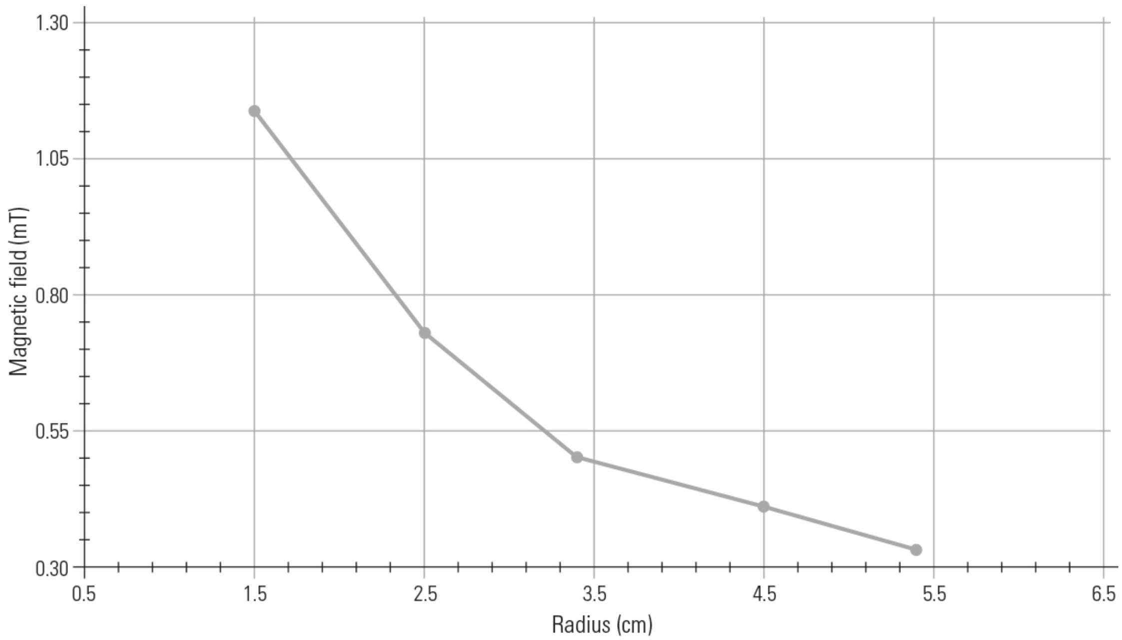
Part 2 – Magnetic Field and Radius

Table 2: Change in magnetic field as the coil radius changes, keeping the current constant

Radius (cm)	1/Radius (cm^{-1})	Magnetic Field (mT)
1.5	0.67	1.14
2.5	0.40	0.73
3.4	0.29	0.50
4.5	0.22	0.41
5.4	0.19	0.33

3. Plot a graph of *magnetic field* versus *radius* in the blank Graph 2 axes. Be sure to label both axes with the correct scale and units.

Graph 2: Magnetic field versus radius of a current-carrying coil with constant current

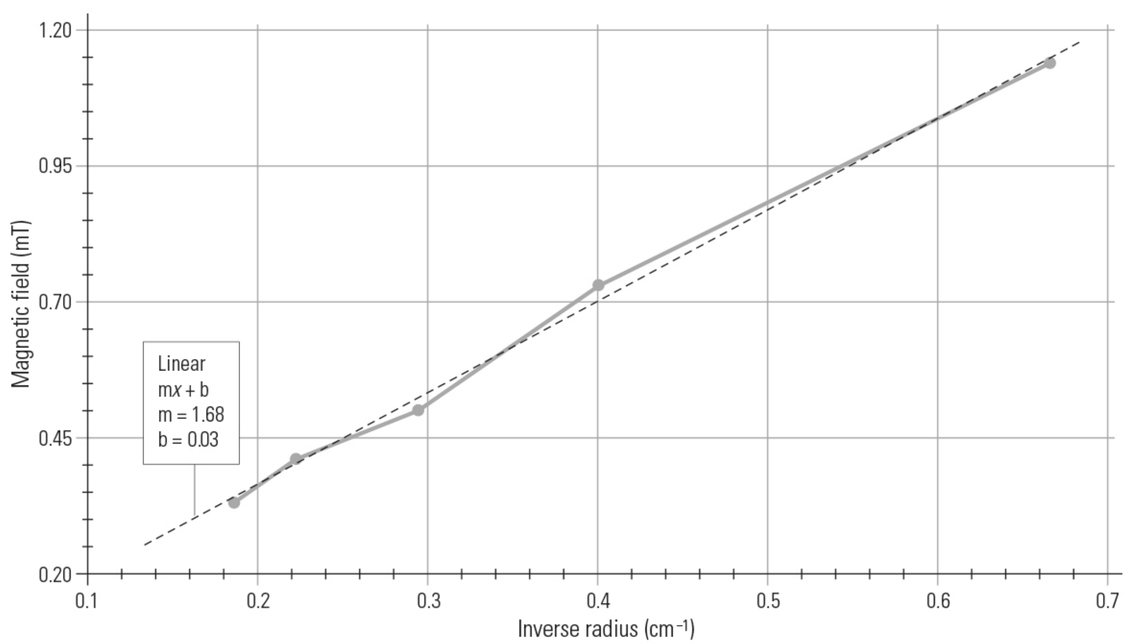


4. How does the magnetic field change as the radius increases?
   As the radius increases, the magnetic field decreases.

5. Calculate the inverse radius (1/radius) for each coil and record the results in Table 2.

6. Plot a graph of *magnetic field* versus 1/*radius* in the blank Graph 3 axes. Be sure to label both axes with the correct scale and units.

Graph 3: Magnetic field versus 1/radius of a current-carrying coil with constant current



7. Draw a line of best fit through your data in Graph 2. Determine and record the equation of the line here:

   Best fit line equation: $B = (1.68\ \text{mT} \cdot \text{cm})\frac{1}{r} + 0.03\ \text{mT}$

Guided Inquiry Questions

Below are sample responses to the Guiding Questions found in the Guided Inquiry version of this lab activity.

1. Based on the driving question and experiment objective, what will be the dependent and independent variables in your exploration?

   Students should identify magnetic field strength as the dependent variable. The two independent variables are the current and the radius of the coil.

2. What equipment will you use to measure these variables? Describe how you will make your measurements using this equipment.

   Students should list a magnetic field sensor (refer to Teacher Tip 3 for using a different magnetic field sensor than the one specified in the Structured version of this activity), wire coils with different radii, a power supply that includes an ammeter or a separate power supply and multimeter or current sensor. A ruler or calipers can be used to measure the radius of each coil.

   Magnetic field measurements should be made by positioning the sensor at the center of the coil and recording the magnetic field with a data collection system, as described in the Structured version of this lab.

   If students use different sizes of beakers for constructing the coils, as in the Structured version, the radius can be obtained by measuring the outside diameter of the beaker used for wrapping a coil or measuring the diameter of the coil directly.

   When the independent variable is current, students should measure the current through a single coil and change the current to obtain multiple data points.

   When the independent variable is radius, students should measure the same current through coils with different radii.

3. How many loops of wire will you use to construct a coil? What are the benefits of using more than one loop? Are there any disadvantages of using too many loops?

   Student answers will vary on the number of loops, but at least 10 loops should be used. The benefit of using more loops is that a greater magnetic field is generated, which is easier to measure. The disadvantage of using too many loops is that the radius of the coil is more difficult to determine due to the coil thickness. Additionally, the coil can become wider due to additional loops, which would generate a field more like that of a solenoid than a coil. Students may also use the argument of practicality, due to the limitation of available materials and time.

4. Magnet wire is coated with an insulating material. How will you remove the insulation in order to make an electrical connection?

   Students should use the provided sandpaper to rub off insulation from magnet wire. If students are using wire with plastic insulation, the insulation can be stripped off using wire strippers.

5. How should the magnetic field sensor be positioned in order to measure the field at the center of a coil?

   The sensor element is located at the end of the magnetic field sensor wand, which means the end of the wand must be placed in the center of the coil. The wand must also point perpendicular to the plane of the coil so that the wand is parallel to the magnetic field lines.

6. How will you change each independent variable while collecting data?

   Students should mention that current can be changed by making adjustments to the power supply and that radius can be changed by creating different sized coils. At least five different data points should be collected for each variable.

Assessment Questions: Sample Responses

Sample responses to the Analysis and Synthesis questions found in each version of the lab activity:

Analysis Questions

1. Based on your data, how is the magnetic field mathematically related to the current in a current-carrying coil? Use terms such as proportional, inversely proportional, linear, or quadratic in your response.

   The magnetic field is proportional to the current in a current-carrying coil.

2. Based on your data, how is the magnetic field mathematically related to the radius of a current-carrying coil? Use terms such as proportional, inversely proportional, linear, or quadratic in your response.

   The magnetic field is inversely proportional to the radius of a current-carrying coil.

3. The magnetic field B created by a long, straight, current-carrying wire is given by the equation

   $$B = \frac{\mu_0}{2\pi}\frac{I}{r}$$

   where I is the current, r is the distance from the wire to the point where the magnetic field is measured, and μ_0 is the vacuum permeability constant. How is your data related to this equation?

   Magnetic field and distance have an inverse relationship in this equation. Students should relate this to the inverse relationship between magnetic field strength and the coil radius discovered in the lab activity. They should also make the connection that as the radius of the coil increases, the distance from the sensor to the wire also increases.

4. The magnetic field at the center of a current-carrying coil is given by the equation

   $$B = N\frac{\mu_0}{2}\frac{I}{r}$$

   where N is the number of loops and r is the radius of the coil. Use the slope value from a best fit line of a magnetic field strength versus inverse radius graph to determine an experimental value for the vacuum permeability constant μ_0.

   Using the Sample Data, $\text{slope} = 1.68\ \text{mT} \cdot \text{cm}$.

   After converting the units to T·m:

   $$\text{slope} = 1.68 \times 10^{-5}\ \text{T} \cdot \text{m} = N\frac{\mu_o I}{2}$$

   $$\mu_o = \frac{2 \cdot 1.68 \times 10^{-5}\ \text{T} \cdot \text{m}}{NI} = \frac{2 \cdot 1.68 \times 10^{-5}\ \text{T} \cdot \text{m}}{(10)(3.0\ \text{A})}$$

   $$\mu_o = 1.12 \times 10^{-6}\ \text{T} \cdot \text{m/A}$$

❷ 5. Calculate the percent error between your experimental value and the actual value of the vacuum permeability constant μ_0 : $4\pi \times 10^{-7}$ T·m/A.

$$\text{Percent error} = \left|\frac{\text{Actual} - \text{Experimental}}{\text{Actual}}\right| \times 100$$

Sample calculation:

$$\text{Percent error} = \left|\frac{4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A} - 1.12 \times 10^{-6}\ \text{T}\cdot\text{m/A}}{4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}}\right| \times 100 = 10.9\%$$

With careful data collection methods and the proper data collection equipment, it is possible for students to obtain an experimental value within 15% of the theoretical value.

❷ 6. What are factors that might have caused error in your experimental value for the vacuum permeability constant?

The primary source of error is due to the magnetic field sensor, since the PASCO 2-Axis Magnetic Field Sensor has an accuracy of 5%. Additionally, a coil made of multiple loops of wire is not a perfect representation of a single coil. As more loops are added to the coil, it becomes more difficult to determine the radius if loops are wrapped over other loops. Adding more loops also cause the coil to become wider, which creates a slightly different magnetic field than a single coil.

Synthesis Questions

❷ 1. A current of 5.0 A is flowing in a 3.0-m long straight wire.

a. If one loop is created with this wire, what would be the magnetic field at the center of the loop?

$$C = 2\pi r \rightarrow r = \frac{C}{2\pi} = \frac{3.0\ \text{m}}{2\pi} = 0.48\ \text{m}$$

$$B = N\frac{\mu_0}{2}\frac{I}{r} = 1\left(\frac{4\pi \times 10^{-7}(\text{T}\cdot\text{m})/A}{2}\right)\left(\frac{5.0\ \text{A}}{0.48\ \text{m}}\right)$$

$$B = 6.5 \times 10^{-6}\ \text{T}$$

b. If the wire is straightened, what would be the magnetic field at a distance from the wire equal to the radius of the loop?

$$B = \frac{\mu_0}{2\pi}\frac{I}{r} = \left(\frac{4\pi \times 10^{-7}(\text{T}\cdot\text{m})/A}{2\pi}\right)\left(\frac{5.0\ \text{A}}{0.48\ \text{m}}\right)$$

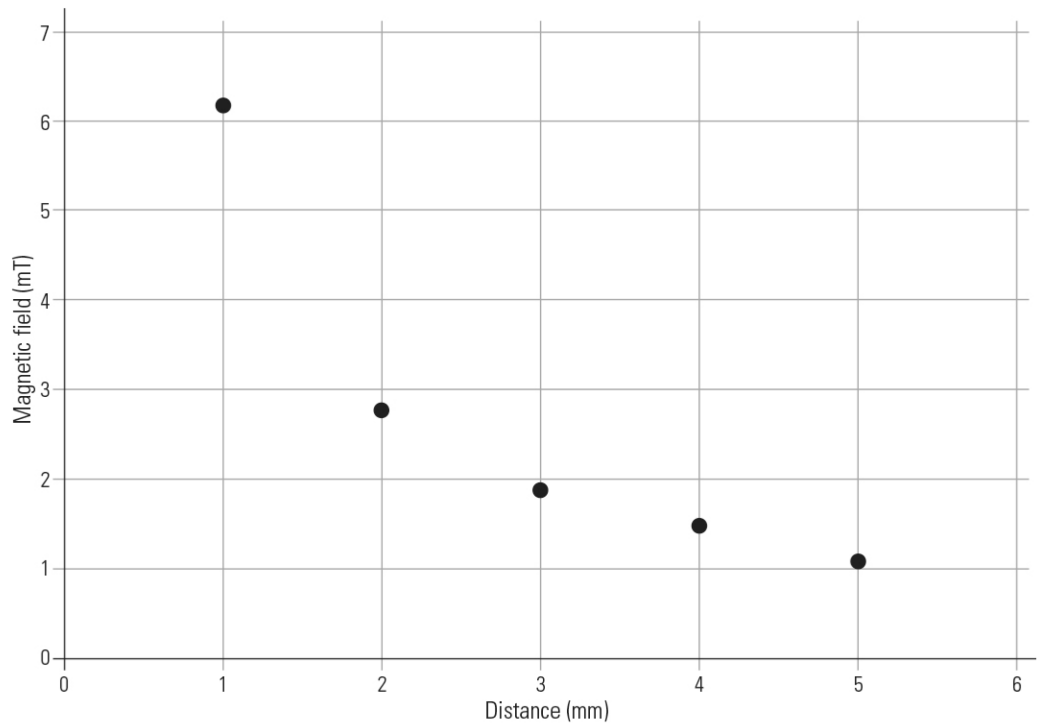
$$B = 2.1 \times 10^{-6}\ \text{T}$$

c. Why is the magnetic field stronger at the center of the loop than at the same distance from the straight wire even though the same length of wire and same current are used?

Students should note that bending the wire into a loop causes the magnetic field from the wire to concentrate at the center of the loop due to the wire completely surrounding the center point on all sides. For a straight wire, at any point in space, there is only a single wire on one side of that point.

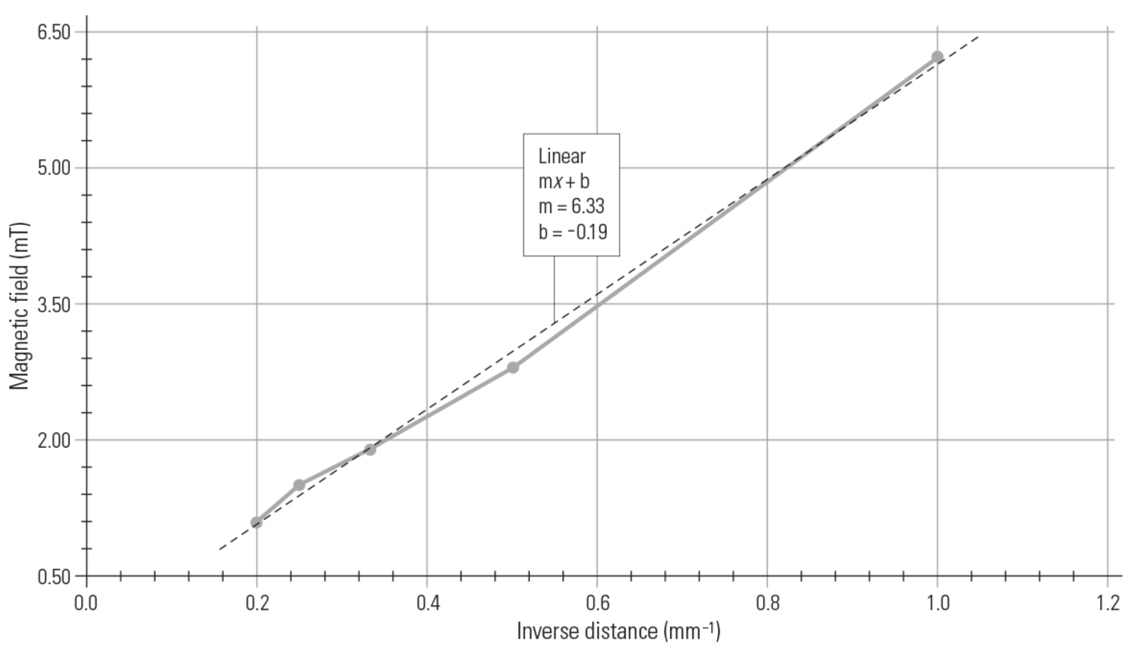
❷ 2. Graph 4 shows sample data of the magnetic field versus distance for the magnetic field created by a long, straight, current-carrying wire.

Graph 4: Sample data of magnetic field versus distance for a long, straight, current-carrying wire



a. Linearize the data in Graph 4 and then plot that data in Graph 5.

Graph 5: Linearized data of magnetic field versus distance for a long, straight, current-carrying wire



b. Draw a line of best fit through your data in Graph 5. Determine and record the equation of the line here:

Best fit line equation: $B = 6.33\ (\mathrm{mT \cdot mm})\frac{1}{r} - 0.19\ \mathrm{mT}$

c. Use the slope in the best fit line equation to determine the current through the wire.

$$B = \frac{\mu_0}{2\pi}\frac{I}{r}$$

$$\text{slope} = 6.33 \times 10^{-6}\ \text{T} \cdot \text{m} = \frac{\mu_o I}{2\pi}$$

$$I = \frac{2\pi\left(6.33 \times 10^{-6}\ \text{T} \cdot \text{m}\right)}{\mu_o} = \frac{2\pi\left(6.33 \times 10^{-6}\ \text{T} \cdot \text{m}\right)}{4\pi \times 10^{-7}\ \text{T} \cdot \text{m/A}}$$

$$I = 31.7\ \text{A}$$

d. How much current would be required in a single loop with a radius of 1.0 mm to create a magnetic field of the same magnitude as that obtained 1 mm from the straight wire?

$$B = N\frac{\mu_0}{2}\frac{I}{r}$$

$$I = \frac{2rB}{N\mu_o} = \frac{2(1.0 \times 10^{-3}\ \text{m})(6.2 \times 10^{-3}\ \text{T})}{1(4\pi \times 10^{-7}\ (\text{T} \cdot \text{m})/\text{A})}$$

$$I = 9.9\ \text{A}$$

e. How much current would be required to create a magnetic field of 1.0 T at a distance of 1.0 mm from the straight wire?

$$B = \frac{\mu_0}{2\pi}\frac{I}{r}$$

$$I = \frac{2\pi rB}{\mu_o} = \frac{2\pi(1.0 \times 10^{-3}\ \text{m})(1.0\ \text{T})}{4\pi \times 10^{-7}\ (\text{T} \cdot \text{m})/\text{A}}$$

$$I = 5000\ \text{A}$$

Extended Inquiry Suggestions

- Have the students explore the relationship between the magnetic field strength at the center of a coil and the number of coil loops. In this case, the students keep the radius and current constant and measure the magnetic field each time a loop of wire is added to the coil.
- Have the students explore the relationship between the magnetic field strength produced by a long, straight wire and the distance from the center of the wire. Due to the small magnetic field produced, a very large current would be required to make quantitative measurements. Make sure to use a larger diameter gauge wire which can handle the higher currents.

NAME PERIOD DATE

11. MAGNETIC FIELD STRENGTH

STRUCTURED

Driving Question | Objective

How is the strength of the magnetic field at the center of a current-carrying coil dependent on the coil current and radius? Experimentally determine a mathematical relationship between the current, radius, and the magnetic field at the center of the current-carrying coil.

Materials and Equipment

- Data collection system
- PASCO 2-Axis Magnetic Field Sensor[1]
- Sensor rod[1]
- PASCO Sensor Extension Cable
- PASCO AC/DC Electronics Laboratory[2]
- Magnet wire or enameled wire, fine gauge (~10 m)
- Power supply, 18-VDC, 3-A
- 4-mm banana plug patch cord (2)
- Wire lead[2]
- Support rod, 45-cm
- Table clamp
- Right angle clamp
- Beakers of different diameter (5)
- Sandpaper
- Scissors or wire cutters
- Ruler

[1]www.pasco.com/ap35
PASCO 2-Axis Magnetic Field Sensor

[2]www.pasco.com/ap04
PASCO AC/DC Electronics Laboratory

Background

Sources of magnetic fields include permanent magnets and moving electrical charges, such as those in a current-carrying wire. The magnitude of a magnetic field created by a current-carrying wire can be easily adjusted by varying the current in the wire or changing the physical properties of the wire. Though a magnetic field is created by a straight, current-carrying wire, coiling the wire into a loop strengthens the magnetic field at the center of the coil. The magnetic field can be made even stronger by increasing the number of loops, which makes the magnetic field easier to measure.

In this activity you use a sensor to measure the magnetic field at the center of a current-carrying coil as both the radius of the coil and the current through the coil are varied. Data from these measurements are then used to determine the mathematical relationship between the radius, current, and strength of the magnetic field.

The SI unit of magnetic field is "tesla." Because the magnetic fields measured in this activity are small, the unit of mT is used for convenience.

Safety

Follow these important safety precautions in addition to your regular classroom procedures:

- Do not allow current to flow through the wires any longer than a few seconds. The wires (and possibly the power supply) can become hot and cause burns and damage equipment.

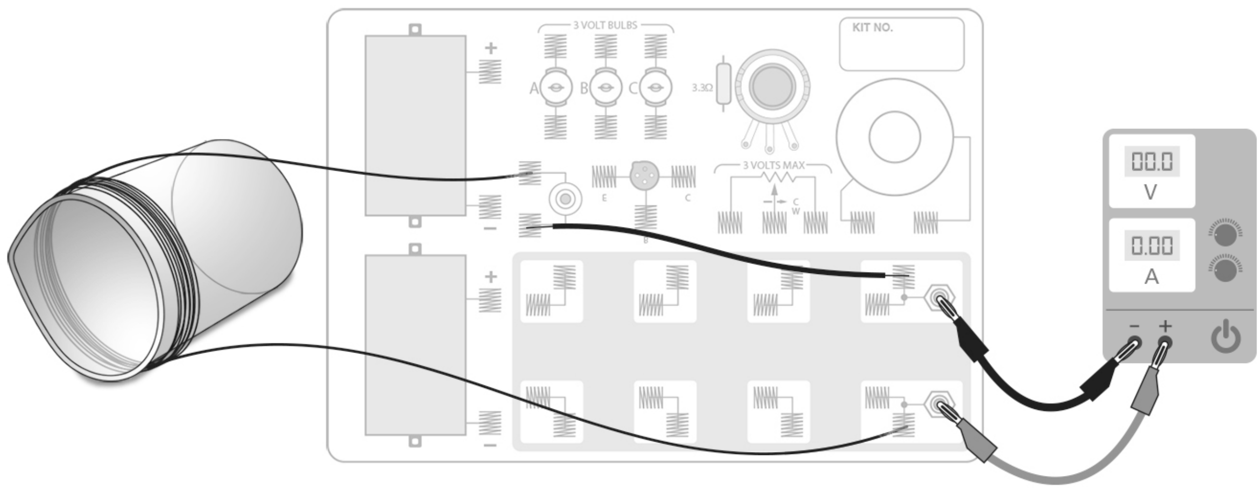
Procedure

Part 1 – Magnetic Field and Current

SET UP

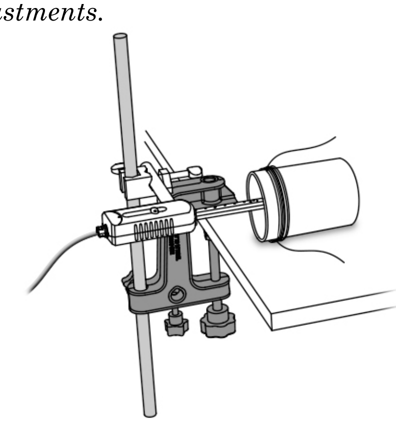
1. Wrap 10 loops of wire around the top of a 100-ml (or smaller) beaker, forming a coil. Leave about 20 cm of straight wire on each end and use sandpaper to sand off about 1 cm of insulation from the tips of each end of the wire.

2. Connect the power supply, coil, and switch on the AC/DC Electronics Board as shown.



3. Turn the power supply on, press and hold the switch on the circuit board, and adjust the power supply so that 0.50 A of current is output to the coil. Release the switch every few seconds and after the adjustment is complete.

   NOTE: Do not hold the switch for more than a few seconds. Repeatedly press the switch to adjust the power supply if needed, waiting for 1–2 seconds between adjustments.



4. Mount the magnetic field sensor using the support rod, the sensor rod, and clamps so the tip of the probe is held at the center of the coil, as shown.

5. Connect the magnetic field sensor to the data collection system using the sensor extension cable and then create a digits display of Magnetic Field Strength (Axial) with units of mT.

COLLECT DATA

6. Begin recording data and then press the Tare button on the magnetic field sensor to zero the measurement.

7. Press the push-button switch to close the circuit, quickly note the magnetic field value, and release the push-button switch.

8. Stop data collection and record the magnetic field value into Table 1.

9. Repeat the data collection steps for current values of 1.0, 1.5, 2.0, 2.5, and 3.0 A. Record each corresponding magnetic field value into Table 1.

Part 2 – Magnetic Field and Radius

SET UP

10. Use the same equipment and setup from Part 1 in Part 2.

11. Build four additional coils with different radii by wrapping wire around the top of four other beakers with different diameters. Use 10 loops of wire in each coil, leave about 20 cm of straight wire on each end, and use sandpaper to sand off about 1 cm of insulation from the tips of each end.

    NOTE: Be sure to wrap the same number of wire loops around each beaker.

12. Connect the coil with the smallest radius to your circuit, and then position the magnetic field sensor so the tip of the probe is held at the center of the coil.

13. Press and hold the switch on the circuit board, releasing it every few seconds as you did before, and adjust the power supply so that 3.0 A of current is output to the coil. Release the switch after the adjustment is complete.

    NOTE: Do not hold the switch for more than a few seconds. Repeatedly press the switch to adjust the power supply if needed, waiting for 1–2 seconds between adjustments.

COLLECT DATA

14. Begin recording data and then press the push-button switch to close the circuit. Hold the push-button switch down for a few seconds.

15. Release the push-button switch and stop data collection.

16. Use the tools on your data collection system to determine the magnetic field while current was flowing through the coil. Record this value in Table 2 in the Data Analysis section.

17. Measure the outside diameter of the beaker to determine the radius of the coil and record this value in Table 2.

18. Repeat the data collection steps for the other four coils. Record all corresponding magnetic field and radius values into Table 2.

Data Analysis

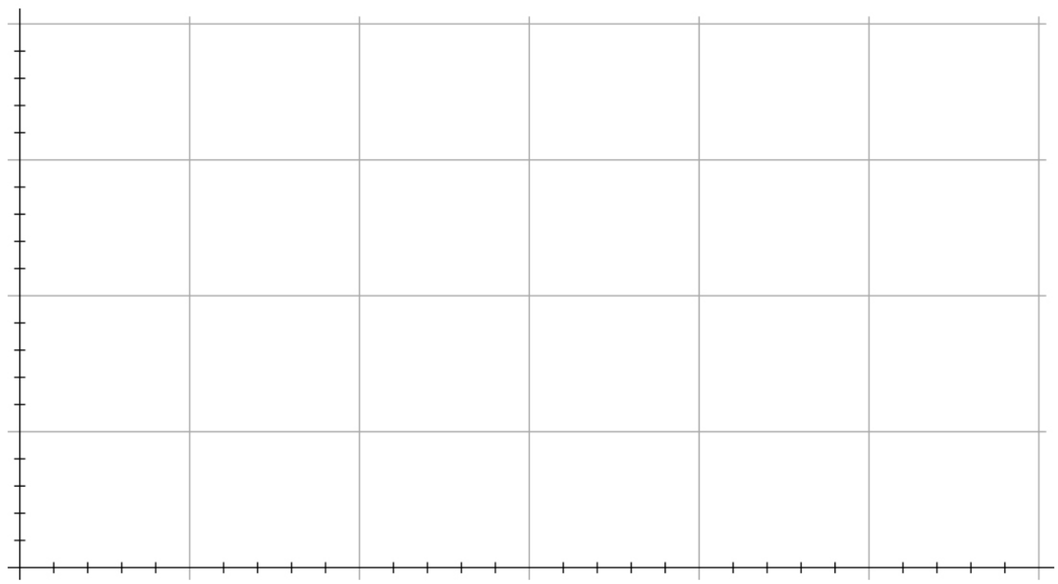
Part 1 – Magnetic Field and Current

Table 1: Magnetic field and current for a current-carrying coil with constant radius

Current (A)	Magnetic Field (mT)
0.50	
1.0	
1.5	
2.0	
2.5	
3.0	

1. Plot a graph of *magnetic field* versus *current* in the blank Graph 1 axes. Be sure to label both axes with the correct scale and units.

Graph 1: Magnetic field versus current of a current-carrying coil with constant radius



2. Draw a line of best fit through your data in Graph 1. Determine and record the equation of the line here:

   Best fit line equation: ____

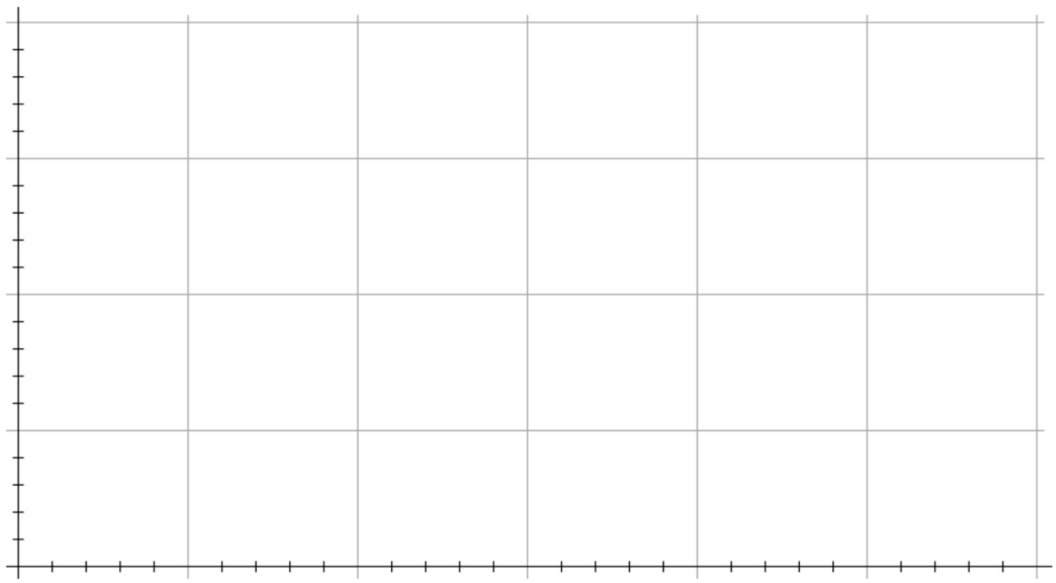
Part 2 – Magnetic Field and Radius

Table 2: Change in magnetic field as the coil radius changes, keeping the current constant

Radius (cm)	1/Radius (cm^{-1})	Magnetic Field (mT)

3. Plot a graph of *magnetic field* versus *radius* in the blank Graph 2 axes. Be sure to label both axes with the correct scale and units.

Graph 2: Magnetic field versus radius of a current-carrying coil with constant current



4. How does the magnetic field change as the radius increases?

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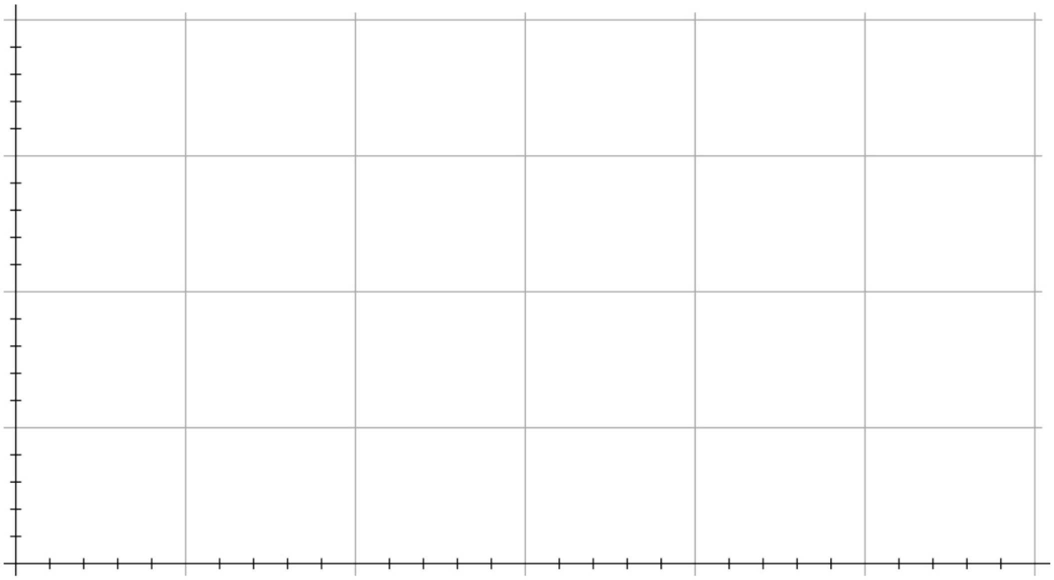
5. Calculate the inverse radius (1/radius) for each coil and record the results in Table 2.

6. Plot a graph of *magnetic field* versus 1/*radius* in the blank Graph 3 axes. Be sure to label both axes with the correct scale and units.

Graph 3: Magnetic field versus 1/radius of a current-carrying coil with constant current



7. Draw a line of best fit through your data in Graph 2. Determine and record the equation of the line here:

   Best fit line equation: ____

Analysis Questions

1. Based on your data, how is the magnetic field mathematically related to the current in a current-carrying coil? Use terms such as proportional, inversely proportional, linear, or quadratic in your response.

2. Based on your data, how is the magnetic field mathematically related to the radius of a current-carrying coil? Use terms such as proportional, inversely proportional, linear, or quadratic in your response.

❓ 3. The magnetic field B created by a long, straight, current-carrying wire is given by the equation

$$B = \frac{\mu_0}{2\pi}\frac{I}{r}$$

where I is the current, r is the distance from the wire to the point where the magnetic field is measured, and μ_0 is the vacuum permeability constant. How is your data related to this equation?

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❓ 4. The magnetic field at the center of a current-carrying coil is given by the equation

$$B = N\frac{\mu_0}{2}\frac{I}{r}$$

where N is the number of loops and r is the radius of the coil. Use the slope value from a best fit line of a magnetic field strength versus inverse radius graph to determine an experimental value for the vacuum permeability constant μ_0.

❓ 5. Calculate the percent error between your experimental value and the actual value of the vacuum permeability constant μ_0: $4\pi \times 10^{-7}$ T·m/A.

$$\text{Percent error} = \left|\frac{\text{Actual} - \text{Experimental}}{\text{Actual}}\right| \times 100$$

❓ 6. What are factors that might have caused error in your experimental value for the vacuum permeability constant?

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Synthesis Questions

1. A current of 5 A is flowing in a 3-m long straight wire.

   a. If one loop is created with this wire, what would be the magnetic field at the center of the loop?

   b. If the wire is straightened, what would be the magnetic field at a distance from the wire equal to the radius of the loop?

   c. Why is the magnetic field stronger at the center of the loop than at the same distance from the straight wire even though the same length of wire and same current are used?

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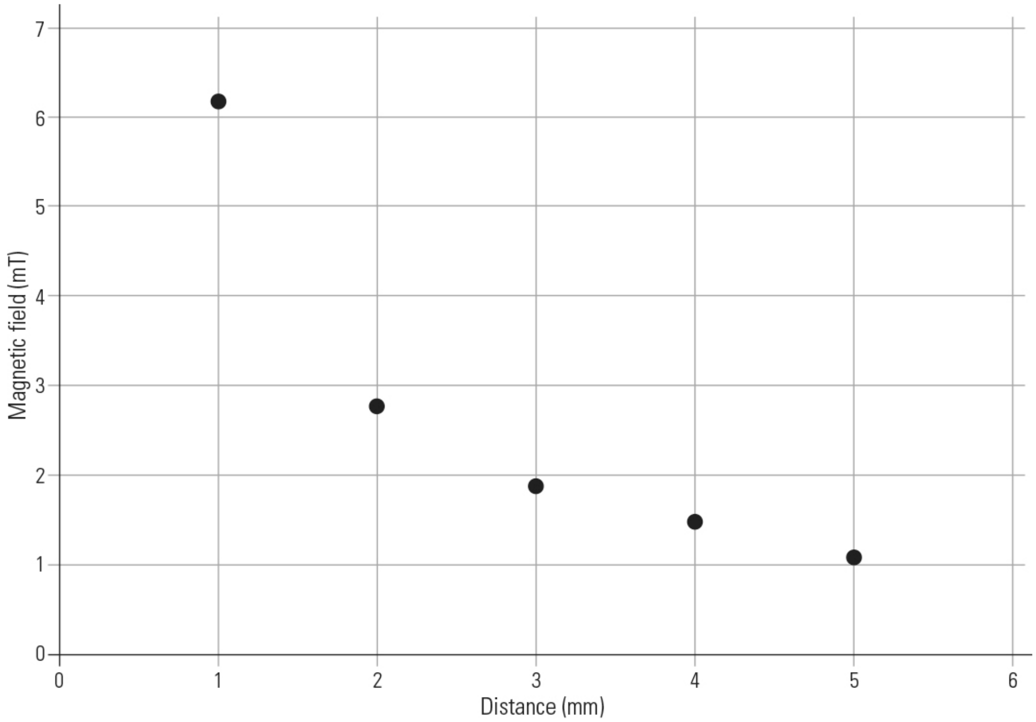
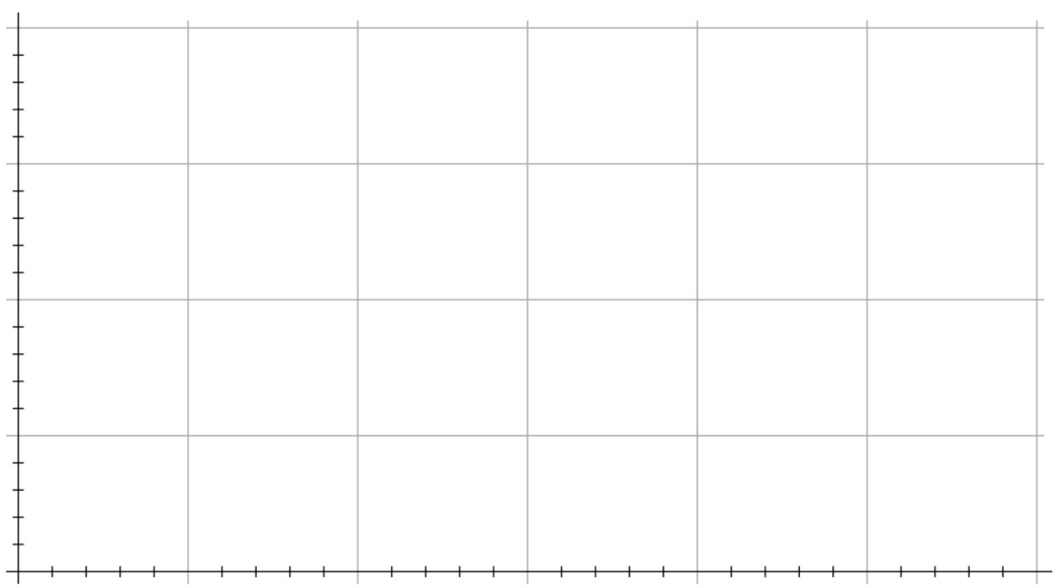
❷ 2. Graph 4 shows sample data of magnetic field versus distance for the magnetic field created by a long, straight, current-carrying wire.

Graph 4: Sample data of magnetic field versus distance for a long, straight, current-carrying wire



a. Linearize the data in Graph 4 and then plot that data in Graph 5.

Graph 5: Linearized data of magnetic field versus distance for a long, straight, current-carrying wire



b. Draw a line of best fit through your data in Graph 5. Determine and record the equation of the line here:

Best fit line equation: ______________________________

c. Use the slope in the best fit line equation to determine the current through the wire.

d. How much current would be required in a single loop with a radius of 1.0 mm to create a magnetic field of the same magnitude as that obtained 1 mm from the straight wire?

e. How much current would be required to create a magnetic field of 1.0 T at a distance of 1.0 mm from the straight wire?

12. ELECTROMAGNETIC INDUCTION

Connections to the AP® Physics 2 Curriculum*

The lab activity correlates to the following pieces of the AP® Physics 2 framework:

Big Idea	4	Enduring Understanding	E	Essential Knowledge	2

Learning Objective 1: The student is able to construct an explanation of the function of a simple electromagnetic device in which an induced emf is produced by a changing magnetic flux through an area defined by a current loop (i.e., a simple microphone or generator) or of the effect on behavior of a device in which an induced emf is produced by a constant magnetic field through a changing area.
Science Practices: 6.4

Time Requirement

Preparation Time: 10 minutes

Lab Activity: 50 minutes

Prerequisites

Students should be familiar with the following concepts:

- *Electromotive force* (emf) refers to the potential difference that causes charge to move (current) through a conducting material, like a wire. Emf is measured in units of volts.
- *Magnetic flux* Φ_B is a measure of the magnetic field $\vec{B}$ that passes through a given surface area $\vec{A}$. It is often referred to as a measure of relative magnetic field strength and can be illustrated as the density of magnetic field lines that pass through a given surface area. If the magnetic field lines of $\vec{B}$ are perpendicular to the surface area $\vec{A}$, magnetic flux Φ_B is:

  $$\Phi_B = BA$$

- The *average* of a measurement, variable, or function f_{ave} over a time interval Δt can be determined by dividing the area under the curve of that measurement, variable, or function versus time, by the time interval Δt:

  $$f_{\text{ave}}(t) = \frac{[\text{Area under curve}]}{\Delta t}$$

Driving Question | Objective

How is the average emf induced in a wire coil affected by the rate at which magnetic flux through the coil changes? Investigate how the rate of change of magnetic flux through a coil affects the magnitude and polarity of the average emf induced in it, and then determine a mathematical relationship between the two.

Procedural Overview

The Structured version of this lab activity is divided into two parts:

Part 1 – Students use a voltage sensor and rotary motion sensor to simultaneously measure the induced emf and angular position of a small coil of wire at the end of a pendulum as it swings through the magnetic field from a permanent magnet. Because the magnetic field is strongest near the magnet, and decreases with increasing distance from the magnet, the coil experiences an increasing magnetic flux as it approaches the magnet, and a decreasing flux as it travels away from the magnet.

Students measure the average emf induced in the coil over a specific angular range in the coil's swing and vary the time it takes the coil to traverse that range in each trial. Assuming the total change in magnetic flux over that angular range is the same in each trial, students vary the rate of magnetic flux change through the coil by varying the time it takes the coil to traverse that specific angular range. Students use their data to plot a graph of average induced emf versus rate of magnetic flux change through the coil and demonstrate that the relationship is proportional.

Part 2 – Using the same setup as Part 1, students use a voltage sensor to measure the induced emf voltage in the coil as it is moved into and out of the magnetic field. By moving the coil into and out of the magnetic field, students create a situation where the flux through the coil is increasing (positive rate change) as the coil enters the magnetic field, and decreasing (negative rate change) as the coil leaves the magnetic field. Students use their graphs of emf voltage to establish that the polarity of emf is opposite (opposite current flow) when the flux through the coil is increasing versus when the flux is decreasing, demonstrating Lenz's Law.

Pre-Lab Discussion and Activity

Before performing this activity, students should have a foundational understanding of fields; more specifically, of magnetic fields and the relative strength and shape of magnetic fields surrounding permanent magnets. A cursory refresher of this information before this activity will be helpful for students performing any version of this lab activity. The Structured version of this activity relies heavily on the understanding that the strength of the magnetic field surrounding the PASCO Variable Gap Magnet decreases as the distance from the magnet increases, and that the plane of the coil in the PASCO Induction Wand is effectively perpendicular to the magnetic field lines as it swings.

Students should also understand the concept of magnetic flux as it relates to magnetic field strength and orientation. If your students need a brief introduction to this topic, the information in the Background section of the Structured student handout can serve as a guide.

Materials and Equipment

- Data collection system
- PASCO Induction Wand[1]
- PASCO Rotary Motion Sensor[2]
- PASCO Variable Gap Magnet[3]
- PASCO Voltage–Current Sensor[4]
- PASCO 2-Axis Magnetic Field Sensor[5]
- PASCO Sensor Extension Cable[5]
- Table clamp or large base
- Support rod, 45-cm
- Right angle clamp
- Additional rod

Probeware Resources

Below are web-link and QR codes that will direct you to instructional video resources for individual pieces of PASCO probeware, sensors, and other hardware used in the lab activity. These same links and codes are provided to students in their activity handouts.

[1]www.pasco.com/ap02
PASCO Induction Wand

[2]www.pasco.com/ap20
PASCO Rotary Motion Sensor

[3]www.pasco.com/ap03
PASCO Variable Gap Magnet

[4]www.pasco.com/ap19
PASCO Voltage–Current Sensor

[5]www.pasco.com/ap35
PASCO 2-Axis Magnetic Field Sensor

Teacher Tips

Tip 1 – When to Use High Sample Rates

- Students performing the Guided Inquiry or Student Designed versions of this lab activity are likely to use voltage and motion or rotary motion sensors to acquire data in their experiments. Remind students to acquire data from their sensors using sample rates suitable for the setups and procedures they choose. Using a sample rate that is too low will reduce the ability to resolve the phenomenon being explored, especially if that phenomenon occurs over a very short time. Students following the procedure outlined in the Structured version of this lab activity are instructed to use a sample rate of 250 Hz on their data collection system. It may be good practice for students using a procedure and equipment similar to that in the Structured version to consider a 250 Hz sample rate as a baseline in their experiment.

  In contrast, students using a conventional motion sensor should be advised not to exceed 100 Hz for nearly all applications. For conventional motion sensors, sample rates above 100 Hz may cause data to be noisy.

Tip 2 – Structured Version: Separating Faraday's Law from Lenz's Law

- Part 1 in the Structured version of this lab activity is an exploration of Faraday's Law, while Part 2 is an exploration of the polarity associated with the induced emf in the coil as it passes into and out of the magnetic field from a permanent magnet: Lenz's Law. Some instructors may choose to separate these topics into two different lab activities, one for Faraday's Law, and one for Lenz's Law. This is easily done by editing the Structured student handout document to separate Parts 1 and 2 in the Procedure and Data Analysis sections of the handout.

- Although it is recommended that students perform the procedure and analyses outlined in Part 2 of the Structured student handout, some instructors may prefer to address Lenz's Law using the data from Part 1 alone.

  The voltage (emf) data acquired in Part 1 will show polarity as the coil swings into and then out of the magnetic field. This difference in polarity could be the basis of a discussion about Lenz's Law. Use questions to prompt your students, such as, “How was the magnetic flux changing as the coil passed through the magnetic field?” “At what points was the magnetic flux increasing, decreasing, and constant?” “How does your data support or not support Lenz's Law?”

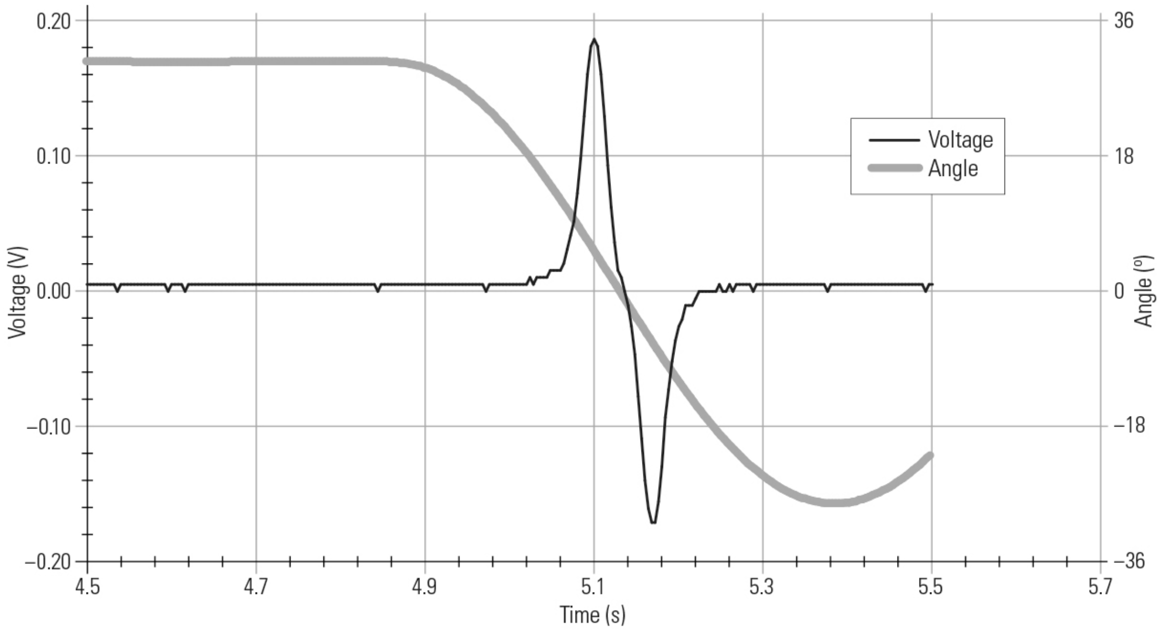
Sample Data

Below are sample data, acquired using the experimental setup and procedure outlined in the Structured version of the lab activity, and answers to questions in the Data Analysis section.

Data Analysis

PART 1 – RATE OF FLUX CHANGE AND AVERAGE EMF

Student graphs of voltage and angle versus time will look similar to this:



Maximum magnetic field (T): 0.0665 T

Table 1: Determination of the average induced emf and the rate of magnetic flux change

Trial	Release Angle (approximate) (°)	Area Under Curve (V·s)	Δt (s)	$\mathcal{E}_{ave}$ (V)	$\Delta\Phi/\Delta t$ (T·m²/s)
1	30	0.0072	0.120	0.060	0.00028
2	45	0.0069	0.076	0.091	0.00043
3	60	0.0068	0.060	0.11	0.00055
4	75	0.0068	0.048	0.14	0.00069
5	90	0.0068	0.044	0.15	0.00075

1. Calculate the average induced emf $\mathcal{E}_{ave}$ for each Part 1 trial. Record your results in Table 1.

$$\mathcal{E}_{ave} = \frac{\text{Area under voltage versus time curve}}{\Delta t}$$

Calculation using sample data for Trial 1:

$$\mathcal{E}_{ave} = \frac{\text{Area under curve}}{\Delta t} = \frac{0.0072\ \text{V} \cdot \text{s}}{0.120\ \text{s}} = 0.060\ \text{V}$$

2. The following information will help with the calculation of the total change of magnetic flux through the coil:

   Assuming the magnetic field was always perpendicular to the plane of the coil as it swung in each trial, and the area of the coil A is constant, then according to Equation 2, the equation describing the total change in magnetic flux $\Delta\Phi_B$ in each trial is:

   $$\Delta\Phi_B = \Delta BA$$

   $$\Delta\Phi_B = \Delta B\pi r^2 \quad (3)$$

   where ΔB is the total change in magnetic field amplitude experienced by the coil and r is the radius of the coil.

   If we assume the magnetic field magnitude was zero for angles greater than 20° and maximum at an angle of 0° (between the pole plates) in each trial, the total change in magnetic field magnitude for all trials is:

   $$\Delta B = B_{\text{f}} - B_{\text{i}} = B_{\text{max}} - 0 = B_{\text{max}}$$

   Plugging the result for ΔB back into Equation 3 gives the equation for the total change in magnetic flux through the coil for all trials:

   $$\Delta\Phi_B = B_{\text{max}}\pi r^2 \quad (4)$$

   where B_{max} is the measured maximum magnetic field value and r is the radius of the coil (in the case of the PASCO Induction Wand, $r = 0.0125$ m).

   Using the above information, calculate the total change in magnetic flux $\Delta\Phi_B$ through the coil for all trials. Record your value here:

   $\Delta\Phi_B$ (T·m²): 3.3×10^{-5} T·m²

   Calculation using the sample data value for B_{max} and $r = 0.0125$ m (the radius of the PASCO Induction Wand coil):

   $$\Delta\Phi_B = B_{\text{max}}\pi r^2 = (0.0665\ \text{T})\pi(0.0125\ \text{m})^2 = 3.3 \times 10^{-5}\ \text{T} \cdot \text{m}^2$$

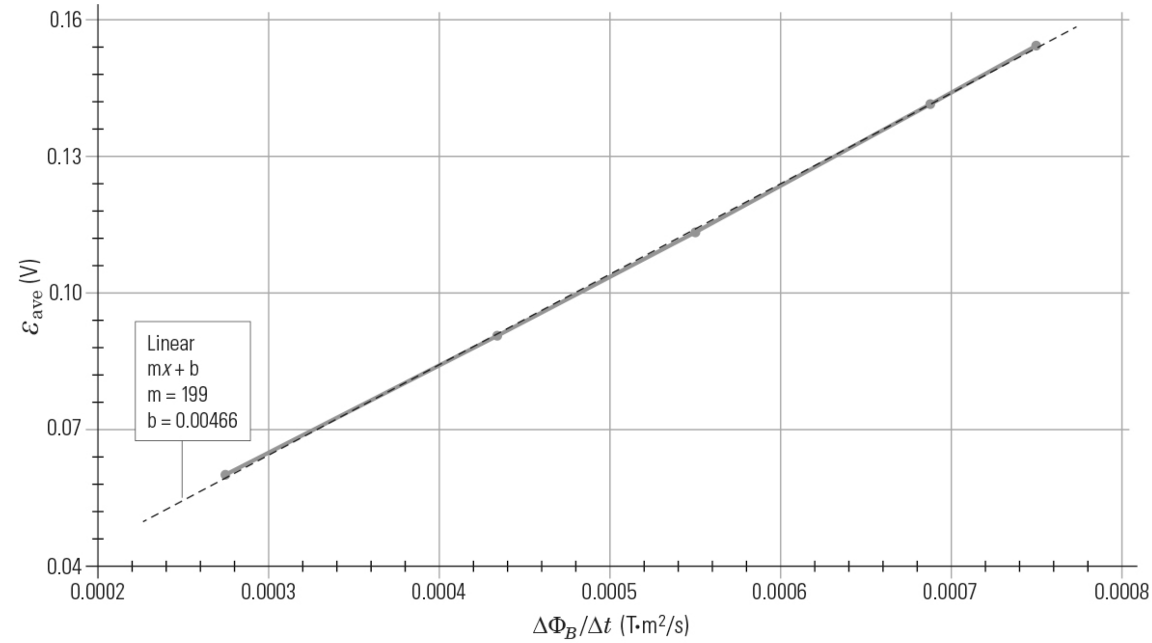
3. Calculate the rate of magnetic flux change $\Delta\Phi_B/\Delta t$ for each trial using the $\Delta\Phi_B$ value from the previous question and the elapsed time Δt values from Table 1. Record the results for each trial in Table 1.

   Calculation using sample data for Trial 1:

   $$\frac{\Delta\Phi_B}{\Delta t} = \frac{3.3 \times 10^{-5}\ \text{T} \cdot \text{m}^2}{0.120\ \text{s}} = 0.00028\ \frac{\text{T} \cdot \text{m}^2}{\text{s}}$$

4. Plot a graph of *average induced emf* $\mathcal{E}_{ave}$ versus *rate of magnetic flux change* $\Delta\Phi_B/\Delta t$ in the blank Graph 1 axes. Be sure to label both axes with the correct scale and units.

Graph 1: Average induced emf versus rate of magnetic flux change through a coil with fixed radius

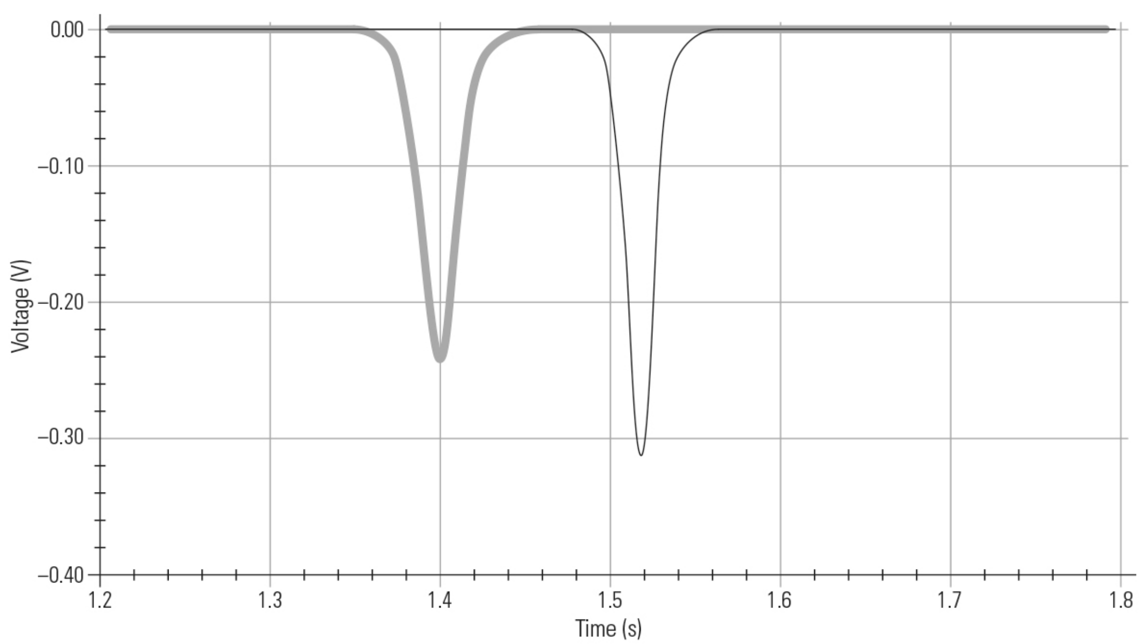


5. Based on Graph 1, how is the average induced emf related to the rate of flux change $\Delta\Phi_B/\Delta t$ through the coil (proportional, inverse, squared, et cetera)?

Student graphs of average induced emf voltage versus rate of magnetic flux change through the coil should show a linear graph, indicating that average induced emf is proportional to the rate of flux change through the coil.

PART 2 – EMF DIRECTION

Graph 2: Induced emf by moving the coil out of a magnetic field



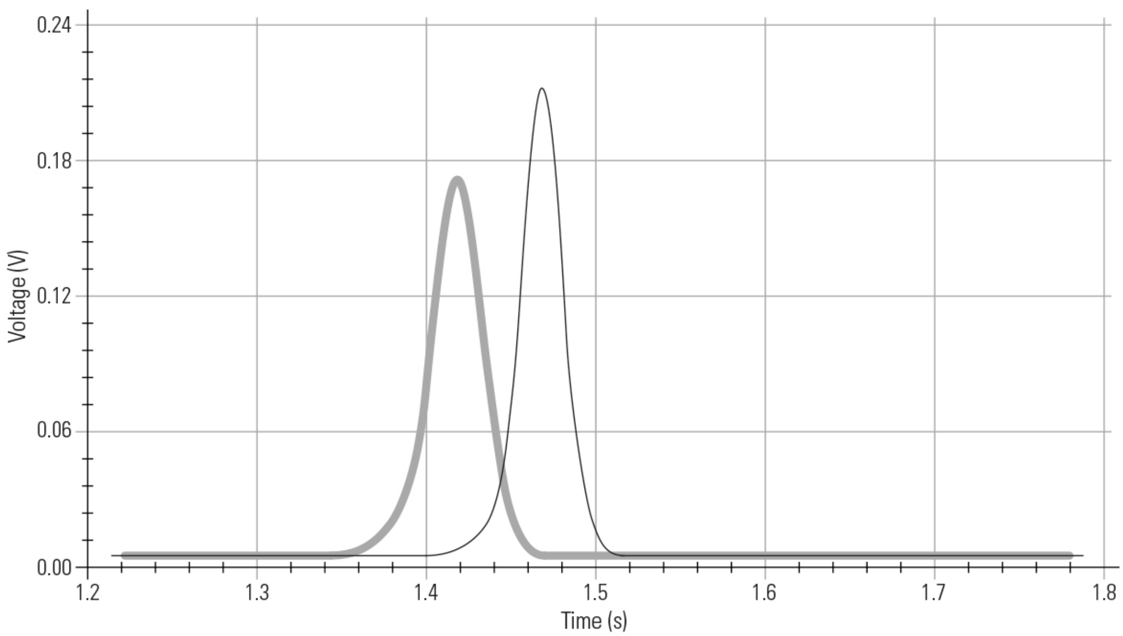
❷ 6. As the coil was moved away from the magnet, was the amount of magnetic flux passing through the coil increasing or decreasing? How do you know?

Magnetic field strength from a permanent magnet decreases with increasing distance from the magnet. According to Equation 2, moving the coil away from the magnet would cause the flux to decrease due to the decreasing magnetic field strength.

❷ 7. Was the induced emf positive or negative as the coil was moved away from the magnet? Did it make a difference if it was moved to the left versus to the right?

Student data may show a positive or negative emf voltage depending on the direction of the magnetic field from their permanent magnet, and how their coil was connected to the voltage–current sensor. Either positive or negative is correct here as long as students identify that polarity of the emf voltage was unchanged when the coil was moved to the left versus to the right.

Graph 3: Induced emf by moving the coil into a magnetic field



❷ 8. As the coil was moved toward the magnet, was the amount of magnetic flux passing through the coil increasing or decreasing? How do you know?

Magnetic field strength from a permanent magnet increases as distance decreases from the magnet. According to Equation 2, moving the coil toward the magnet would cause the flux to increase due to the increasing magnetic field strength.

❷ 9. Was the induced emf voltage positive or negative as the coil was moved toward the magnet? Did it make a difference if it was moved toward the center from the left versus from the right?

Either positive or negative emf voltage is a correct response here as long as the polarity identified in this response is opposite that identified when the coil was moved away from the magnet. Students should also indicate that the polarity of the emf voltage was unchanged when the coil was moved toward the center from the left or from the right.

Guided Inquiry Questions

Below are sample responses to the Guiding Questions found in the Guided Inquiry version of this lab activity.

❷ 1. List two factors that will affect the induced emf in a coil.

Based on the lab objective, students should identify factors that affect the rate of change of magnetic flux through a coil. These factors should be related to or include:

- The rate of change of magnetic field strength through the coil.
- The rate of change of the angle formed between the magnetic field direction and the vector normal to the coil area.
- The rate of change of the coil area.
- The direction of the coil moving through the magnetic field.

Some students may also mention that the number of turns in the coil will affect the induced emf. Although this is correct, for the purpose of this lab activity, students should be limited to those factors that affect the rate of change of magnetic flux through a coil.

❷ 2. Do either of the factors listed above change the magnetic flux through a coil? If yes, how does each factor change the magnetic flux through the coil?

Student responses will vary depending on the factors identified in their response to the previous question. In each case, students must identify how each factor changes at least one of the three variables in Equation 1:

$$\Phi_B = \left|\vec{B}\right| \cos\theta \left|\vec{A}\right|$$

where $\vec{B}$ is the vector representing the magnitude and direction of the magnetic field through the coil area, $\vec{A}$ is the normal vector to the coil, with magnitude equal to the coil area A, and θ is the angle between the two vectors. For magnetic field lines that pass perpendicularly through the coil area, Equation 1 can be simplified to:

$$\Phi_B = BA$$

❷ 3. How will you set up your equipment to produce a changing magnetic flux through the coil?

Students should use a setup in which the equipment that they have chosen can be used to vary any of the three variables in Equation 1. For example, in the Structured version of this lab activity, students create a varying magnetic field through their coil by passing the coil through the field generated by a permanent magnet. Because the field from the magnet decreases with increasing distance from the magnet, the coil experiences changing magnetic field magnitude through it as it moves toward or away from the magnet. This, in turn, creates a changing magnetic flux.

❷ 4. Assuming each factor listed above can be tested experimentally, what should the dependent and independent variables be when testing each factor?

The objective statement refers to two variables: rate of change of magnetic flux through a coil and average induced emf. Students will explore how the rate of change of magnetic flux through a coil affects the average induced emf in the coil. Average induced emf should be identified as the dependent variable and rate of change of magnetic flux should be the independent variable.

Students may choose to divide their experiment into sections that isolate each of the factors outlined in the responses to the previous questions as individual variables. Students should still identify *average induced emf* as the dependent variable and each of the factors they listed that will affect the induced emf in a coil as independent variables, and then reconcile the data from each section showing how each factor affects the rate of change of magnetic flux through the coil.

❷ 5. What equipment do you have at your disposal to measure each variable, and how can you set up this equipment to measure them?

Students should be guided to use a voltage sensor to measure the magnitude and polarity of the induced emf in their coil. Their sensors should be connected in parallel across the input and output terminals of their coils. Students using the PASCO Induction Wand should pay close attention to the wiring diagram at the top of the wand handle. This diagram shows the direction in which the coil is wound, which may play an important role in their experiment design.

Students may use a variety of other equipment and sensors to measure the factors outlined in their responses to the previous questions. Guide students to choose equipment setups and measurement tools that will measure their variables directly to help avoid inaccuracies and error in indirect measurements and calculations.

6. How will you change each independent variable while collecting data? What steps will you take, and should you change more than one variable at a time? Explain your answer.

   Responses will vary. Some examples of changing the independent variable include:

   - Vary the rate at which the angle between the magnetic field and the normal vector to the coil area is changing by rotating the coil in a magnetic field at different angular speeds.
   - Vary the rate at which the magnetic field strength through a coil is changing by using an electromagnet driven by a variable power supply.
   - Vary the rate at which the magnetic field strength through a coil is changing by passing the coil at different speeds through the varying magnetic field surrounding a permanent magnet.
   - Vary the direction in which the coil is moving through the magnetic field.

   Each independent variable should be changed one at a time while the others are held constant. For example, hold the total magnetic flux change constant while varying the time over which the coil experiences that total flux change.

Assessment Questions: Sample Responses

Sample responses to the Analysis and Synthesis questions found in each version of the lab activity:

Analysis Questions

1. In this experiment, what steps did you take to change the magnetic flux through the coil of wire?

   In the Structured version of this lab activity, students vary the magnetic flux through the coil by moving the coil through the magnetic field of a permanent magnet (toward and away from the magnet). The magnetic field strength varies relative to the distance from the magnet.

2. Did the rate of magnetic flux change $\Delta\Phi_B/\Delta t$ affect the induced emf in the coil? If yes, how did it affect it?

   Student data should show that the rate of magnetic flux change affects both the maximum and the average induced emf in their coil. Some groups will show increases in positive emf with increasing rate of flux change, while other groups will show increasing negative emf with increasing rate of flux change. In either case, data should show that the average induced emf in the coil is proportional to the rate of magnetic flux change through the coil.

3. Faraday's Law of Electromagnetic Induction is written:

$$\mathcal{E}_{\text{ave}} = -N\frac{\Delta\Phi_B}{\Delta t} \qquad (5)$$

   where N is the number of turns in the coil. How does your data support Faraday's Law?

   Student data should show that the average induced emf in the coil is proportional, either positive or negative, to the rate of magnetic flux change through the coil, supporting the mathematic model in the Faraday's Law equation above.

   Students who used the PASCO Induction Wand may recognize that the proportionality constant in their data is approximately equal to the number of turns in its coil: 200. This supports Faraday's Law, as the proportionality constant *N* is equal to the number of turns in the coil.

4. How was the emf different when the magnetic flux through the coil was increasing versus when it was decreasing?

   The polarity of the induced emf while the flux was increasing or decreasing will depend on the direction of the magnetic field used in each experiment setup. In either case, the polarity of the emf while the flux was increasing should be opposite that when the flux was decreasing.

❷ 5. The negative sign in Faraday's Law is due to Lenz's Law, which states that the emf induced in a coil will generate current in the coil that produces a magnetic field opposing the change in flux. How does your data support Lenz's Law?

Student data showing that the polarity of the induced emf is opposite when the change in magnetic flux through the coil is increasing versus when it is decreasing is evidence supporting Lenz's Law.

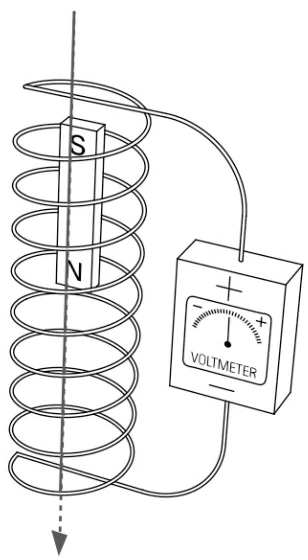
Students performing the Structured version of this lab activity can use their data from Part 2 to support Lenz's Law:

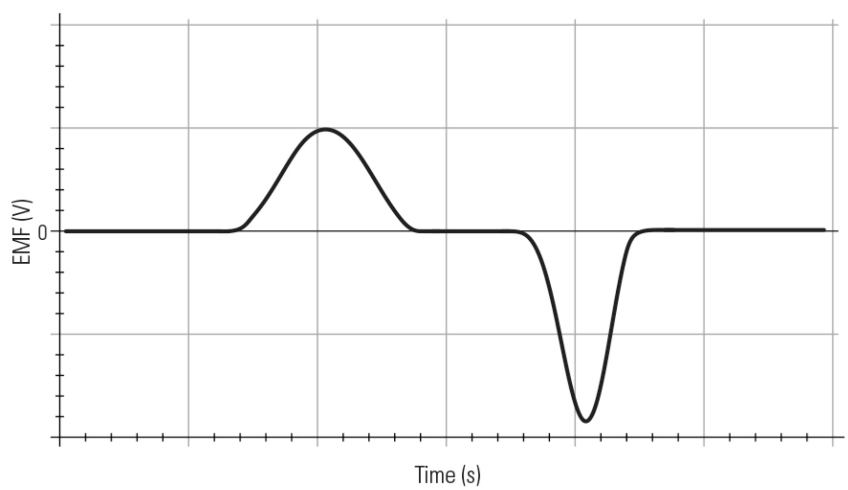
Observe the sample data shown above in Graph 2: As the coil swings out of the magnetic field in both runs, a negative emf is generated that drives current in the direction opposite to the diagram on the induction wand. This current creates a magnetic field that aligns itself with the field from the permanent magnet, in an attempt to keep the total flux through the coil constant.

Observe the sample data shown above in Graph 3: As the coil swings into the magnetic field, a positive emf is generated that drives current in the same direction as the diagram on the induction wand. This current creates a magnetic field that opposes the field from the permanent magnet in an attempt to keep the total flux through the coil constant.

Synthesis Questions

❷ 1. A 4-cm long bar magnet is dropped from 2 cm above a coil of wire. If the falling bar magnet passes through the coil, north pole first (as in the diagram below), what would the graph of emf versus time look like? Sketch your answer in the blank graph axes below, starting from the time at which the magnet is dropped, and ending after the magnet has fallen out of the coil.





Emf increases smoothly from 0 V to some positive value as the north pole on the magnet enters the coil: The downward magnetic flux through the coil increases as the north pole of the magnet approaches the coil. According to Lenz’s Law, a current is generated in the coil that produces a magnetic field that opposes the change in flux. If the change in flux is increasing in the downward direction, the magnetic field from the coil will be increasing in the opposite direction. This is only possible when the voltage is positive.

The emf decreases smoothly to 0 V and remains 0 V as the entire magnet enters, and travels through the coil: The total magnetic flux in the coil is constant as the entire magnet travels through it.

The emf decreases smoothly from 0 V to some negative value as the magnet leaves the coil: The downward magnetic flux through the coil decreases as the south pole of the magnet leaves, and moves away from the coil. According to Lenz’s Law, a current is generated in the coil that produces a magnetic field that opposes the change in flux. If the change in flux is decreasing in the downward direction, the magnetic field from the coil will be increasing in the same direction. This is only possible when the voltage is negative.

The emf increases smoothly to 0 V and remains 0 V as the magnet falls away from the coil: The magnetic flux effectively goes to zero once the distance between the coil and magnet is large enough.

The second peak is taller (negative) because the speed of the magnet is increasing as it falls, but the average flux (the area under the curves) will be the same for both peaks.

❷ 2. A round coil of wire with 10 turns sits in a uniform magnetic field whose field lines pass perpendicularly through the coil area. The magnetic field magnitude increases at a constant rate from 1 T to 10 T over some amount of time. This produces an average 4 V emf voltage in the coil over that time.

a. What would be the average emf voltage if the same coil experienced the same constant change in magnetic field magnitude but in half the time? Explain your answer.

If the total change in magnetic field strength and the area of the coil do not change, the total change in magnetic flux through the coil does not change. Based on Equation 5, if the number of turns in the coil and the total change in magnetic flux are constant, but the time over which the change occurs is halved, the average emf will be double, 8 V.

b. What would be the average emf voltage if the coil had 30 turns instead of 10 and experienced the original change in magnetic field magnitude over time? Explain your answer.

If the total change in magnetic field strength and the area of the coil do not change, the total change in magnetic flux through the coil does not change. Based on Equation 5, if the time over which the change occurs is constant, but the coil has three times as many turns, the average emf will be three times as large, 12 V.

c. What would be the average emf voltage if the coil experienced the original change in magnetic field magnitude over time, but with a radius twice as large? Explain your answer.

Increasing the radius of the coil increases the magnitude of the area through which the magnetic flux change occurs, thus increasing the total change in magnetic flux. If the radius is twice as large, the area of the round coil is four times as large. Based on Equation 3, if the total change in magnetic field magnitude is the same, but the area of the coil is four times as large, the total change in magnetic flux though the coil is four times as large. If the number of turns in the coil and the elapsed time are the same, the average emf will be four times as large as well, 16 V.

d. What would be the average emf voltage if the number of turns and radius of the coil were unchanged but the magnetic field magnitude changed at a constant rate from 10 T to 1 T over the same amount of time? Explain your answer.

The absolute value of the total change in magnetic field magnitude is the same, as are all the other variables, thus making the magnitude of the induced emf the same; however, the change in magnetic field strength is now negative, which implies that the polarity of the new emf is opposite the original emf polarity, making the new average emf equal and opposite, −4 V.

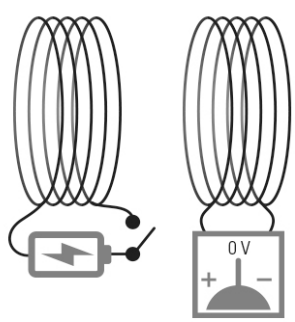
Extended Inquiry Suggestions

Extend this lab with a discussion regarding inductance between two current-carrying coils:

- Ask your students, "What happens to a coil when a current is driven through it?"

  Students should answer that a current-carrying coil generates a magnetic field around it and through its core when a current is driven through it.

Draw the diagram below where two identical coils are placed near each other. One is connected to a battery with a switch, and the other is connected to a voltmeter.



- Ask students to predict how the voltmeter attached to the coil on the right will change, and what its polarity will be when the switch on the left coil is closed.

  When the switch is closed on the left coil, the coil generates a magnetic field that permeates the area around the right coil, increasing the magnetic flux through the right coil from zero (before the switch was thrown) to some value *B* (after the switch was thrown). This flux change in turn induces an emf in the right coil (Faraday's Law). The emf will be negative so that the induced current generates a magnetic field that opposes the field from the left coil (Lenz's Law). The voltmeter changes from 0 V to a negative voltage.

- Ask students, "The voltmeter responded when the switch was thrown, but how will the voltmeter respond when the switch stays thrown? Will the voltmeter stay the same?" Have students predict how the voltmeter will react and what its polarity will be.

  After the switch is thrown on the left coil, the current in the coil quickly becomes steady-state, which generates a constant magnetic field. When the magnetic field is constant, the flux through the right coil is also constant, which, according to Faraday's Law, implies that there will be zero emf. The voltmeter will return to 0 V.

- Ask students, "What happens when the switch is opened?" Have students predict how the voltmeter will react and what its polarity will be.

  When the switch is opened on the left coil, the steady-state magnetic field that permeates the right coil is turned off, reducing the magnetic flux through the right coil from *B* (before the switch was opened) to zero (after the switch is open). This magnetic flux change in turn induces an emf in the right coil (Faraday's Law). The emf will be positive, so the induced current generates a magnetic field in the same direction as the original field from the left coil (Lenz's Law). The voltmeter changes from 0 V to a positive voltage.

Explain to students that without the alternating current in the left coil, the right coil won't produce an emf, and that this is one of the fundamental reasons coil transformers use alternating current.

12. ELECTROMAGNETIC INDUCTION

STRUCTURED

Driving Question | Objective

How is the average emf induced in a wire coil affected by the rate at which magnetic flux through the coil changes? Investigate how the rate of change of magnetic flux through a coil affects the magnitude and polarity of the average emf induced in it, and then determine a mathematical relationship between the two.

Materials and Equipment

- Data collection system
- PASCO Induction Wand[1]
- PASCO Rotary Motion Sensor[2]
- PASCO Variable Gap Magnet[3]
- PASCO Voltage–Current Sensor[4]
- PASCO 2-Axis Magnetic Field Sensor[5]
- PASCO Sensor Extension Cable[5]
- Table clamp or large base
- Support rod, 45-cm
- Right angle clamp
- Additional rod

[1]www.pasco.com/ap02
PASCO Induction Wand

[2]www.pasco.com/ap20
PASCO Rotary Motion Sensor

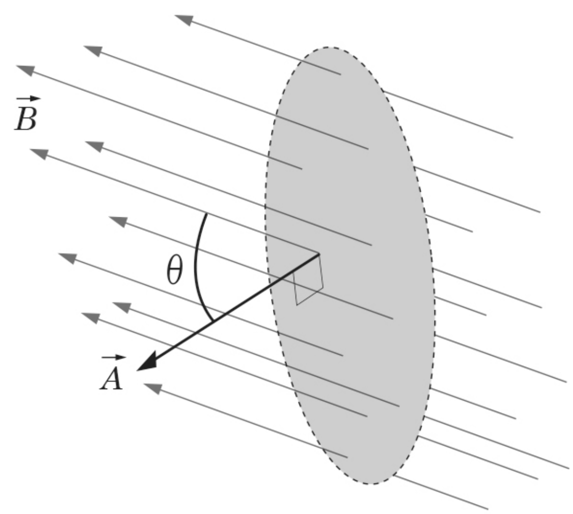
[3]www.pasco.com/ap03
PASCO Variable Gap Magnet

[4]www.pasco.com/ap19
PASCO Voltage–Current Sensor

[5]www.pasco.com/ap35
PASCO 2-Axis Magnetic Field Sensor

Background

Magnetic flux Φ_B is a measure of the amount of magnetic field that passes through a given surface area. It is often referred to as a measure of relative magnetic field strength and can be demonstrated in a diagram as the density of magnetic field lines that pass through a given surface area.



If a uniform magnetic field $\vec{B}$ passes through a flat uniform surface, the equation for magnetic flux can be written:

$$\Phi_B = \vec{B} \bullet \vec{A}$$

$$\Phi_B = \left|\vec{B}\right| \cos\theta \left|\vec{A}\right| \qquad (1)$$

where $\vec{B}$ is the vector representing the magnitude and direction of the magnetic field at the surface, $\vec{A}$ is the normal vector to the surface with area A through which the magnetic field passes, and θ is the angle between the two vectors. For magnetic field lines that pass perpendicularly through the surface, Equation 1 can be simplified to:

$$\Phi_B = BA \qquad (2)$$

When the magnetic flux through a coil of wire changes ($\Delta\Phi_B$), an electromotive force (emf) is induced within the coil. This emf, in turn, generates current flow in the wire and a measureable emf voltage $\mathcal{E}$.

In this lab activity, you will explore how varying the rate of magnetic flux change $\Delta\Phi_B/\Delta t$ through a coil affects the average induced emf voltage $\mathcal{E}_{\text{ave}}$ in the coil, make conclusions about the directionality and magnitude associated with the average induced emf, and use your data to support those conclusions.

RELEVANT EQUATIONS

$$\Phi_B = BA \qquad (2)$$

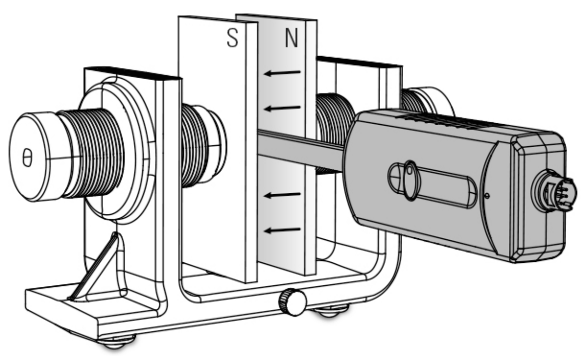
Procedure

Part 1 – Rate of Flux Change and Average Emf

SET UP: MEASURE MAXIMUM MAGNETIC FIELD MAGNITUDE

1. Place the variable gap magnet on the lab table, increase the gap spacing so you can insert the two flat iron pole plates (to provide a uniform magnetic field), and adjust either plate so that the gap between the pole plates is 2 cm.

2. Connect the magnetic field sensor to the data collection system using the sensor extension cable, and then create a digits display of perpendicular magnetic field strength.



3. Hold the sensor with its tip directly between the magnet pole plates. Turn the sensor so that the "perpendicular" measurement axis is aligned with, and in the same direction as, the magnetic field between the plates (from north pole to south pole), similar to the figure.

   NOTE: A diagram on the sensor case indicates the direction of the perpendicular axis.

COLLECT DATA: MEASURE MAXIMUM MAGNETIC FIELD MAGNITUDE

4. Start recording data. If the value is negative, rotate the sensor so the Tare button on the sensor faces the other pole plate. This should change the measurement polarity to positive. Measured values are positive when the perpendicular axis is in the same direction as the magnetic field (north pole to south pole).

5. Determine the magnetic field strength (magnitude) at the center between the pole plates on the magnet. Record this as the maximum magnetic field, in units of tesla, above Table 1 in the Data Analysis section below.

6. Stop recording data, disconnect the magnetic field sensor and extension cable, and delete any data on your data collection system.

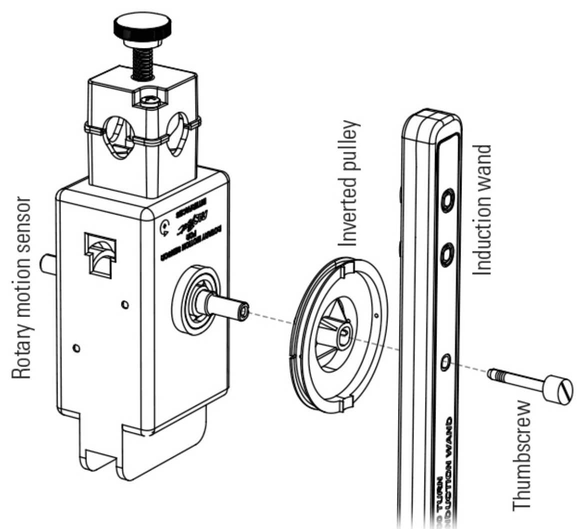
SET UP: MEASURE INDUCED EMF

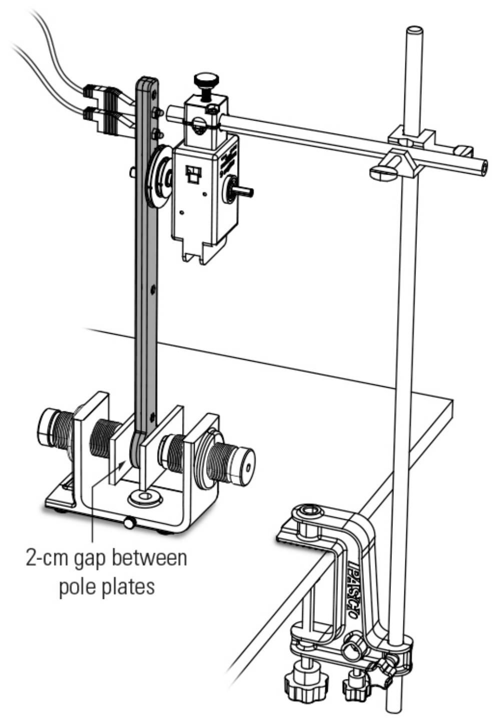
7. Assemble the equipment as shown in the figure:

    - Invert the pulley on the rotary motion sensor so the smallest pulley step faces the sensor. Use the thumbscrew accompanying the induction wand to attach the wand to the axle on the rotary motion sensor.

    - The gap between the pole plates on the magnet should be 2 cm wide.

    - Adjust the height of the rotary motion sensor and the position of the variable gap magnet so the center of the coil at the bottom end of the induction wand can swing freely between the pole plates on the magnet, but hangs freely and motionless exactly between the poles on the magnet.





8. Connect the voltage sensor leads to the top end of the induction wand: connect the red lead to the top port, connect the black lead to the bottom port.

    NOTE: Do not allow the voltage sensor cords to impede the swing of the induction wand.

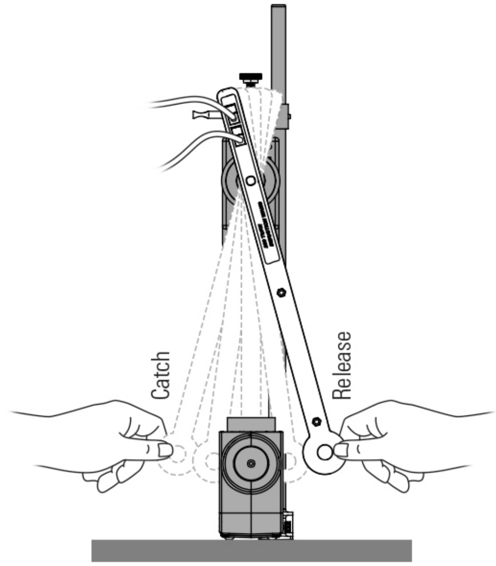
9. Connect the voltage sensor and rotary motion sensor to the data collection system.

10. Create one graph display with two y-axes versus time: one y-axis showing voltage from the voltage sensor, and the second y-axis showing angle (°) from the rotary motion sensor. Lock the graph so that both y-axes share the origin.

11. Set the sample rate on the data collection system to 250 Hz.

12. In your data collection system's sensor settings, configure the rotary motion sensor to *NOT* automatically zero the sensor's measurements on start. Then, with the wand hanging freely and motionless between the poles on the magnet, on the data collection system select the "Zero Sensor Now" button to associate the current position of the wand to an angular position of 0° for all trials.

COLLECT DATA: MEASURE INDUCED EMF



13. Start recording data, and then rotate the wand to the right (counterclockwise) approximately 30°. Hold it in place for a short moment and then release it to swing freely, once, through the pole plates on the magnet.

    Catch the wand on the other side of the magnet before it swings back through the pole plates.

14. Stop recording data.

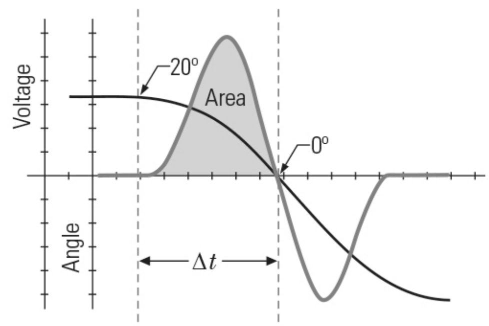
15. Repeat the data collection steps four additional times, each time rotating the wand in the same direction and increasing the rotation angle by approximately 15°. When you are finished, you will have a total of five trials of data.

16. Use the tools on your data collection system to obtain the following values in each trial:

    a. The area under your voltage versus time curve between the angles 20° and 0° (the area may be positive or negative). Record each area value for the corresponding trial in Table 1.

    b. The change in time Δt between the angles 20° and 0°. Record each Δt value for the corresponding trial in Table 1.

    NOTE: Each trial may not have angle or voltage data points that align exactly with 20° or 0°. In this case, use the data points closest to 20° or 0°.

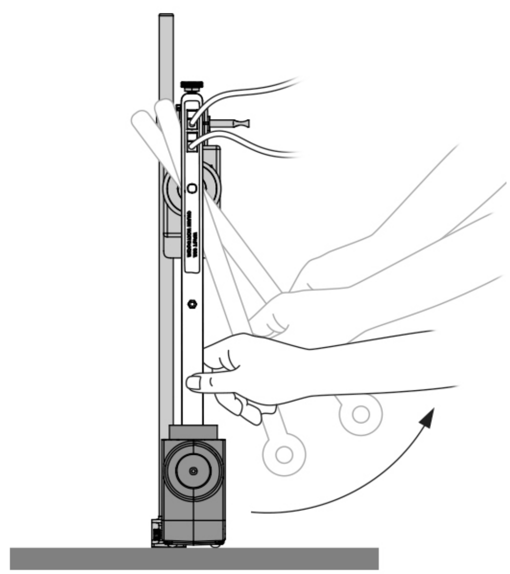


Part 2 – Emf Direction

SET UP

17. Keep the equipment setup identical to that of the previous part, and create a new graph display of only voltage versus time. Hide any data so that your graph is blank.

COLLECT DATA



18. With the wand hanging freely and motionless between the poles of the magnet, begin recording data.

19. Using your hand, and without taking your hand off the wand, quickly rotate the wand to the right, moving the coil completely out from between the magnet plates. Hold it in place, and then stop recording data.

20. Again, with the wand hanging freely and motionless between the magnet poles, begin recording a second run of data: quickly rotate the wand to the left, out from between the magnet plates, and hold it in place.

21. Stop recording data and sketch a copy of your *voltage* versus *time* graph, with these two runs, in the blank Graph 2 axes in the Data Analysis section. Be sure to label both axes with the correct scale and units.

22. Hide your data so the voltage versus time graph is blank.

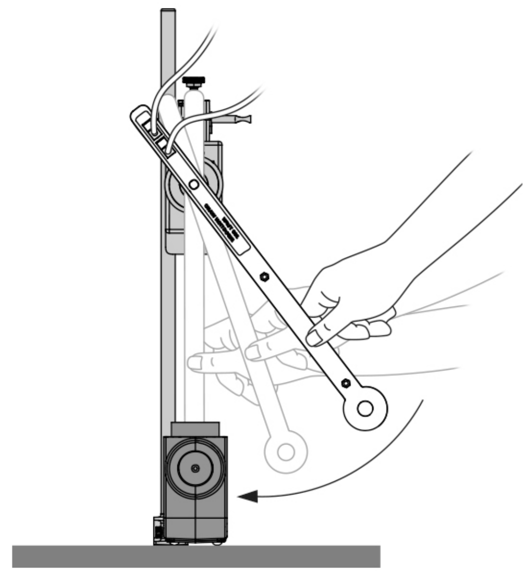
23. Use your hand to rotate the wand to the right and hold it in place so that the coil is completely out from between the magnet plates.

24. Begin recording data, and without the wand leaving your hand, quickly rotate it back and stop when the wand is vertical and the coil is directly between the centers of the pole plates on the magnet.

25. Stop recording data.

26. Use your hand to rotate the wand to the left and hold it in place so that the coil is completely out from between the magnet plates and then repeat the two previous data collection steps.

27. Stop recording data. Sketch a copy of your graph with these two runs into the blank Graph 3 axes in the Data Analysis section. Be sure to label both axes with the correct scale and units.



Data Analysis

Part 1 – Rate of Flux Change and Average Emf

Maximum magnetic field (T): ____________________

Table 1: Determination of the average induced emf and the rate of magnetic flux change

Trial	Release Angle (approximate) (°)	Area Under Curve (V·s)	Δt (s)	$\mathcal{E}_{ave}$ (V)	$\Delta\Phi/\Delta t$ ($T \cdot m^2/s$)
1	30				
2	45				
3	60				
4	75				
5	90				

1. Calculate the average induced emf $\mathcal{E}_{ave}$ for each Part 1 trial. Record your results in Table 1.

$$\mathcal{E}_{ave} = \frac{\text{Area under voltage versus time curve}}{\Delta t}$$

2. The following information will help with the calculation of the total change of magnetic flux through the coil:

   Assuming the magnetic field was always perpendicular to the plane of the coil as it swung in each trial, and the area of the coil A is constant, then according to Equation 2, the equation describing the total change in magnetic flux $\Delta\Phi_B$ in each trial is:

   $$\Delta\Phi_B = \Delta BA$$

   $$\Delta\Phi_B = \Delta B\pi r^2 \qquad (3)$$

   where ΔB is the total change in magnetic field amplitude experienced by the coil and r is the radius of the coil.

   If we assume the magnetic field magnitude was zero for angles greater than 20° and maximum at an angle of 0° (between the pole plates) in each trial, the total change in magnetic field magnitude for all trials is:

   $$\Delta B = B_{\text{f}} - B_{\text{i}} = B_{\text{max}} - 0 = B_{\text{max}}$$

   Plugging the result for ΔB back into Equation 3 gives the equation for the total change in magnetic flux through the coil for all trials:

   $$\Delta\Phi_B = B_{\text{max}}\pi r^2 \qquad (4)$$

   where B_{max} is the measured maximum magnetic field value and r is the radius of the coil (in the case of the PASCO Induction Wand, $r = 0.0125$ m).

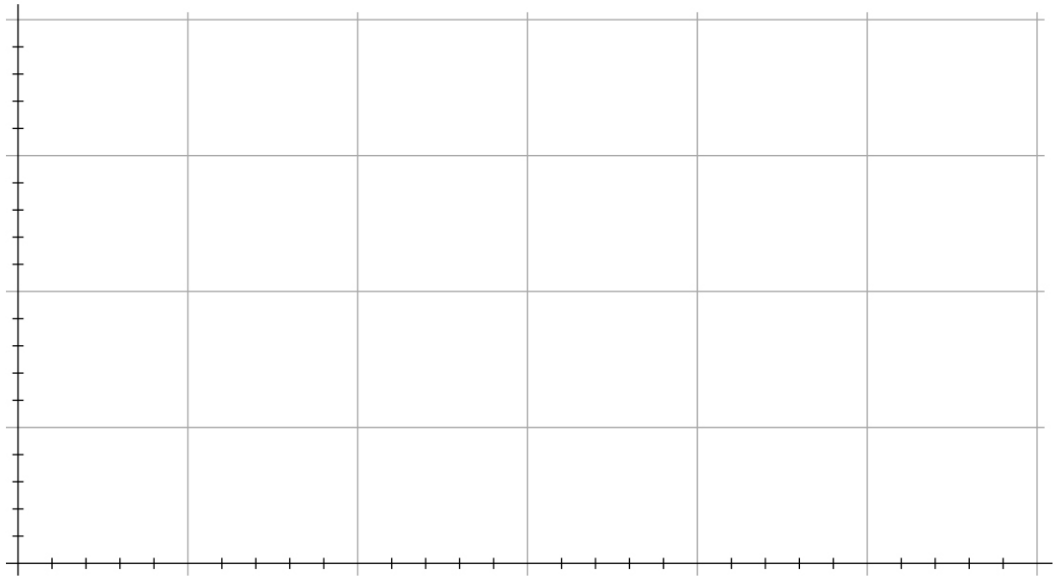
   Using the above information, calculate the total change in magnetic flux $\Delta\Phi_B$ through the coil for all trials. Record your value here:

   $\Delta\Phi_B$ (T·m^2): ________________________________

3. Calculate the rate of magnetic flux change $\Delta\Phi_B/\Delta t$ for each trial using the $\Delta\Phi_B$ value from the previous question and the elapsed time Δt values from Table 1. Record the results for each trial in Table 1.

4. Plot a graph of *average induced emf* $\mathcal{E}_{ave}$ versus *rate of magnetic flux change* $\Delta\Phi_B/\Delta t$ in the blank Graph 1 axes. Be sure to label both axes with the correct scale and units.

Graph 1: Average induced emf versus rate of magnetic flux change through a coil with fixed radius



5. Based on Graph 1, how is the average induced emf related to the rate of flux change $\Delta\Phi_B/\Delta t$ through the coil (proportional, inverse, squared, et cetera)?

____

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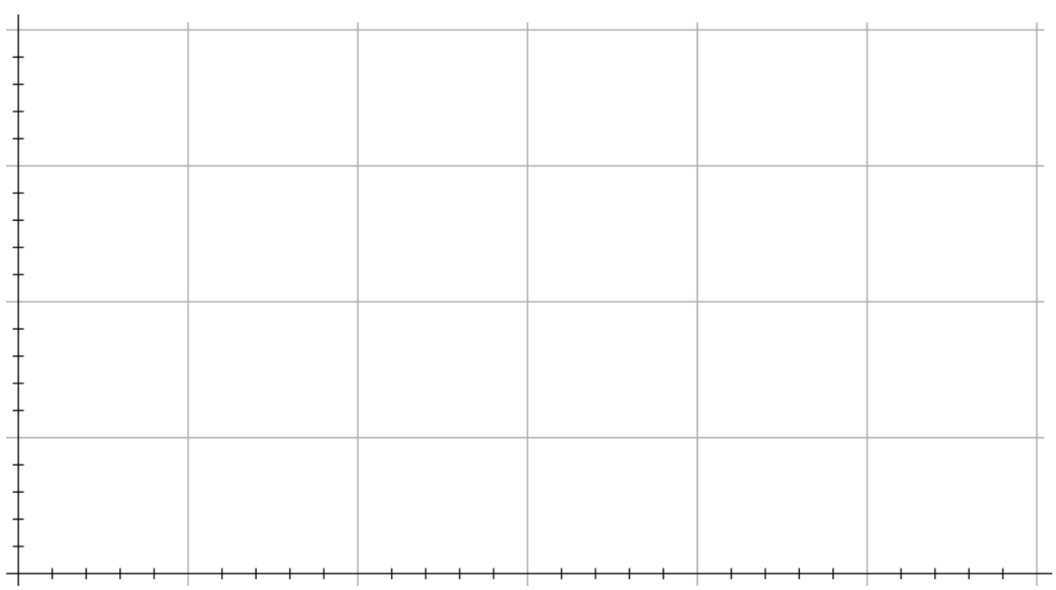
Part 2 – Emf Direction

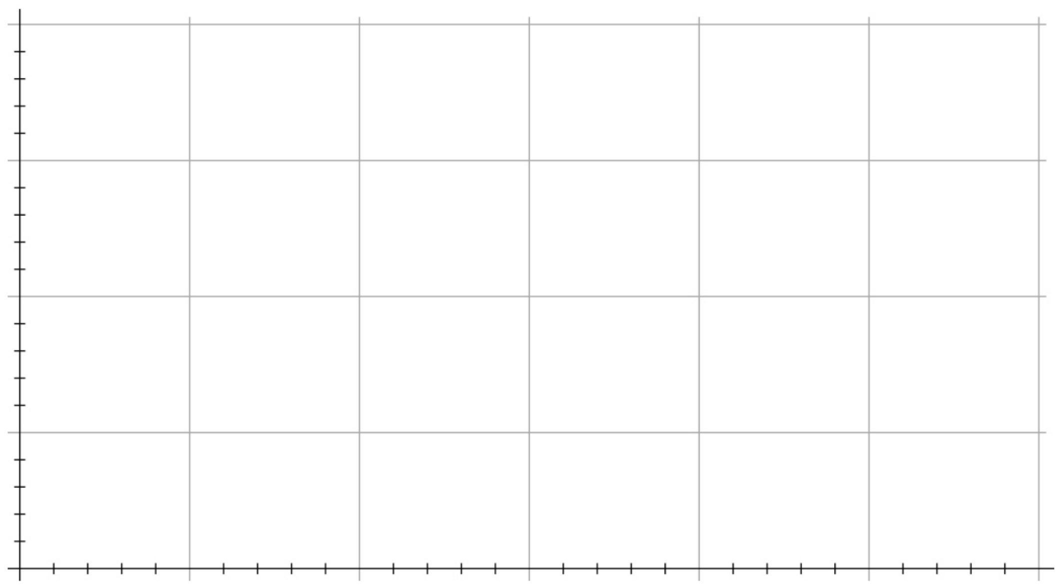
Graph 2: Induced emf by moving the coil out of a magnetic field



6. As the coil was moved away from the magnet, was the amount of magnetic flux passing through the coil increasing or decreasing? How do you know?

7. Was the induced emf positive or negative as the coil was moved away from the magnet? Did it make a difference if it was moved to the left versus to the right?

Graph 3: Induced emf by moving the coil into a magnetic field



8. As the coil was moved toward the magnet, was the amount of magnetic flux passing through the coil increasing or decreasing? How do you know?

9. Was the induced emf voltage positive or negative as the coil was moved toward the magnet? Did it make a difference if it was moved toward the center from the left versus from the right?

Analysis Questions

1. In this experiment, what steps did you take to change the magnetic flux through the coil of wire?

2. Did the rate of magnetic flux change $\Delta\Phi_B/\Delta t$ affect the induced emf in the coil? If yes, how did it affect it?

3. Faraday's Law of Electromagnetic Induction is written:

$$\mathcal{E}_{\text{ave}} = -N\frac{\Delta\Phi_B}{\Delta t} \qquad (5)$$

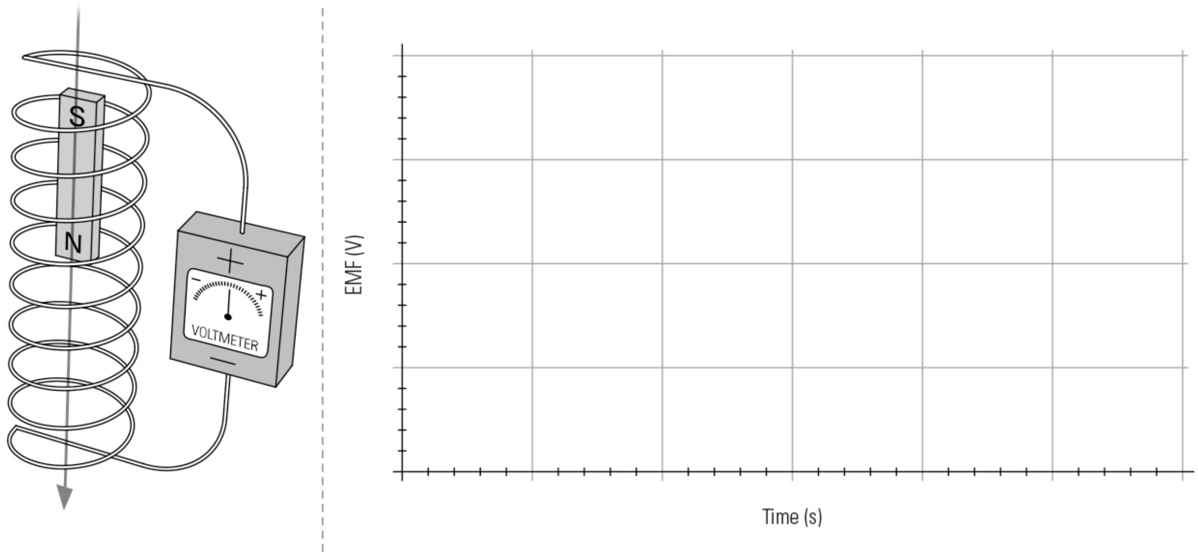
where N is the number of turns in the coil. How does your data support Faraday's Law?

4. How was the emf different when the magnetic flux through the coil was increasing versus when it was decreasing?

5. The negative sign in Faraday's Law is due to Lenz's Law, which states that the emf induced in a coil will generate current in the coil that produces a magnetic field opposing the change in flux. How does your data support Lenz's Law?

Synthesis Questions

1. A 4-cm long bar magnet is dropped from 2 cm above a coil of wire. If the falling bar magnet passes through the coil, north pole first (as in the diagram below), what would the graph of emf versus time look like? Sketch your answer in the blank graph axes below, starting from the time at which the magnet is dropped, and ending after the magnet has fallen out of the coil.



2. A round coil of wire with 10 turns sits in a uniform magnetic field whose field lines pass perpendicularly through the coil area. The magnetic field magnitude increases at a constant rate from 1 T to 10 T over some amount of time. This produces an average 4 V emf voltage in the coil over that time.

   a. What would be the average emf voltage if the same coil experienced the same constant change in magnetic field magnitude but in half the time? Explain your answer.

   b. What would be the average emf voltage if the coil had 30 turns instead of 10 and experienced the original change in magnetic field magnitude over time? Explain your answer.

   c. What would be the average emf voltage if the coil experienced the original change in magnetic field magnitude over time, but with a radius twice as large? Explain your answer.

d. What would be the average emf voltage if the number of turns and radius of the coil were unchanged but the magnetic field magnitude changed at a constant rate from 10 T to 1 T over the same amount of time? Explain your answer.

13. CAPACITOR FUNDAMENTALS

Connections to the AP® Physics 2 Curriculum*

The lab activity correlates to the following pieces of the AP® Physics 2 framework:

Big Idea	4	Enduring Understanding	E	Essential Knowledge	4

Learning Objective 2: The student is able to design a plan for the collection of data to determine the effect of changing the geometry and/or materials on the resistance or capacitance of a circuit element and relate results to the basic properties of resistors and capacitors.
Science Practices: 4.1, 4.2

Learning Objective 3: The student is able to analyze data to determine the effect of changing the geometry and/or materials on the resistance or capacitance of a circuit element and relate results to the basic properties of resistors and capacitors.
Science Practices: 5.1

Time Requirement

Preparation Time: 10 minutes

Lab Activity: 45 minutes

Prerequisites

Students should be familiar with the following concepts:

- The difference between an electrical conductor and an insulator.
- Electric force and how electric charges interact.

Driving Question | Objective

How do physical properties of a parallel-plate capacitor affect its ability to store electric charge? Experimentally determine the relationship of the capacitor-plate area and the distance between plates to its ability to store electric charge (capacitance).

Procedural Overview

In the Structured version of this lab activity, students construct parallel-plate capacitors using aluminum foil sheets as capacitor plates separated by insulation (in the Structured version of this lab students use paper, but students in Student Designed or Guided Inquiry labs might use different materials). Using a digital multimeter, students measure the capacitance of the capacitors and observe the effects that plate size and plate spacing have on the capacitance value. Students then use their data to make qualitative arguments regarding how these physical properties affect charge storage within a parallel-plate capacitor.

The Structured version of this lab activity is divided into two parts. In Part 1, students construct a capacitor and measure its capacitance as they vary the plate size by folding the aluminum foil sheets to decrease plate area. In Part 2, students construct a second capacitor and measure capacitance as they vary the plate spacing by placing additional sheets of paper between the capacitor plates. Students then plot graphs of their data and use those graphs to determine the mathematical relationship between the capacitance and each variable.

Pre-Lab Discussion and Activity

Build a large capacitor similar to the one the students will build in the activity, only place a polystyrene sheet in-between the aluminum foil sheets instead of paper. Charge the capacitor by rubbing a piece of wool against a section of a PVC pipe while the piece of pipe touches one of the capacitor plates. Repeat this several times while having a student volunteer touch the tab on the other plate. After charging the capacitor for several seconds, ask the student to discharge the capacitor by touching the tab on the other plate with another finger. They should be able to feel and hear a spark!

Explain how the intensity of the spark is dependent on the amount of charge transferred between the plates, and that the amount of charge is dependent on the capacitance of the capacitor. Ask the students what could be changed about the capacitor in order to increase the capacitance (and create a more intense spark). The goal is for students to suggest changing the surface area and the separation of the plates, which will be the basis of their investigation, but they may also suggest other things, such as using different materials for the conductors and insulators. These suggestions may make good topics to investigate as an extension activity. Discuss why they believe changing these items on the capacitor would cause the capacitance to increase.

Materials and Equipment

- Digital capacitance meter/multimeter, 0.01-nF resolution
- 4-mm banana plug patch cord (2)
- 4-mm banana plug patch cord alligator clip (2)
- Aluminum foil sheets (4), approx. 8 1/2" × 11"
- Paper (6 sheets), 8 1/2" × 11"
- Scissors
- Ruler
- Heavy textbook

Safety

Follow these important safety precautions in addition to your regular classroom procedures:

- Follow the instructions in the user manual to correctly operate the digital multimeter (DMM).
- Use caution when working with sharp objects such as scissors, hobby knives, and serrated cutting edges.

Teacher Tips

Tip 1 – Choosing a Digital Multimeter

- Any standard multimeter that can measure capacitance in nanofarads will work for this experiment. You could also use a meter dedicated to measuring capacitance.

Tip 2 – Choosing Paper

- Standard printer paper works well for this experiment. Graph paper may be handy for determining the surface area instead of using the ruler, but is not necessary.

Tip 3 – Measuring Capacitance

- The capacitance of a parallel plate capacitor is very sensitive to plate spacing. It is important that the students use the same textbook to press the layers of their capacitors together for each measurement. The capacitance will increase significantly when a student applies their weight onto the textbook, pressing the capacitor plates closer together. It is recommended to press down on the textbook a few times to flatten the layers as much as possible before recording measurements.

Tip 4 – Variance in Measurements

- Expect a large degree of variance in capacitance values between the student groups, even if the plate sizes and spacings are the same. Regardless of the variance, each group should be able to reliably obtain data showing a proportional and an inverse relationship.

Sample Data

Below are sample data, acquired using the experimental setup and procedure outlined in the Structured version of the lab activity, and answers to questions in the Data Analysis section.

Data Analysis

Part 1 – Varying the Area of Parallel Conducting Plates

Table 1: Capacitance of a parallel-plate capacitor with varying plate area

Length (c*m*)	Width (c*m*)	Area (c*m*^{2})	Capacitance (nF)
27.2	20.4	555	6.26
13.6	20.4	277	3.40
13.6	10.2	139	2.28
6.8	10.2	69	1.06
6.8	5.1	34	0.42

1. Calculate the area of the capacitor plates used in each trial. Record the result for each trial in Table 1.

   Calculation using sample data from Table 1:

   $\text{Area} = \text{Length} \times \text{Width}$

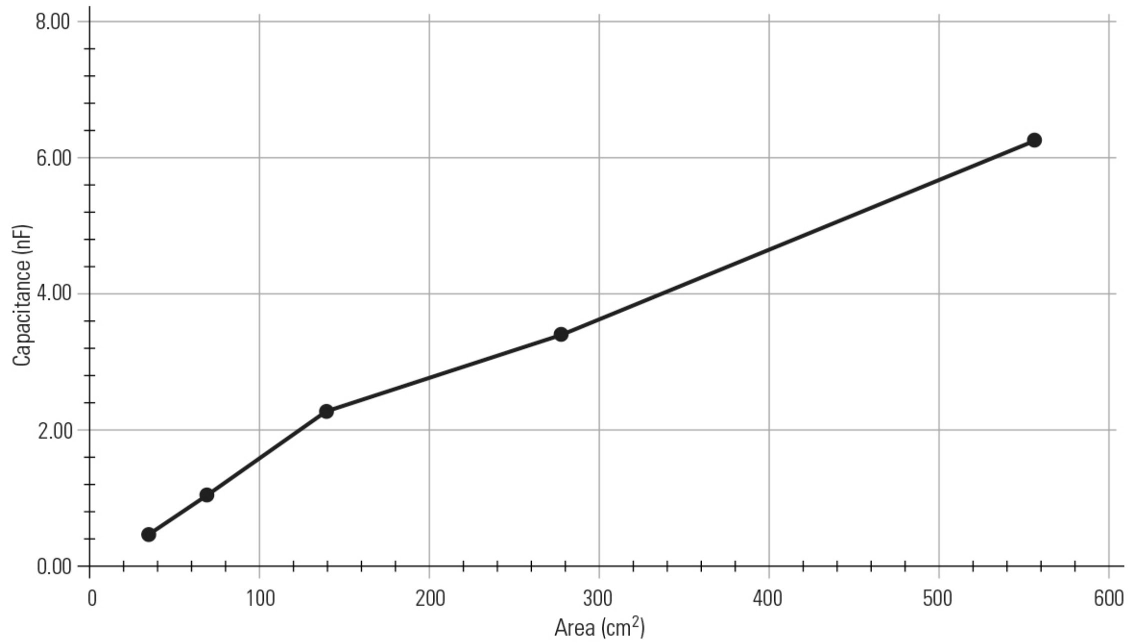
   $\text{Area} = 27.2\text{ cm} \times 20.4\text{ cm} = 555\text{ cm}^2$

❷ 2. Qualitatively, what happens to the capacitor's ability to store charge (capacitance) as the area of each plate decreases?

   A parallel-plate capacitor's ability to store charge (capacitance) decreases as the area of its plates decreases. This is a result of the inability to physically store the same number of charges on the smaller surface of the plates without increasing the surface charge density.

3. Plot a graph of *capacitance* versus *area* in the blank Graph 1 axes. Be sure to label both axes with the correct scale and units.

Graph 1: Capacitance versus area of a parallel-plate capacitor



❷ 4. How are the variables in Graph 1 related to each other mathematically? Use terms such as proportional, inversely proportional, linear, or quadratic in your response.

Student graphs should show a directly proportional relationship between plate area and capacitance.

Part 2 – Varying the Distance between Parallel Conducting Plates

Table 2: Capacitance of a parallel-plate capacitor with varying plate spacing

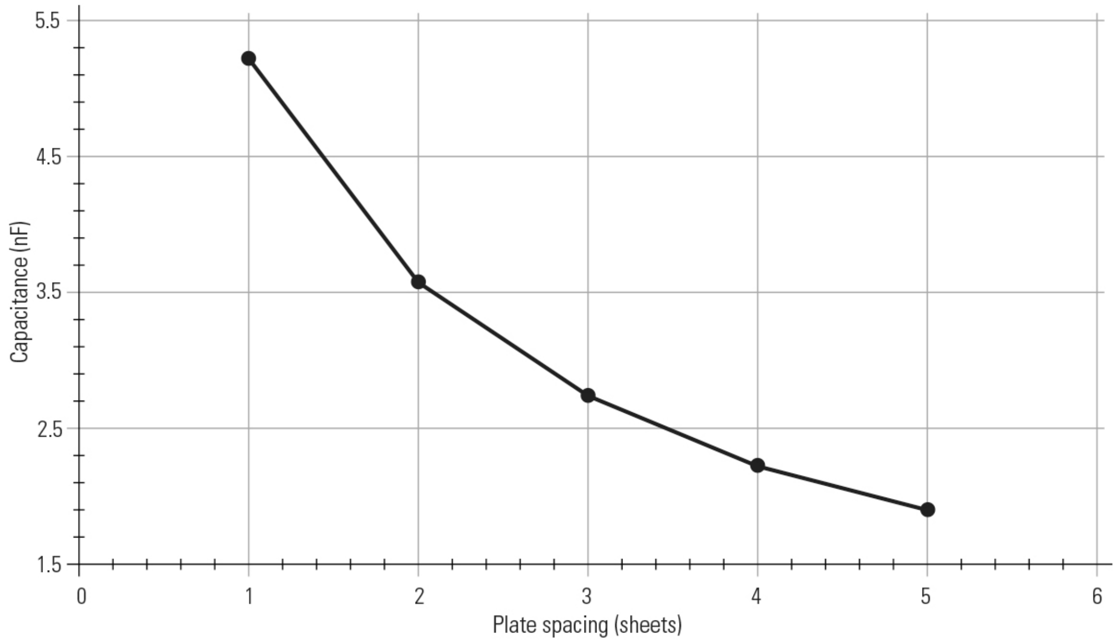
Plate Spacing (sheets of paper)	Capacitance (nF)
1	5.24
2	3.60
3	2.76
4	2.25
5	1.92

❷ 5. Qualitatively, what happens to the capacitor's ability to store charge (capacitance) as the spacing between the plates increases?

A parallel-plate capacitor's ability to store charge decreases as the distance between its plates increases. This is due to the decreasing electric field (at a given potential difference) between the plates as they are separated. As the field strength decreases, so does the capacitor's ability to store charge on the plates.

6. Plot a graph of *capacitance* versus *plate spacing* in the blank Graph 2 axes. For plate spacing, use the number of sheets of paper as the unit. Be sure to label both axes with the correct scale and units.

Graph 2: Capacitance versus plate spacing for parallel-plate capacitor



7. How are the variables in Graph 2 related to each other mathematically? Use terms such as proportional, inversely proportional, linear, or quadratic in your response.

   The capacitance is inversely proportional to the plate spacing.

Guided Inquiry Questions

Below are sample responses to the Guiding Questions found in the Guided Inquiry version of this lab activity.

1. Based on the driving question and experiment objective, what will be the dependent and independent variables in your exploration?

   Students should measure, directly or indirectly, the capacitance (the dependent variable) of their constructed parallel-plate capacitor as they manipulate the area of its capacitor plates (the first independent variable) and the spacing between the plates (the second independent variable).

2. What equipment will you use to measure these variables?

   Students should measure capacitance with a digital multimeter or dedicated capacitance meter. Capacitor plate area should be calculated using measurements of length and width made with a ruler or meter stick. Plate spacing can be measured using high resolution measurement tools such as vernier calipers or a micrometer. Students may also choose a measurement strategy similar to that outlined in the Structured version of this lab activity, where plate separation is measured by counting the number of sheets of paper that separate the plates.

3. How will you construct your parallel-plate capacitor(s)? What materials will you use to create the capacitor plates? What materials will you use to separate the plates? Explain your reasoning.

   Student capacitor designs will vary. In all cases, each parallel-plate capacitor should have two identically-shaped and sized flat capacitor plates separated by an insulating material. Capacitor plates can be constructed of any flat conducting surface with a sufficiently large surface area, separated by an insulating material that is not too thick, so it does not decrease the capacitance value below measurable quantities.

   Students must also use a material that allows them to separately manipulate the area of the plates and the spacing between them. Students following the procedure outlined in the Structured version of this lab activity construct parallel-plate capacitors using two flat sheets of aluminum foil separated by multiple sheets of insulating printer paper.

4. How will you change each independent variable while collecting data? Should you change more than one variable at a time? Describe the steps you will take to change each variable.

   Students can change the area of the plates by either folding or cutting the aluminum foil into different sizes. Plate spacing can be changed by adding additional sheets of paper (or other chosen material) between the plates. Only one variable should be changed at a time so students can observe how a single variable affects the capacitance.

5. What are some sources of experimental error in this experiment? What will you do in order to minimize error?

   Students will find that the sheets of foil are not completely flat and can have minor gaps between them that affect the capacitance measurement. To minimize this error, a student can use a flat object like a textbook and repeatedly press down on the capacitor to press the capacitor layers together. Additionally, the foil can be smoothed by rubbing a textbook across the foil.

Assessment Questions: Sample Responses

Sample responses to the Analysis and Synthesis questions found in each version of the lab activity:

Analysis Questions

1. Consider the materials you used to make your parallel-plate capacitor. Classify the materials as conductors or insulators and discuss the relative mobility of the electrons in each type of material.

   Regardless of the materials chosen, student groups should classify conducting materials as those that allow electrons within the material to move freely through and about its surface, and insulating materials as those that impede the free movement of electrons, not allowing current to flow.

2. What are the physical properties of a parallel-plate capacitor? In this experiment, which of these physical properties did you vary?

   The physical properties of a parallel-plate capacitor include the area of the plates, the spacing between the plates, and the material of the insulator separating the plates. In this experiment, students manipulate the area of and spacing between the plates while observing the effects on overall capacitance.

3. For each of the properties you varied, what was the effect on the capacitance you measured?

   Student data should show that decreasing capacitor plate area or increasing capacitor plate spacing decreases the overall capacitance of a parallel-plate capacitor.

4. Why do you think the change in each of these physical properties changed the capacitance the way it did?

   Decreasing the area of the plates does not allow as much physical space for charges to gather, which in turn decreases the capacitance. Increasing the spacing between the plates increases the distance between the positive and negative charges present on the plates. As the spacing increases, the attractive force between the charges decreases, and fewer charges gather on the plates, decreasing the capacitance.

❷ 5. The capacitance C for a parallel-plate capacitor is given by the equation:

$$C = \kappa\varepsilon_0 \frac{A}{d}$$

where A is the area of each plate, d is the plate spacing, and κ and ε_0 are constants. How does your data support this definition of capacitance?

Student data should show that the capacitance of a parallel-plate capacitor is directly proportional to plate area (as area increases or decreases, capacitance proportionally increases or decreases, respectively), and is inversely proportional to plate spacing (as plate spacing increases or decreases, capacitance decreases or increases, respectively), which is what the equation indicates.

❷ 6. Consider the configurations you tested. Overall, what combination of conditions would yield the largest value for capacitance? What conditions would yield the lowest value?

A capacitor with the greatest area and the smallest plate spacing would yield the greatest value for capacitance. A capacitor with the smallest area and with the greatest plate spacing would yield the smallest value for capacitance.

Synthesis Questions

❷ 1. If you wanted to build a capacitor with twice the maximum capacitance you measured in this experiment, how would you design your new capacitor?

To obtain twice the maximum capacitance, the physical properties of a capacitor should be changed so the capacitor-plate area-to-distance ratio (A/d) is doubled. For example, doubling the area of the capacitor plates or halving the spacing between them will double the overall capacitance.

❷ 2. The SI unit of capacitance is a derived unit known as the farad (named after Michael Faraday). One farad is defined as one coulomb per volt: 1 F = 1 C/V. Based on the definition of this unit, write an equation that relates capacitance, charge, and potential difference and then solve this equation for charge.

Since the farad, coulomb, and volt relate to capacitance C, charge Q, and potential difference ΔV, respectively, 1 F = 1 C/V translates to:

$$C = \frac{Q}{\Delta V}$$

Solving this equation for charge yields:

$$Q = C\Delta V$$

❷ 3. An engineer needs a 1.0 μF parallel-plate capacitor with a spacing of 0.050 mm between its plates. She will use the capacitor in the design of a portable electronic device.

a. What would the area of the plates need to be to make this capacitor? Assume that $\kappa = 1$ and $\varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/(N \cdot m^2)}$.

NOTE: 1 V = 1 J/C

$$C = \kappa\varepsilon_0 \frac{A}{d}$$

$$A = \frac{Cd}{\kappa\varepsilon_0} = \frac{\left(1.0 \times 10^{-6}\ \mathrm{F}\right)\left(5.0 \times 10^{-5}\ \mathrm{m}\right)}{(1.00)\left(8.85 \times 10^{-12}\ \mathrm{C^2/(N \cdot m^2)}\right)} = 5.6\ \mathrm{m^2}$$

b. Discuss the practicality of this capacitor in the context of its intended use.

This is not a practical capacitor since it would be many times larger than the person using it and it is intended for use in a portable device!

c. If the portable electronic device is designed to use a 1.5 V battery, what is the maximum charge the capacitor can store?

$$Q = C\Delta V = \left(1.0 \times 10^{-6}\ \mathrm{F}\right)(1.5\ \mathrm{V}) = 1.5 \times 10^{-6}\ \mathrm{C}$$

Extended Inquiry Suggestions

- Have students use different insulators between the plates to measure the effect on capacitance. Some suggested materials include wax paper, polystyrene sheets, and overhead transparency film.
- Electrolytic capacitors are able to be manufactured with high capacitance values in a compact package by rolling the capacitor plates together. Have the students make a large parallel plate capacitor, measure the capacitance, roll it up, and then measure the capacitance again. They will most likely find that the capacitance increases since the layers are pressed closer together when rolled. The students may want to try gluing the foil to the paper. Not only will this help the layers stay together as they are rolled, it will dramatically increase the capacitance of the capacitor!

13. CAPACITOR FUNDAMENTALS

STRUCTURED

Driving Question | Objective

How do physical properties of a parallel-plate capacitor affect its ability to store electric charge? Experimentally determine the relationship of the capacitor-plate area and the distance between plates to its ability to store electric charge (capacitance).

Materials and Equipment

- Digital capacitance meter/multimeter, 0.01-nF resolution
- 4-mm banana plug patch cord (2)
- 4-mm banana plug patch cord alligator clip (2)
- Aluminum foil sheets (4), approx. 8 1/2" × 11"
- Paper (6 sheets), 8 1/2" × 11"
- Scissors
- Ruler
- Heavy textbook

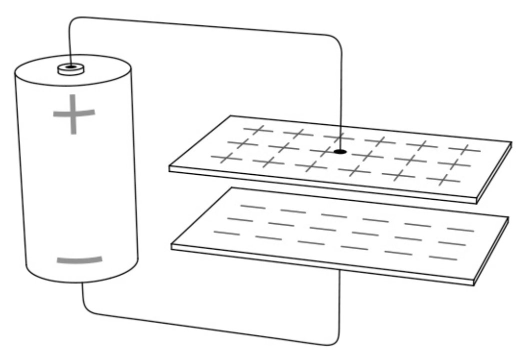
Background

The static electric effects of a Van de Graaff generator, of the laundry taken from the clothes dryer, and of a plastic wrapper pulled off a package are examples of positive and negative charges that have been separated from each other. Electrical potential energy is stored in those separated charges, since work has been done to separate them. Storing potential energy in the form of separated charges in an electric circuit is the function of a capacitor.

When allowed a conducting path, charges that have been separated readily flow back together. Charges in motion have kinetic energy. In a circuit, flowing charges are known as electric current.

The type of capacitor most easily analyzed consists of two parallel conductive plates separated by an insulating material. When a potential difference is applied (by a battery, for instance), the insulating material acts as a barrier to electrons and prevents them from jumping across the plates.

Instead, negative charges, attracted to the positive terminal of the battery, leave one plate with a net positive charge while the other plate is left with a net negative charge due to electrons forced onto the plate by the battery's negative terminal. As more and more electrons "crowd" on the negative plate, it gets more difficult to force them onto the plate. Once the potential difference between the two conductive plates equals the voltage source, electrons no longer flow. At this point, the capacitor is said to be "fully charged."



A 1.5 V battery acts as a source in a circuit with a capacitor.

The potential difference across the conducting wires causes charges to flow until the same potential difference exists across the two parallel plates of the capacitor.

A capacitor is used to separate and store charge that can be discharged through a circuit where it does useful work as electric current. A capacitor's ability to store charge is known as *capacitance* and is dependent on the physical properties of the capacitor. In this investigation you will construct several parallel-plate capacitors using aluminum foil as the plates to investigate how certain physical properties of the capacitor affect its capacitance.

Safety

Follow these important safety precautions in addition to your regular classroom procedures:

- Follow the instructions in the user manual to correctly operate the digital multimeter (DMM).
- Use caution when working with sharp objects such as scissors, hobby knives, and serrated cutting edges.

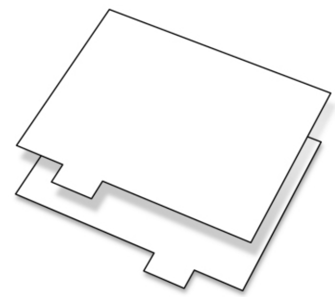
Procedure

Part 1 – Varying the Area of Parallel Conducting Plates

SET UP

1. Cut two rectangular sheets of aluminum foil each with dimensions slightly less than the size of a standard sheet of paper. It is important that each piece of foil be the same dimensions.

2. Cut a small tab into one side of each piece of foil. The tab will be used to make an electrical connection. The tabs should be offset from each other so they do not touch.


Cut sheets with offset tabs to prevent the tabs from touching when stacked.

3. Insert one piece of paper between the two foil sheets. You now have a capacitor with parallel conductive plates separated by an insulator.

   NOTE: Make sure the two foil sheets have no contact with each other.

4. Place one textbook on top of the capacitor to press the layers together so they are flat. Do not cover the tabs with the textbook.

COLLECT DATA

5. Measure the capacitance of the foil capacitor.

   a. Adjust the digital multimeter (DMM) to read capacitance in nanofarads (nF).

   b. Connect the two leads to the appropriate ports of the DMM.

   c. Connect one lead to one of the tabs on the foil sheets and connect the other lead to the other tab.

   d. Record the capacitance of the foil capacitor in Table 1 in the Data Analysis section below.

6. Remove the textbook and use the ruler to measure the dimensions of the foil sheets (excluding the tabs). Record the dimensions in Table 1.

7. Carefully fold each sheet of foil so the area of each sheet decreases by the same factor. For example, make each sheet half of its original area. Be sure not to fold over the foil tabs.

8. Cut the paper to be slightly larger than the capacitor plates. Insert the paper between the plates and use the same textbook to press the layers together.

   NOTE: Do not cover the tabs with the textbook.

9. Measure the capacitance and the dimensions of the foil capacitor. Record all values in Table 1.

10. Repeat the data collection steps three more times, decreasing the area of the capacitor's plates in each trial. Record the capacitance and dimensions for each trial in Table 1.

Part 2 – Varying the Plate Spacing Between Parallel Conducting Plates

SET UP

11. Cut two new rectangular sheets of aluminum foil each with dimensions slightly less than the size of a standard sheet of paper. The sheets should have the same dimensions and they should have offset tabs.

12. Insert one sheet of paper between the two foil sheets, and then place the textbook on top of the capacitor to press the layers together so they are flat. Do not cover the tabs with the textbook.

COLLECT DATA

13. Measure the capacitance of the aluminum foil capacitor with one sheet of paper between the plates. Record the result in Table 2 in the Data Analysis section below.

14. Increase the spacing between the parallel plates by adding a second sheet of paper between them.

15. Measure the capacitance of the foil capacitor and record the results in Table 2.

16. Repeat the data collection steps three more times, increasing the spacing between the parallel plates by adding one sheet of paper in each trial. Record each capacitance value in Table 2.

Data Analysis

Part 1 – Varying the Area of Parallel Conducting Plates

Table 1: Capacitance of a parallel-plate capacitor with varying plate area

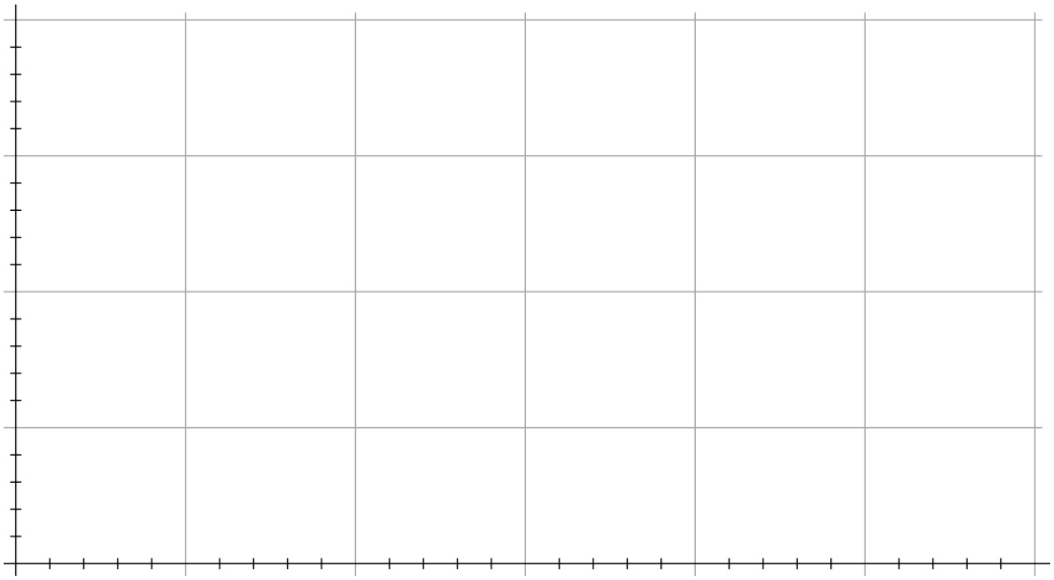
Length (c*m*)	Width (c*m*)	Area (cm^2)	Capacitance (nF)

1. Calculate the area of the capacitor plates used in each trial. Record the result for each trial in Table 1.

2. Qualitatively, what happens to the capacitor's ability to store charge (capacitance) as the area of each plate decreases?

3. Plot a graph of *capacitance* versus *area* in the blank Graph 1 axes. Be sure to label both axes with the correct scale and units.

Graph 1: Capacitance versus area for parallel-plate capacitor



4. How are the variables in Graph 1 related to each other mathematically? Use terms such as proportional, inversely proportional, linear, or quadratic in your response.

Part 2 – Varying the Distance between Parallel Conducting Plates

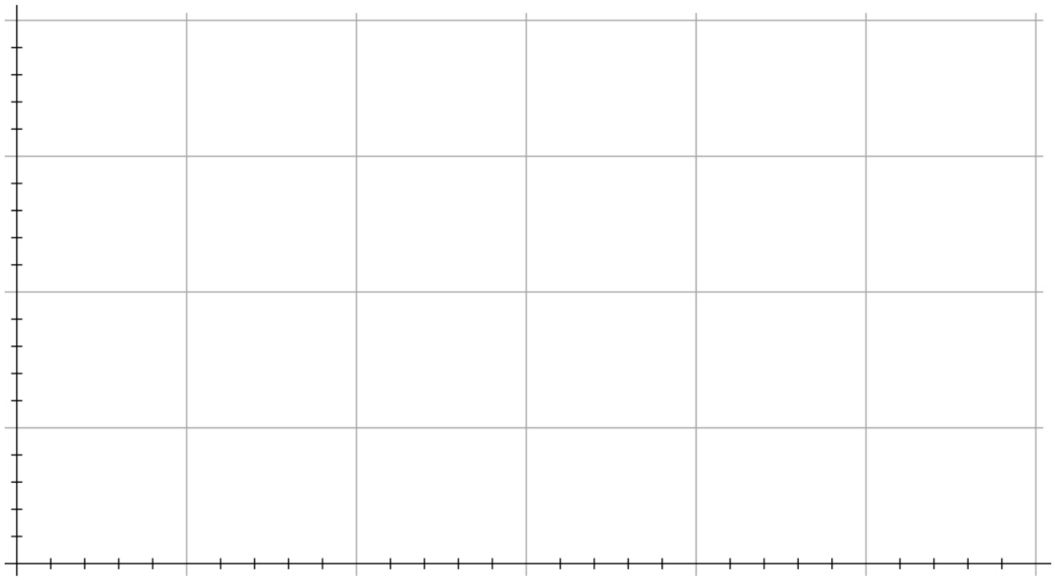
Table 2: Capacitance of a parallel-plate capacitor with varying plate spacing

Plate Spacing (sheets of paper)	Capacitance (nF)
1	
2	
3	
4	
5	

5. Qualitatively, what happens to the capacitor's ability to store charge (capacitance) as the spacing between the plates increases?

6. Plot a graph of *capacitance* versus *plate spacing* in the blank Graph 2 axes. For plate spacing, use the number of sheets of paper as the unit. Be sure to label both axes with the correct scale and units.

Graph 2: Capacitance versus plate spacing for parallel-plate capacitor



7. How are the variables in Graph 2 related to each other mathematically? Use terms such as proportional, inversely proportional, linear, or quadratic in your response.

Analysis Questions

1. Consider the materials you used to make your parallel-plate capacitor. Classify the materials as conductors or insulators and discuss the relative mobility of the electrons in each type of material.

2. What are the physical properties of a capacitor? In this experiment, which of these physical properties did you vary?

❓ 3. For each of the properties you varied, what was the effect on the capacitance you measured?

❓ 4. Why do you think the change in each of these physical properties changed the capacitance the way it did?

❓ 5. The capacitance C for a parallel-plate capacitor is given by the equation:

$$C = \kappa\varepsilon_0 \frac{A}{d} \quad (1)$$

where A is the area of each plate, d is the plate spacing, and κ and ε_0 are constants. How does your data support this definition of capacitance?

❓ 6. Consider the configurations you tested. Overall, what combination of conditions would yield the largest value for capacitance? What conditions would yield the lowest value?

Synthesis Questions

❓ 1. If you wanted to build a capacitor with twice the maximum capacitance you measured in this experiment, how would you design your new capacitor?

❓ 2. The SI unit of capacitance is a derived unit known as the farad (named after Michael Faraday). One farad is defined as one coulomb per volt: 1 F = 1 C/V. Based on the definition of this unit, write an equation that relates capacitance, charge, and potential difference and then solve this equation for charge.

❷ 3. An engineer needs a 1.0 μF parallel-plate capacitor with a spacing of 0.050 mm between its plates. She will use the capacitor in the design of a portable electronic device.

a. What would the area of the plates need to be to make this capacitor? Assume that $\kappa = 1$ and $\varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/(N \cdot m^2)}$.

NOTE: 1 V = 1 J/C

b. Discuss the practicality of this capacitor in the context of its intended use.

c. If the portable electronic device is designed to use a 1.5 V battery, what is the maximum charge the capacitor can store?

14. SERIES AND PARALLEL CAPACITORS

Connections to the AP® Physics 2 Curriculum*

The lab activity correlates to the following pieces of the AP® Physics 2 framework:

Big Idea	4	Enduring Understanding	E	Essential Knowledge	5

Learning Objective 3: The student is able to plan data collection strategies and perform data analysis to examine the values of currents and potential differences in an electric circuit that is modified by changing or rearranging circuit elements, including sources of emf, resistors, and capacitors.
Science Practices: 2.2, 4.2, 5.1

Big Idea	5	Enduring Understanding	B	Essential Knowledge	9

Learning Objective 5: The student is able to use conservation of energy principles (Kirchhoff's loop rule) to describe and make predictions regarding electrical potential difference, charge, and current in steady-state circuits composed of various combinations of resistors and capacitors.
Science Practices: 6.4

Time Requirement

Preparation Time: 10 minutes

Lab Activity: 45 minutes

Prerequisites

Students should be familiar with the following concepts:

- Series circuits are constructed by connecting components end-to-end, forming one path for current to flow.
- Parallel circuits are constructed by connecting each component to two common points, allowing multiple paths for current to flow.
- Capacitors are constructed of two parallel conductive plates separated by an insulator.
- Capacitors store charge when a potential difference is placed across the capacitor plates.

Driving Question | Objective

How does connecting capacitors in series or parallel affect the equivalent capacitance of a circuit? Experimentally determine the mathematical relationship between equivalent capacitance and capacitors connected in series and parallel.

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Procedural Overview

The Structured version of this lab activity is divided into two parts. In Part 1, the students build parallel circuits using only capacitors with the same capacitance and measure the equivalent capacitance of the circuit with a digital multimeter. The students start by measuring the capacitance of a single capacitor and then measure the capacitance again after adding another capacitor to the circuit. This procedure is repeated until the circuit comprises five capacitors connected in parallel. Part 2 follows the same procedure as Part 1, except that students wire the capacitors in series.

After collecting data, the students graph equivalent capacitance versus the number of capacitors to determine the mathematical relationship for capacitors in parallel and capacitors in series.

Pre-Lab Discussion and Activity

Give each lab group three capacitors with the same capacitance and ask them to connect the capacitors in as many arrangements as they can imagine. Draw each student arrangement on the board and ask the students to rank the equivalent capacitance of the circuits from lowest to highest. Discuss why they believe these arrangements would cause the circuit to have a low or high equivalent capacitance.

Materials and Equipment

- PASCO AC/DC Electronics Laboratory[1]
- Wire lead[1] (6)
- Digital capacitance meter/multimeter, 1-µF resolution
- Capacitor (5), 100-µF
- 4-mm banana plug patch cord (2)
- 4-mm banana plug patch cord alligator clip (2)

Probeware Resources

Below are web-link and QR codes that will direct you to instructional video resources for individual pieces of PASCO probeware, sensors, and other hardware used in the lab activity. These same links and codes are provided to students in their activity handouts.

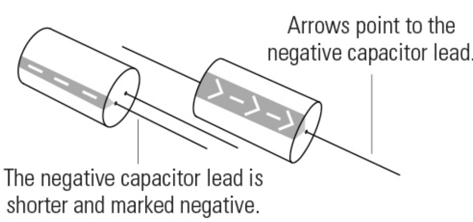
[1]www.pasco.com/ap04

PASCO AC/DC
Electronics Laboratory

Safety

Follow these important safety precautions in addition to your regular classroom procedures:

- If you are using polarized capacitors, always connect the common or negative lead of the DMM to the negative lead of the capacitor, as shown.



Teacher Tips

Tip 1 – Choosing a Digital Multimeter

- Any standard multimeter that can measure capacitance will work for this experiment. You could also use a meter dedicated to measuring capacitance. Inexpensive meters will have varying accuracy and you may find that different meters measure a different value for the same capacitor. For this reason, it is important that lab groups use the same multimeter for the entire experiment.

Tip 2 – Capacitor Values

- Though 100-µF capacitors are used for this experiment, any capacitor value will work as long as each capacitor is the same value. It may be beneficial to give each student group a different capacitor value so that each group has unique data. If possible, avoid using polarized capacitors to eliminate the possibility of students wiring the capacitors incorrectly and damaging them.

Tip 3 – Verifying Capacitance Values

- Since capacitance values labeled on capacitors typically have a tolerance of ±20%, it may be beneficial to have the students measure the capacitance of each capacitor with a digital meter to demonstrate random error in their experiment. This will also help explain to the students why the slopes of their graphs do not exactly match the capacitance written on the capacitors.

Sample Data

Below are sample data, acquired using the experimental setup and procedure outlined in the Structured version of the lab activity, and answers to questions in the Data Analysis section.

Data Analysis

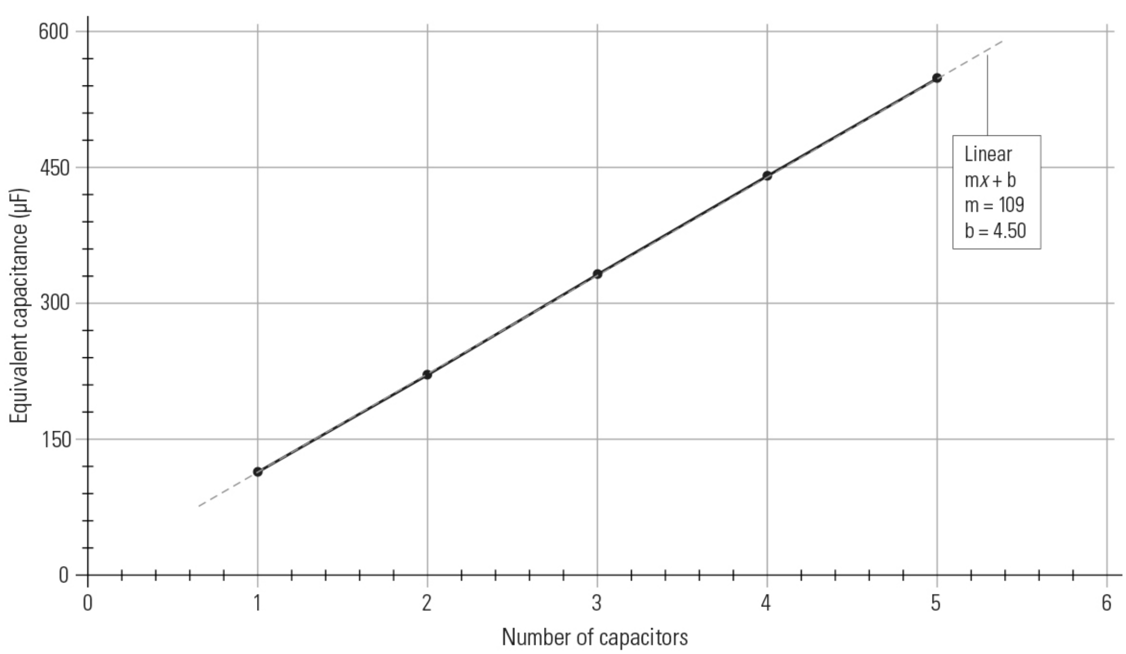
Part 1 – Capacitors in Parallel

Table 1: Equivalent capacitance for capacitors connected in parallel

Number of Capacitors	Equivalent Capacitance (µF)
1	113
2	221
3	332
4	440
5	547

1. Plot a graph of *equivalent capacitance* versus *number of capacitors* in the blank Graph 1 axes. Be sure to label both axes with the correct scale and units.

Graph 1: Equivalent capacitance versus number of capacitors connected in parallel



2. Draw a line of best fit through your data in Graph 1. Determine and record the equation of the line here:

   Best fit line equation: Equivalent capacitance = (109 μF)(Number of capacitors) + 0.38 μF

3. What does the slope of your best fit line represent?

   The slope represents the capacitance value of a single capacitor.

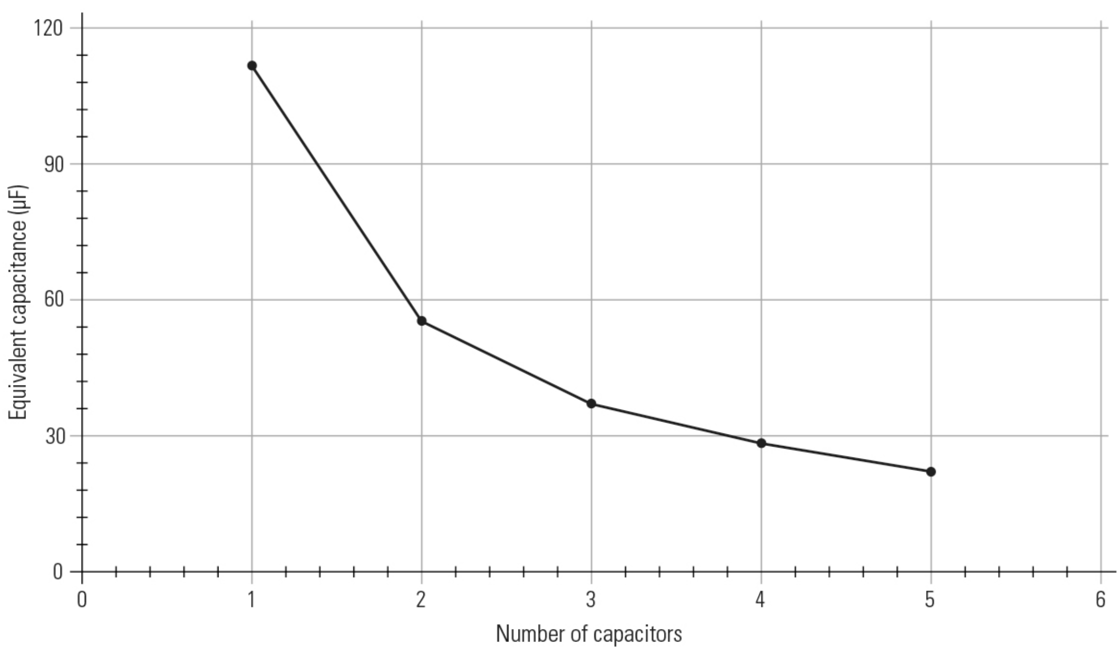
Part 2 – Capacitors in Series

Table 2: Equivalent capacitance for capacitors connected in series

Number of Capacitors	1/Number of Capacitors	Equivalent Capacitance (μF)
1	1.00	112
2	0.50	56
3	0.33	38
4	0.25	29
5	0.20	23

4. Plot a graph of *equivalent capacitance* versus *number of capacitors* in the blank Graph 2 axes. Be sure to label both axes with the correct scale and units.

Graph 2: Equivalent capacitance versus number of capacitors connected in series



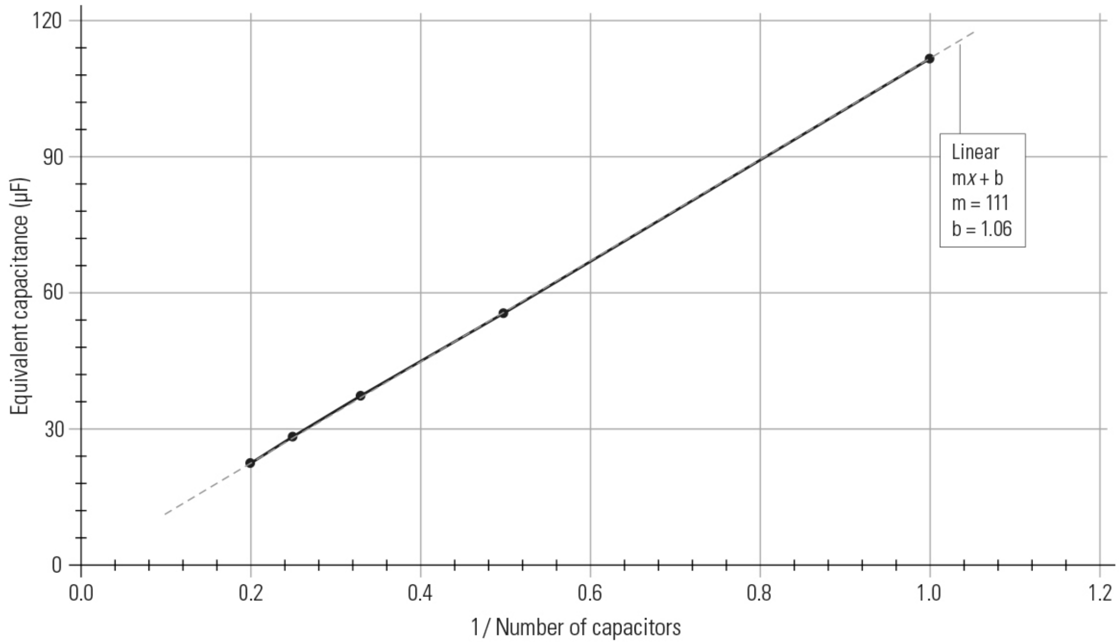
5. How does the equivalent capacitance for series capacitors change as more capacitors are added?

   The equivalent capacitance decreases as more capacitors are added.

6. Calculate the inverse of the number of capacitors for each trial in Part 2. Record the results in Table 2.

7. Plot a graph of *equivalent capacitance* versus 1/*number of capacitors* in Graph 3. Be sure to label both axes with the correct scale and units

Graph 3: Equivalent capacitance versus 1/number of capacitors connected in series



8. Draw a line of best fit through your data in Graph 3. Determine and record the equation of the line here:

Best fit line equation: Equivalent capacitance = (111 μF)(1/Number of capacitors) + 1.0 μF

9. What does the slope of the best fit line represent? How does this value compare to the slope of the best fit line for Graph 1? Explain the connection between these two values.

The slope represents the capacitance value of a single capacitor. When considering the error in measurement of the capacitance, the value of this slope can be considered essentially the same as the slope in Graph 1. Since both of the slopes represent the capacitance value of a single capacitor, the values are expected to be the same. If students are not convinced, have them measure the capacitance of each capacitor as suggested in Teacher Tip 3.

Guided Inquiry Questions

Below are sample responses to the Guiding Questions found in the Guided Inquiry version of this lab activity.

1. How would you connect the capacitors so that they are connected in series? In parallel?

To connect the capacitors in series, the capacitors need to be connected end-to-end. To connect the capacitors in parallel, each capacitor must connect to two common points. If polarized capacitors are being used, the direction of the positive to negative ends should always be the same.

2. How would you connect a meter to your circuit in order to measure the equivalent capacitance?

The meter needs to be connected in parallel to the capacitors. For capacitors in series, connect one lead of the meter to the unconnected end of the first capacitor and the other lead to the unconnected end of the last capacitor. For the capacitors in parallel, connect one lead from the meter to one side of one of the capacitors and the other lead to the other side of the same capacitor.

3. What should be the dependent and independent variables in your experiment based on the objective of the experiment?

The dependent variable is the equivalent capacitance. The independent variable is the number of capacitors in the circuit.

❷ 4. What are some sources of experimental error in this experiment?

The main source of random error is the capacitance of the capacitors. Though the capacitors are labeled to have the same capacitance, the capacitance values typically vary ±20%. Also, the meter used in the experiment has accuracy limitations, which can be found in the meter's manual.

Assessment Questions: Sample Responses

Sample responses to the Analysis and Synthesis questions found in each version of the lab activity:

Analysis Questions

❷ 1. How is the equivalent capacitance mathematically related to the number of capacitors connected in parallel? Use terms such as proportional, inversely proportional, linear, or quadratic in your response.

The equivalent capacitance is proportional to the number of capacitors connected in parallel.

❷ 2. What would be the approximate equivalent capacitance of your circuit if you connected ten 100-μF capacitors in parallel? Explain how you arrived at your answer.

The equivalent capacitance of ten 100-μF capacitors in parallel would be1000 μF. This value can be determined by multiplying the capacitance of a single capacitor by ten, the number of capacitors in the circuit. This value can also be determined by taking the sum of ten 100-μF capacitors.

❷ 3. The equivalent capacitance C_p for capacitors connected in parallel is given by the equation:

$$C_p = \sum_i C_i \tag{1}$$

where C_i is the capacitance of each capacitor added to the circuit. How does your data support this equation?

Student data should show that the equivalent capacitance can be determined by taking the sum of all of the capacitors in parallel.

❷ 4. How is the equivalent capacitance mathematically related to the number of capacitors connected in series? Use terms such as proportional, inversely proportional, linear, or quadratic in your response.

The equivalent capacitance is inversely proportional to the number of capacitors in series.

❷ 5. What would be the approximate equivalent capacitance of your circuit if you connected ten 100-μF capacitors in series? Explain how you arrived at your answer.

The equivalent capacitance of ten 100-μF capacitors in series would be 10 μF. This value can be determined by dividing the capacitance of a single capacitor by ten, the number of capacitors in the circuit.

❷ 6. The equivalent capacitance C_s for capacitors connected in series is given by the equation:

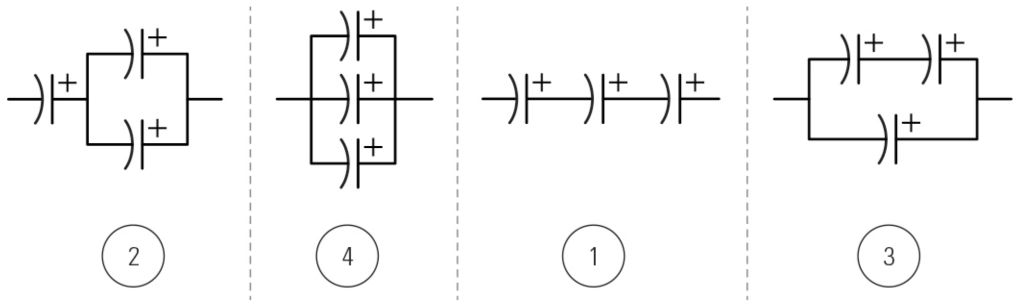
$$\frac{1}{C_s} = \sum_i \frac{1}{C_i} \tag{2}$$

where C_i is the capacitance of each capacitor added to the circuit. How does your data support this equation?

Student data should show that the reciprocal equivalent capacitance can be determined by taking the sum of the reciprocal of each capacitor in series.

Synthesis Questions

❷ 1. Each circuit configuration shown below contains three capacitors with the same capacitance. Rank the circuits from least to greatest equivalent capacitance by writing the numbers "1" to "4" (where "1" indicates the configuration with the least equivalent capacitance) in the circles below. Explain how you can answer this question *without* using Equations 1 or 2.



The circuit with three capacitors in series has the least equivalent capacitance due to the equivalent capacitance being inversely proportional to the number of capacitors in series. The circuit with one capacitor in series and two in parallel has the next lowest since it is equivalent to a circuit that has two capacitors in series. The three capacitors in parallel have the greatest equivalent capacitance due to equivalent capacitance being proportional to the number of capacitors of the same capacitance in parallel.

❷ 2. The AC/DC Electronics Laboratory includes capacitors with the values of 1.00 μF, 10.0 μF, 47.0 μF, 100 μF, 330 μF, and 470 μF. Use these capacitor values to answer the following questions.

a. Use Equation 1 to determine the equivalent capacitance of three circuits that use any three different combinations of capacitors in parallel.

Answers will vary. Below are some examples:

$$C_p = \sum_i C_i$$

$$C_p = 1.00\ \mu F + 10.0\ \mu F + 47.0\ \mu F = 58.0\ \mu F$$

$$C_p = 10.0\ \mu F + 47.0\ \mu F + 100\ \mu F = 157\ \mu F$$

$$C_p = 100\ \mu F + 330\ \mu F + 470\ \mu F = 900\ \mu F$$

b. Use Equation 2 to determine the equivalent capacitance of three circuits that use any three different combinations of capacitors in series.

Answers will vary. Below are some examples:

$$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$$

$$\frac{1}{C_s} = \frac{1}{1.00\ \mu F} + \frac{1}{10.0\ \mu F} + \frac{1}{47.0\ \mu F} = 1.12\ \mu F^{-1} \Rightarrow C_s = 0.891\ \mu F$$

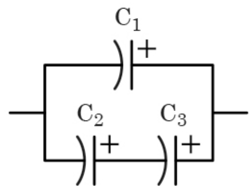
$$\frac{1}{C_s} = \frac{1}{10.0\ \mu F} + \frac{1}{47.0\ \mu F} + \frac{1}{100\ \mu F} = 0.131\ \mu F^{-1} \Rightarrow C_s = 7.62\ \mu F$$

$$\frac{1}{C_s} = \frac{1}{100\ \mu F} + \frac{1}{330\ \mu F} + \frac{1}{470\ \mu F} = 0.0152\ \mu F^{-1} \Rightarrow C_s = 66.0\ \mu F$$

c. Based on the circuits you analyzed in the previous step, what is always true about the equivalent capacitance of a series circuit compared to the capacitance of each individual capacitor in the circuit?

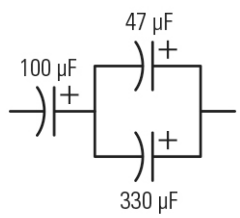
The equivalent capacitance is always less than the capacitance of any individual capacitor in the circuit.

d. The circuit shown below is made up of three different capacitors with an equivalent capacitance of approximately 92 µF. Determine the values of C_1, C_2, and C_3. Explain how you arrived at your answer.



C_1 = 10 µF, C_2 = 100 µF, C_3 = 470 µF (C_2 and C_3 can be switched). Capacitor C_1 must be less than 92 µF, which means C_2 and C_3 must be either100 µF, 330 µF, or 470 µF. A 100 µF and 470 µF capacitor in series has an equivalent capacitance of approximately 82 µF, which can be connected with a 10 µF capacitor in parallel for an equivalent capacitance of 92 µF.

3. Design a circuit with an equivalent capacitance of approximately 79 µF using only one 47-µF, one 100-µF, and one 330-µF capacitor. Draw and label your circuit. Explain how you arrived at your answer.



Four circuit arrangements are possible when using three capacitors (see Synthesis Question 1). For the given capacitors, the circuit arrangement shown above is the only arrangement possible to achieve an equivalent capacitance of 79 µF. The capacitor in series must be greater than 79 µF since the capacitance of each capacitor in series is greater than the equivalent capacitance of the circuit. This leaves us with two choices: 100 µF and 330 µF. The answer can be determined by testing and then eliminating one of those possible combinations.

Extended Inquiry Suggestions

- Have students repeat the data collection steps using five capacitors of different values. The slope of the line will be meaningless since each capacitor has a different value. However, students can still observe the inverse and proportional relationships discovered in the original experiment.
- Have the students measure the equivalent capacitance of the circuits discussed in the Pre-lab Discussion and Activity and compare the results with their original answers.

NAME PERIOD DATE

14. SERIES AND PARALLEL CAPACITORS

STRUCTURED

Driving Question | Objective

How does connecting capacitors in series or parallel affect the equivalent capacitance of a circuit? Experimentally determine the mathematical relationship between equivalent capacitance and capacitors connected in series and parallel.

Materials and Equipment

- PASCO AC/DC Electronics Laboratory[1]
- Wire lead[1] (6)
- Digital capacitance meter/multimeter, 1-µF resolution
- Capacitor (5), 100-µF
- 4-mm banana plug patch cord (2)
- 4-mm banana plug patch cord alligator clip (2)

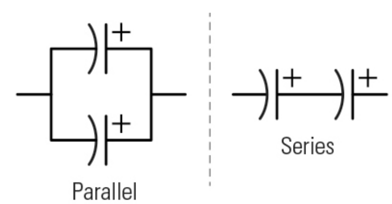
[1]www.pasco.com/ap04

PASCO AC/DC Electronics Laboratory

Background

Capacitance is a measure of the ability of a system to store charge at a given potential difference. Capacitors are devices designed to store charge. When multiple capacitors are electrically connected within a circuit, the overall capacitance—known as *equivalent capacitance*—within the circuit changes depending on how the capacitors are connected.

Capacitors with their positively charged plates connected together, and their negatively charged plates connected together, are said to be connected in *parallel*. Capacitors connected together in a straight line with the positively charged plate of one capacitor directly connected to the negatively charged plate of the next capacitor are said to be connected in *series*.

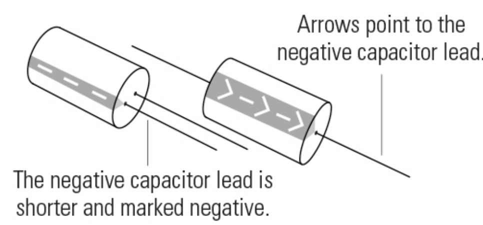


In this activity, you will investigate simple series and parallel circuits with multiple capacitors to experimentally determine a mathematical expression that describes the equivalent capacitance of each circuit type.

Safety

Follow these important safety precautions in addition to your regular classroom procedures:

- If you are using polarized capacitors, always connect the common or negative lead of the DMM to the negative lead of the capacitor, as shown.



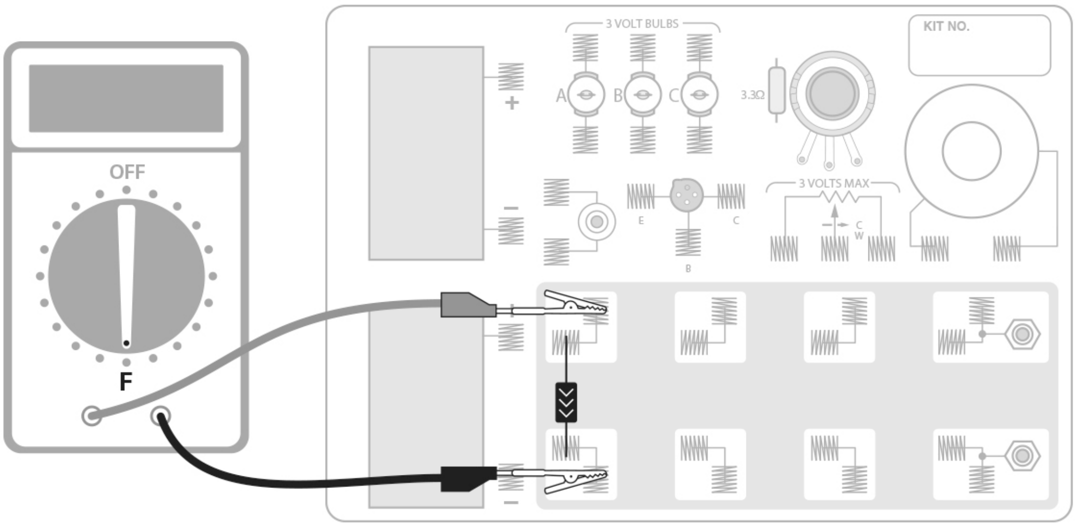
Procedure

Part 1 – Capacitors in Parallel

SET UP

1. Attach one capacitor between two springs on the AC/DC Electronics Laboratory board.

2. Adjust the dial on the digital multimeter (DMM) to measure capacitance in microfarads (μF), and then connect the leads from the DMM across the capacitor, as shown.

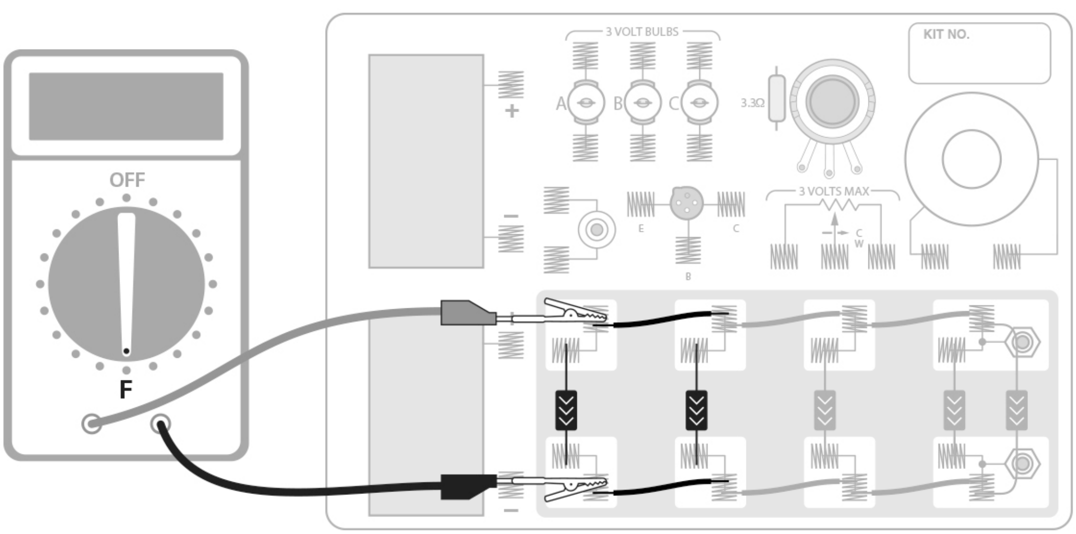


COLLECT DATA

3. Measure the capacitance of the capacitor and record the value in Table 1 in the Data Analysis section below.

4. Remove the DMM leads from the capacitor and connect a second capacitor in parallel with the first capacitor.

5. Connect the DMM across both capacitors.



6. Measure the equivalent capacitance of the two capacitors in parallel and record the value in Table 1.

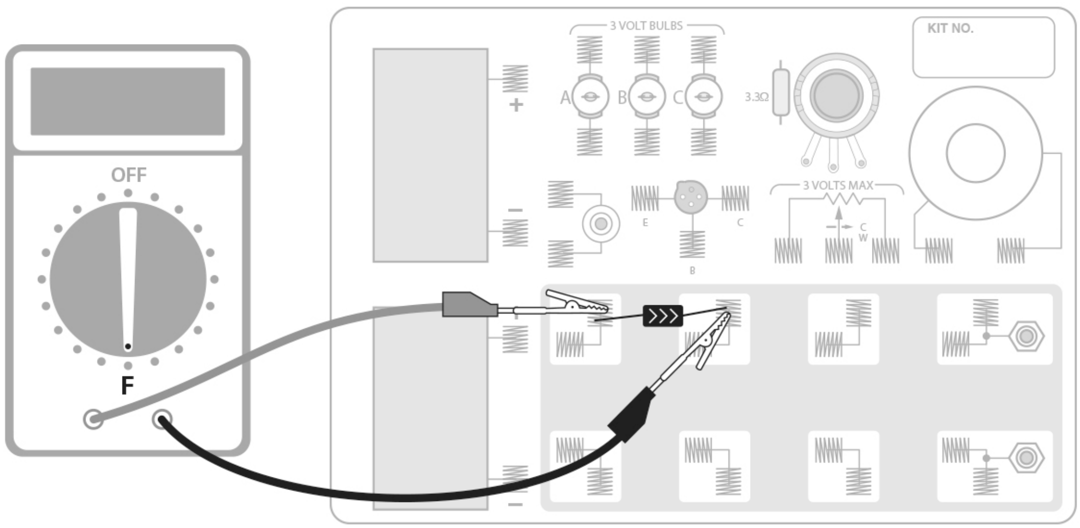
7. Repeat the data collection steps for three, four, and five capacitors in parallel. Record the number of capacitors and the equivalent capacitance for each trial in Table 1.

Part 2 – Capacitors in Series

SET UP

8. Remove all capacitors from the board and replace one as shown in the figure.

9. Connect the DMM leads across the capacitor.

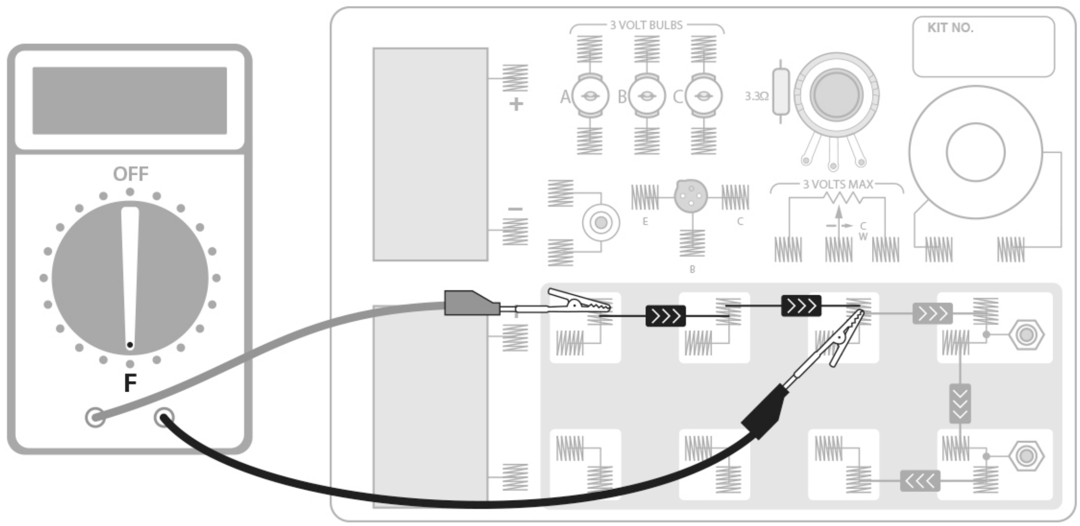


COLLECT DATA

10. Measure the capacitance of the capacitor and record the value in Table 2 in the Data Analysis section below.

11. Remove the DMM leads from the capacitor and connect a second capacitor in series with the first capacitor.

12. Connect the DMM leads across both capacitors.



13. Measure the equivalent capacitance of the two series capacitors and record the value in Table 2.

14. Repeat the data collection steps for three, four, and five capacitors in series. Record the number of capacitors and the equivalent capacitance for each trial in Table 2.

Data Analysis

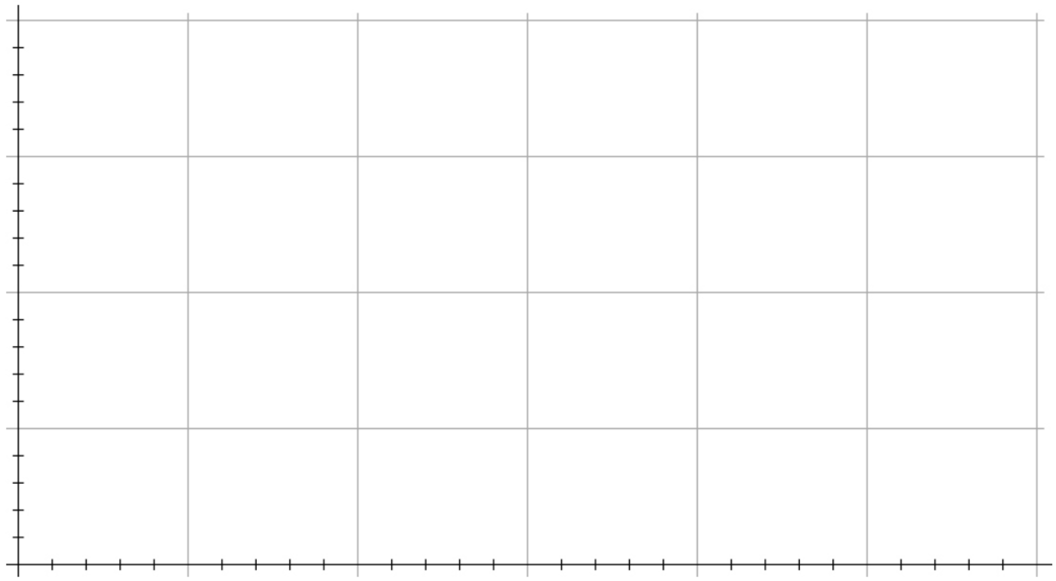
Part 1 – Capacitors in Parallel

Table 1: Equivalent capacitance for capacitors connected in parallel

Number of Capacitors	Equivalent Capacitance (μF)

1. Plot a graph of *equivalent capacitance* versus *number of capacitors* in the blank Graph 1 axes. Be sure to label both axes with the correct scale and units.

Graph 1: Equivalent capacitance versus number of capacitors connected in parallel



2. Draw a line of best fit through your data in Graph 1. Determine and record the equation of the line here:

   Best fit line equation: ____

3. What does the slope of your best fit line represent?

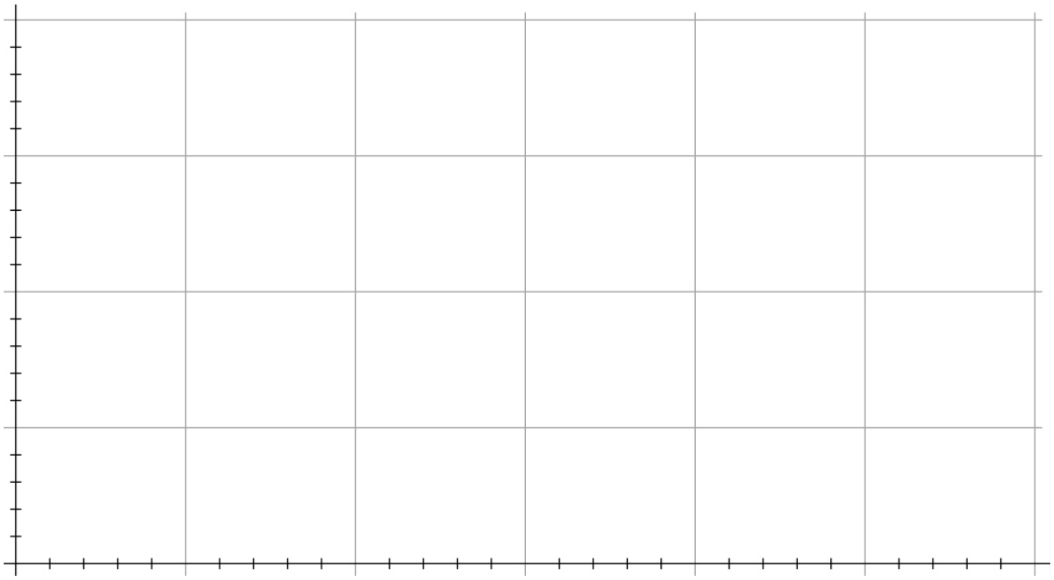
Part 2 – Capacitors in Series

Table 2: Equivalent capacitance for capacitors connected in series

Number of Capacitors	1/Number of Capacitors	Equivalent Capacitance (μF)

4. Plot a graph of *equivalent capacitance* versus *number of capacitors* in the blank Graph 2 axes. Be sure to label both axes with the correct scale and units.

Graph 2: Equivalent capacitance versus number of capacitors connected in series



5. How does the equivalent capacitance for series capacitors change as more capacitors are added?

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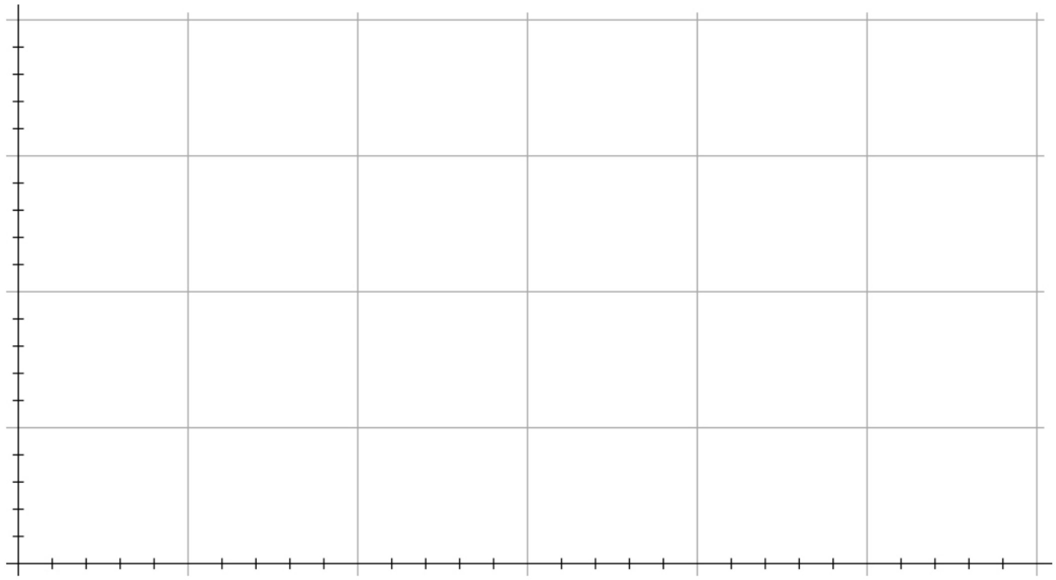
6. Calculate the inverse of the number of capacitors for each trial in Part 2. Record the results in Table 2.

7. Plot a graph of *equivalent capacitance* versus 1/*number of capacitors* in Graph 3. Be sure to label both axes with the correct scale and units

Graph 3: Equivalent capacitance versus 1/number of capacitors connected in series



8. Draw a line of best fit through your data in Graph 3. Determine and record the equation of the line here:

   Best fit line equation: ____________________________

9. What does the slope of the best fit line represent? How does this value compare to the slope of the best fit line for Graph 1? Explain the connection between these two values.

Analysis Questions

1. How is the equivalent capacitance mathematically related to the number of capacitors connected in parallel? Use terms such as proportional, inversely proportional, linear, or quadratic in your response.

2. What would be the approximate equivalent capacitance of your circuit if you connected ten 100-µF capacitors in parallel? Explain how you arrived at your answer.

3. The equivalent capacitance C_p for capacitors connected in parallel is given by the equation:

$$C_p = \sum_i C_i \tag{1}$$

where C_i is the capacitance of each capacitor added to the circuit. How does your data support this equation?

4. How is the equivalent capacitance mathematically related to the number of capacitors connected in series? Use terms such as proportional, inversely proportional, linear, or quadratic in your response.

5. What would be the approximate equivalent capacitance of your circuit if you connected ten 100-µF capacitors in series? Explain how you arrived at your answer.

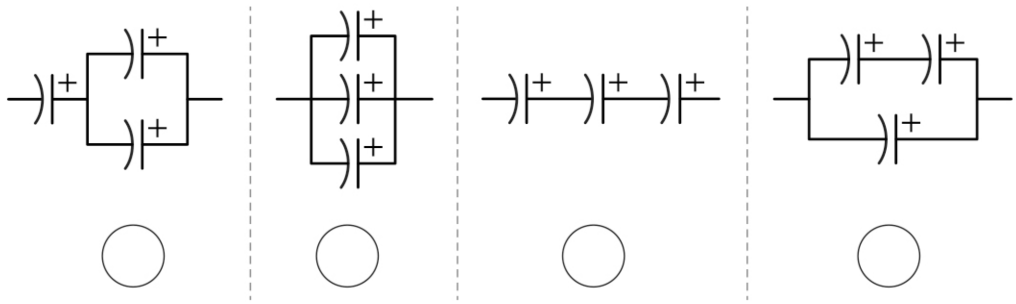
6. The equivalent capacitance C_s for capacitors connected in series is given by the equation:

$$\frac{1}{C_s} = \sum_i \frac{1}{C_i} \tag{2}$$

where C_i is the capacitance of each capacitor added to the circuit. How does your data support this equation?

Synthesis Questions

❓ 1. Each circuit configuration shown below contains three capacitors with the same capacitance. Rank the circuits from least to greatest equivalent capacitance by writing the numbers "1" to "4" (where "1" indicates the configuration with the least equivalent capacitance) in the circles below. Explain how you can answer this question *without* using Equations 1 or 2.



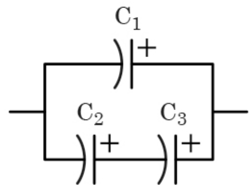
❓ 2. The AC/DC Electronics Laboratory includes capacitors with the values of 1 μF, 10 μF, 47 μF, 100 μF, 330 μF, and 470 μF. Use these capacitor values to answer the following questions.

a. Use equation 1 to determine the equivalent capacitance of three circuits that use any three different combinations of capacitors in parallel.

b. Use Equation 2 to determine the equivalent capacitance of three circuits that use any three different combinations of capacitors in series.

c. Based on the circuits you analyzed in the previous step, what is always true about the equivalent capacitance of a series circuit compared to the capacitance of each individual capacitor in the circuit?

d. The circuit shown below is made up of three different capacitors with an equivalent capacitance of approximately 92 μF. Determine the values of C_1, C_2, and C_3. Explain how you arrived at your answer.



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3. Design a circuit with an equivalent capacitance of approximately 79 μF using only one 47 μF, one 100 μF, and one 330 μF capacitor. Draw and label your circuit. Explain how you arrived at your answer.

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15. RC CIRCUITS

Connections to the AP® Physics 2 Curriculum*

The lab activity correlates to the following pieces of the AP® Physics 2 framework:

Big Idea	4	Enduring Understanding	E	Essential Knowledge	5

Learning Objective 1: The student is able to make and justify a quantitative prediction of the effect of a change in values or arrangements of one or two circuit elements on the currents and potential differences in a circuit containing a small number of sources of emf, resistors, capacitors, and switches in series and/or parallel.
Science Practices: 2.2, 6.4

Learning Objective 2: The student is able to make and justify a qualitative prediction of the effect of a change in values or arrangements of one or two circuit elements on currents and potential differences in a circuit containing a small number of sources of emf, resistors, capacitors, and switches in series and/or parallel.
Science Practices: 6.1, 6.4

Learning Objective 3: The student is able to plan data collection strategies and perform data analysis to examine the values of currents and potential differences in an electric circuit that is modified by changing or rearranging circuit elements, including sources of emf, resistors, and capacitors.
Science Practices: 2.2, 4.2, 5.1

Time Requirement

Preparation Time: 10 minutes

Lab Activity: 60 minutes

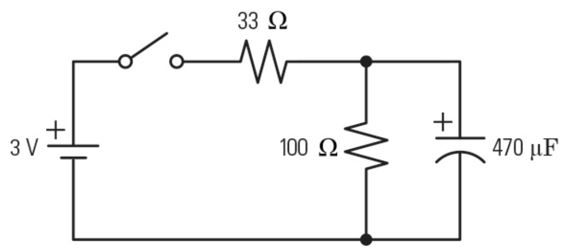
Prerequisites

Students should be familiar with the following concepts:

- Constructing circuits with components in series and parallel.
- Connecting a voltage sensor in parallel and a current sensor in series with the component being measured.
- Identifying resistor values using the color bands printed on the resistors.
- Resistors are used to control voltage and current, and capacitors are used to store electric charge.

Driving Question | Objective

How do the potential differences across the resistors and capacitor in a simple RC circuit differ when the capacitor is charging, discharging, and fully charged, and how do these differences affect the current through each component in the circuit? Build a circuit like the one shown below, and use it to answer this driving question and to understand the behavior of the potential differences and currents in an RC circuit.



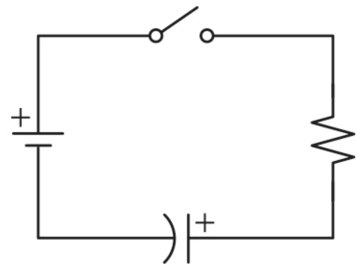
Procedural Overview

The Structured version of this lab activity is divided into three parts. In Part 1, the voltage across and the current through the capacitor are measured as the capacitor is charging, when it is at a steady state, and when it is discharging. Then students sketch graphs of voltage versus time and current versus time, labeling the regions where the capacitor was charging, at a steady state, and discharging. Values of voltage and current are recorded in a data table while the capacitor is at a steady state.

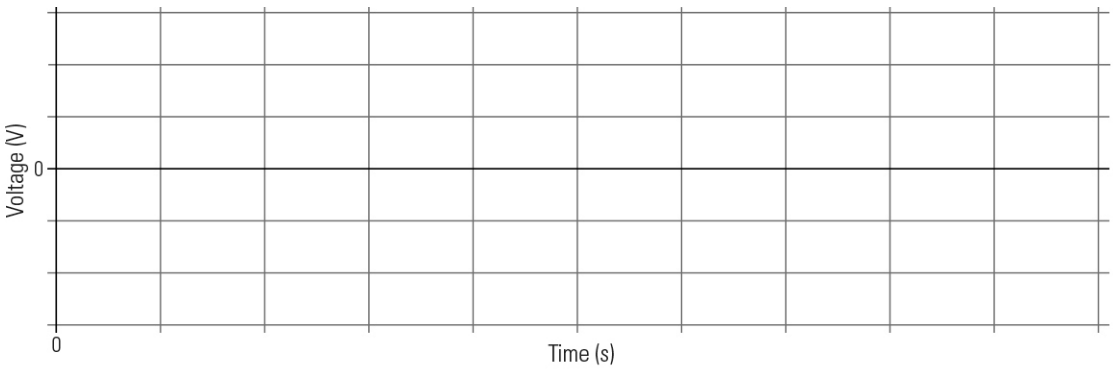
This procedure is repeated in Part 2 for the resistor in series (33 Ω) and in Part 3 for the resistor in parallel (100 Ω). The data is then analyzed, comparing how the voltage and current change across each component while the capacitor is charging and discharging and also comparing the steady state voltage and current values of each component.

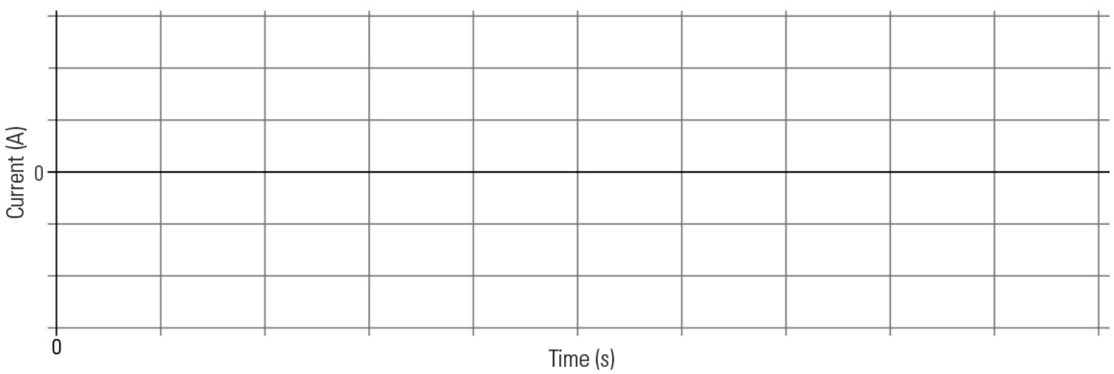
Pre-Lab Discussion and Activity

Build a circuit containing a voltage source (two batteries), a switch, a 100-Ω resistor, and a 470-µF capacitor in series.

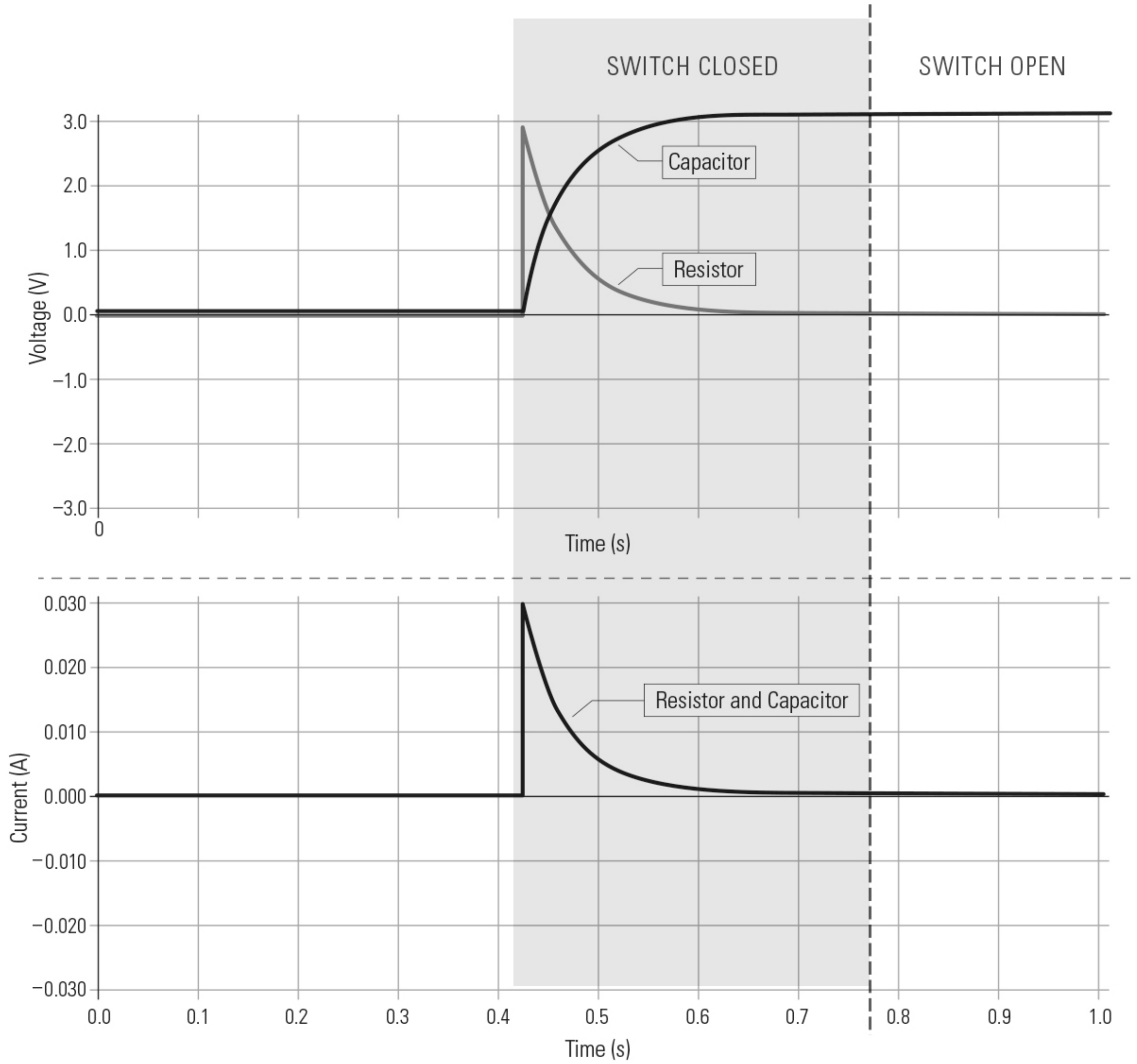


Have the students predict what will happen to the voltage across and current through the resistor and the capacitor when you close the switch and then open the switch. You may want to provide students with the following template to draw their predictions:





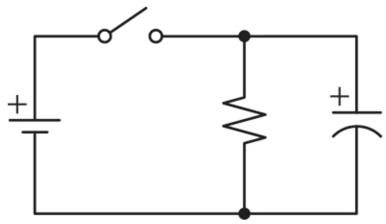
Once the students have drawn their predictions, use your data collection system to create two graph displays, one of voltage versus time and one of current versus time, and then measure the voltage across and the current through each component. To see voltage and current changes, use a sample rate of at least 1000 Hz. You should obtain graphs similar to those shown below:



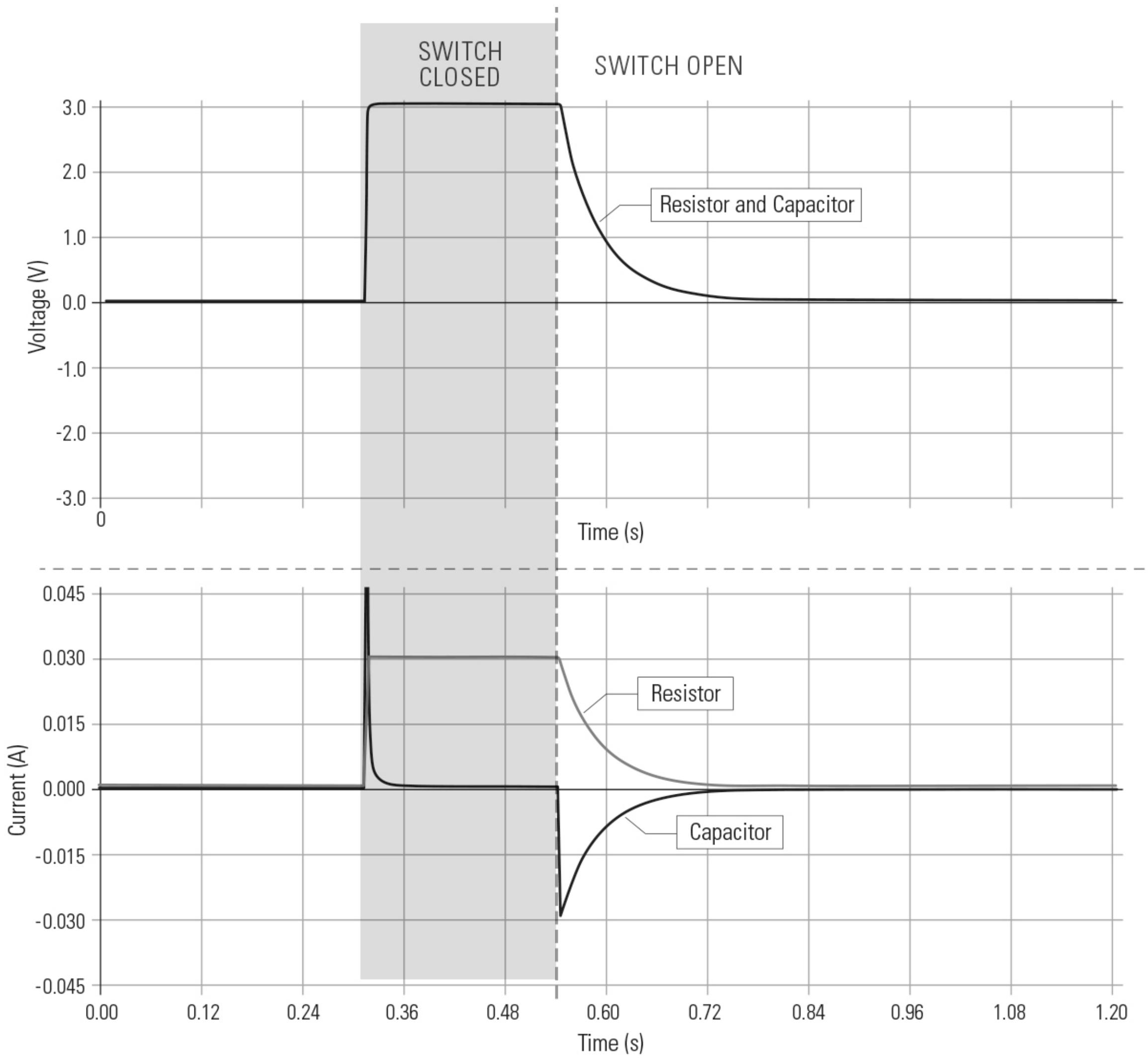
Important discussion points may include:

- As the voltage across the capacitor increases, the voltage across a resistor in series decreases since the voltage drop across each component is always equal to the voltage gain of the battery.
- The current through each component spikes when the switch is closed and slowly decreases to zero as the capacitor charges.
- After the voltage across the capacitor increases, it remains equal to the voltage of the battery, indicating that charge is stored even when the switch is opened again.

Repeat the demonstration with the resistor and capacitor in parallel as shown here:



You should obtain graphs similar to this:



Important discussion points may include:

- It takes the capacitor longer to discharge than charge since current must flow through the resistor when the capacitor is discharging.
- The current is negative through the capacitor when the capacitor is discharging since the current reverses direction as it discharges.
- Current flows through the resistor but not the capacitor when the capacitor is fully charged.

Materials and Equipment

- Data collection system
- PASCO Voltage–Current Sensor[1]
- Banana plug patch cord[1] (2), 4-mm
- Banana plug patch cord alligator clip[1] (4), 4-mm
- PASCO AC/DC Electronics Laboratory[2]
- Resistor[2], 33-Ω
- Resistor[2], 100-Ω
- Capacitor[2], 470-µF
- Wire lead[2] (6)
- Battery (2), D-cell

Probeware Resources

Below are web-link and QR codes that will direct you to instructional video resources for individual pieces of PASCO probeware, sensors, and other hardware used in the lab activity. These same links and codes are provided to students in their activity handouts.

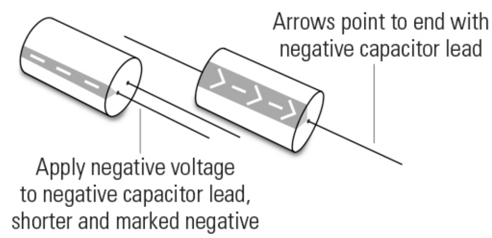
[1]www.pasco.com/ap19 — PASCO Voltage–Current Sensor

[2]www.pasco.com/ap04 — PASCO AC/DC Electronics Laboratory

Safety

Follow these important safety precautions in addition to your regular classroom procedures:

- Do not connect the terminals of a battery without a load; this will cause a short circuit.
- If you are using polarized capacitors, be sure to always apply negative voltage to the marked side of the capacitor as shown.



Teacher Tips

Tip 1 – Alternate Component Values

- The components in this lab were carefully chosen from the PASCO AC/DC Electronics Laboratory to draw a current high enough to be measureable by a current sensor and to create an RC time constant great enough to view the capacitor charging and discharging. If you wish to extend the charge and discharge time, it is recommended to use a larger capacitor, which can be obtained from any electronics supplier. To increase the current, you can use resistors of smaller values. Be mindful of the voltage ratings of the capacitors and the power ratings of the resistors when using alternate components.

Tip 2 – Sample Rate

- A high sample rate is required in order to see the charging and discharging of the capacitor. This may not be obvious to students using the Student Designed version of the experiment, so you may need to provide this as a guiding tip. Though the lab uses 1000 Hz, a sample rate of 500 Hz will work as well.

Tip 3 – Using Light Bulbs

- To give the student the opportunity to see a capacitor charge or discharge, you can use a light bulb in lieu of a resistor. However, this will not produce the characteristic RC curves as seen in textbooks, due to the resistance of the bulb changing as the voltage changes. This also requires a large capacitance capacitor, such as the SE-8626 1-Farad Capacitor sold by PASCO.

Sample Data

Below are sample data, acquired using the experimental setup and procedure outlined in the Structured version of the lab activity, and answers to questions in the Data Analysis section.

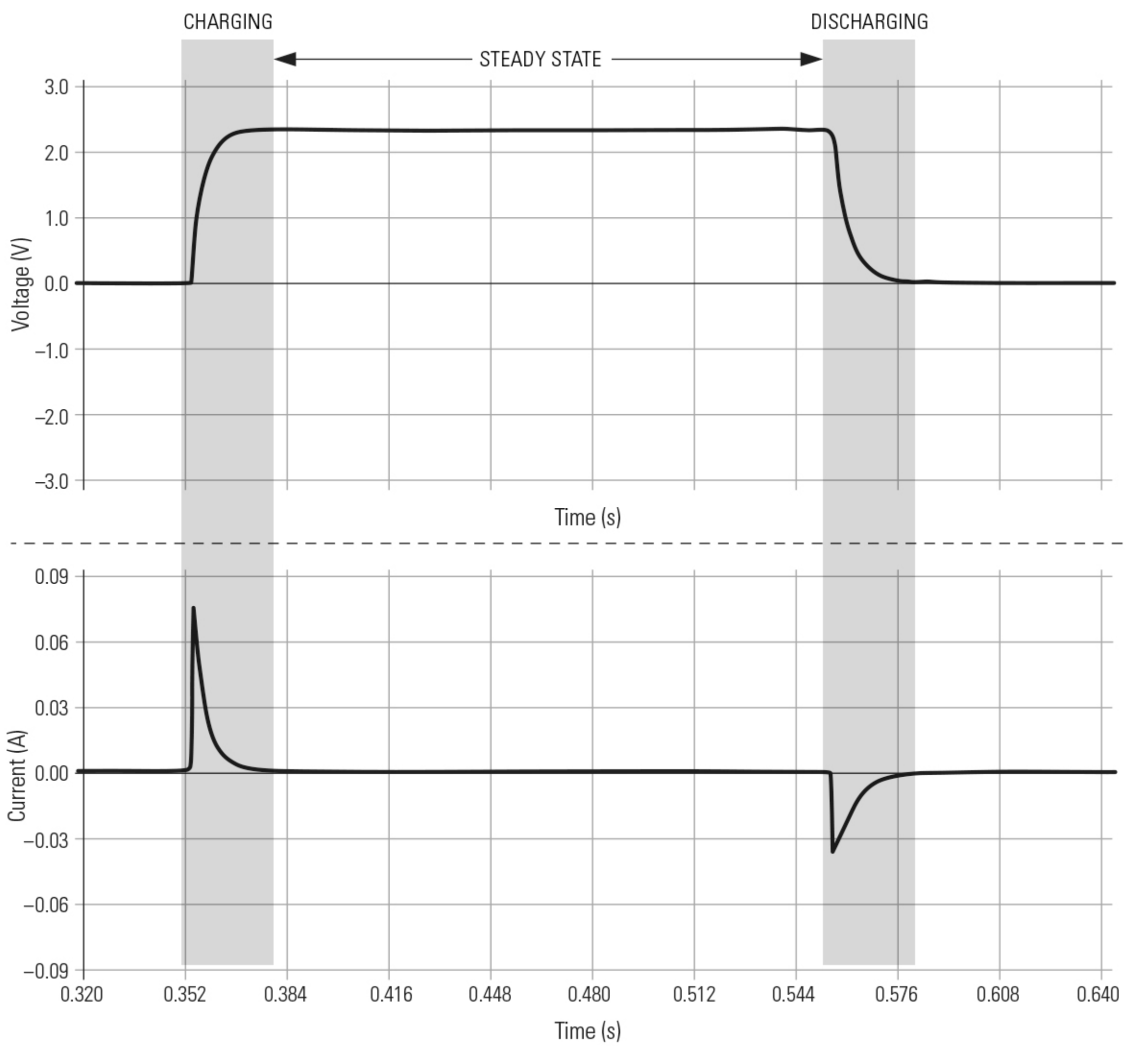
Data Analysis

Table 1: Potential difference and current in an RC circuit at a steady state

Component	Potential Difference (V)	Current (A)
Capacitor	2.31	0
Resistor in series	0.78	0.023
Resistor in parallel	2.29	0.023

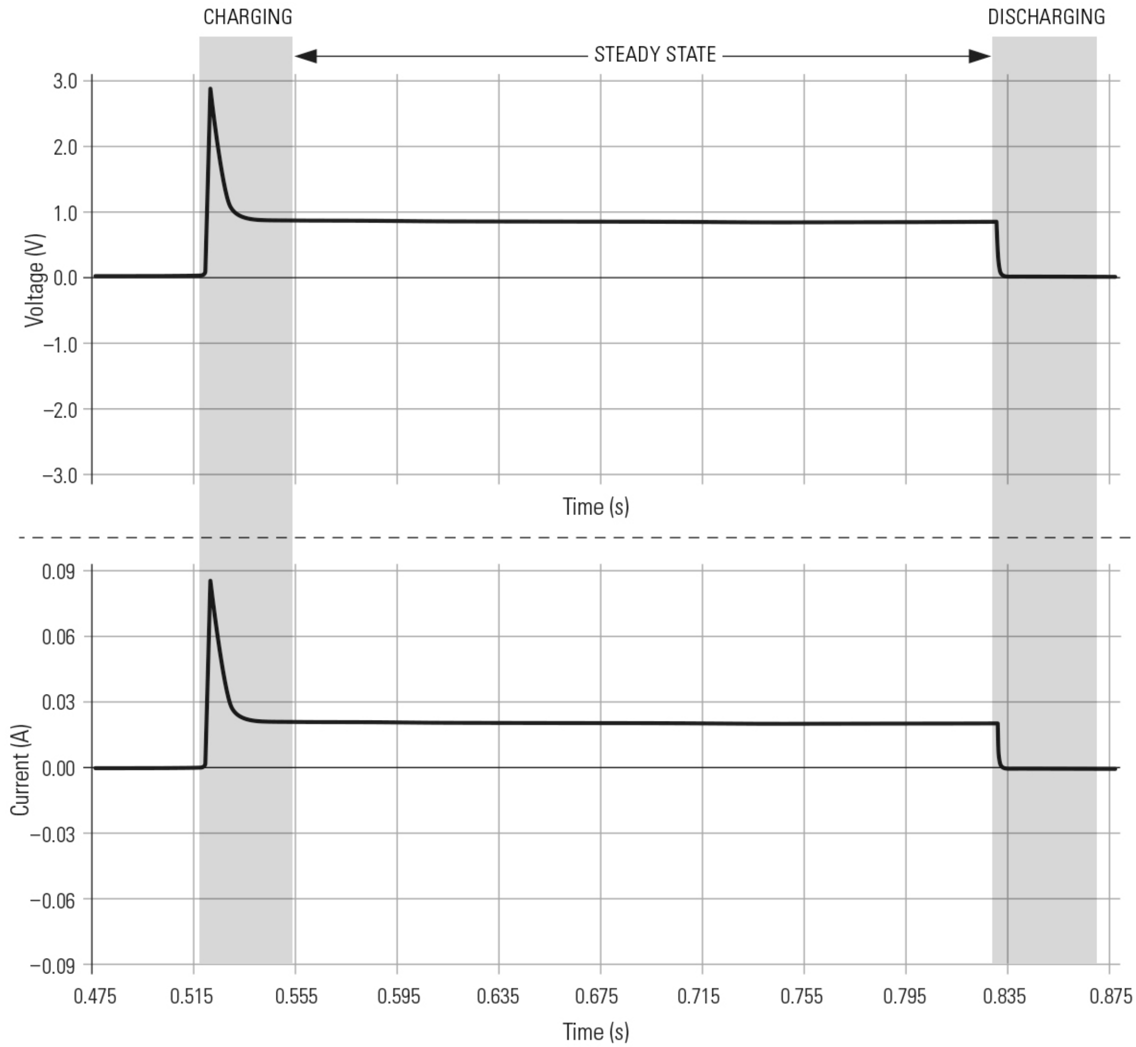
Part 1 – Capacitor (470-μF)

Graph 1: Voltage and current versus time for the 470-μF capacitor in an RC circuit during capacitor charge and discharge



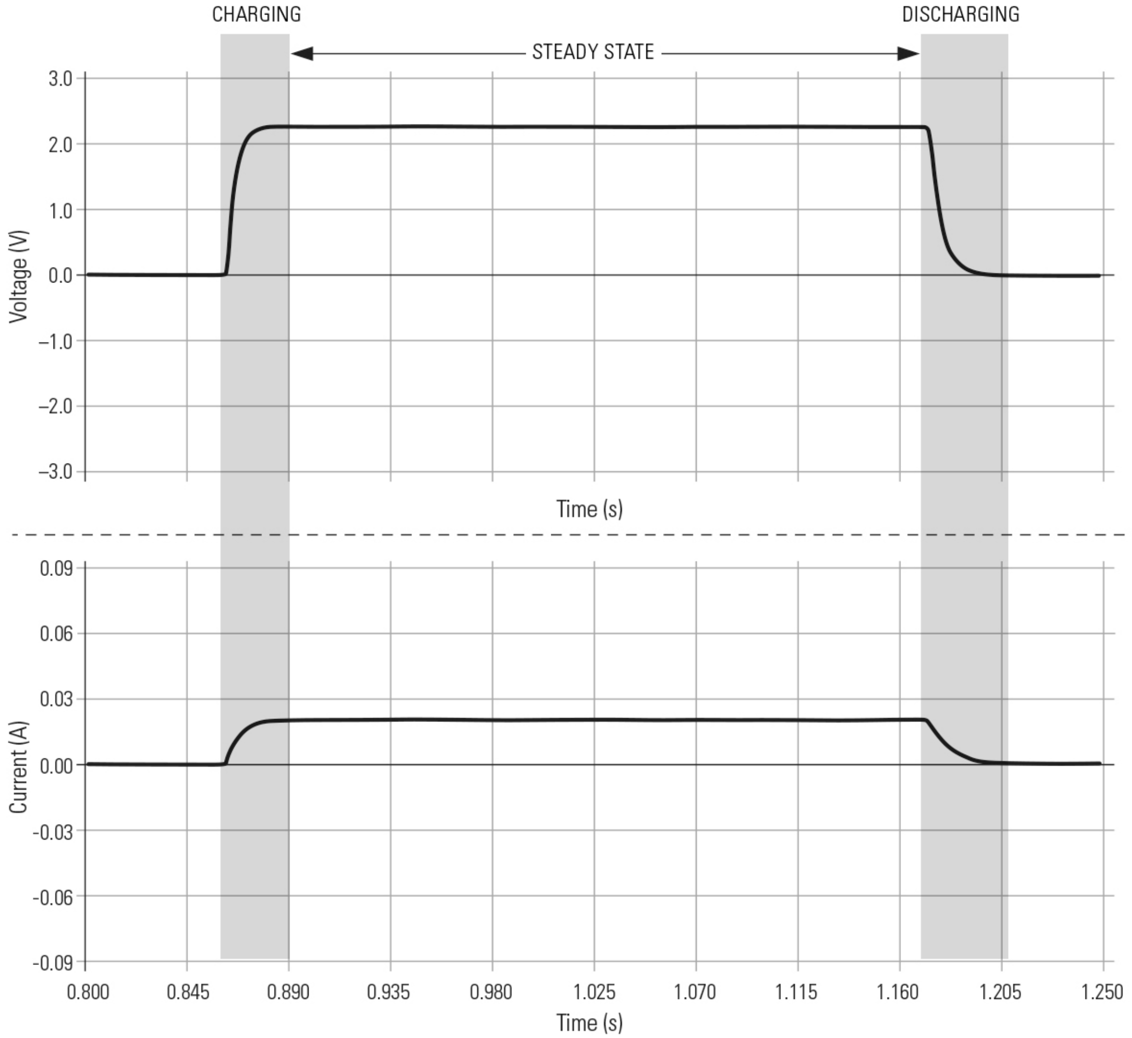
Part 2 – Series Resistor (33-Ω)

Graph 2: Voltage and current versus time for the 33-Ω series resistor in an RC circuit during capacitor charge and discharge



Part 3 – Parallel Resistor (100-Ω)

Graph 3: Voltage and current versus time for the 100-Ω parallel resistor in an RC circuit during capacitor charge and discharge



Guided Inquiry Questions

Below are sample responses to the Guiding Questions found in the Guided Inquiry version of this lab activity.

❷ 1. How should you connect the voltage sensor in order to measure the voltage across a component? How should you connect the current sensor in order to measure the current through a component?

The voltage sensor needs to be connected in parallel to the component. The current sensor needs to be connected in series with the component.

❷ 2. The voltage and current readings can change quickly in this experiment as the capacitor charges and discharges. Which setting could you adjust on your data collection system to accommodate for this?

The sample rate needs to be set to a high value, such as 1,000 Hz.

3. What would be the best way to record your data (graphs, tables, or single values) in order to observe changes in current and voltage when the capacitor is charging and discharging? What would be the best way to record your data when the capacitor is at a steady state?

   A graph display should be used when observing changes in data. A table display should be used when the capacitor is at a steady state.

Assessment Questions: Sample Responses

Sample responses to the Analysis and Synthesis questions found in each version of the lab activity:

Analysis Questions

1. When the capacitor in your circuit is charging, how is the potential difference across it changing? How is the potential difference across the series and parallel resistors changing during the same time?

   The potential difference across the capacitor increases. The potential difference across the series resistor decreases. The potential difference across the parallel resistor increases, similar to the capacitor.

2. When the capacitor in your circuit is charging, how is the current through the capacitor branch changing? How is the current through the series and parallel resistors changing during the same time?

   The current through the capacitor branch decreases toward zero. The current through the series resistor decreases. The current through the parallel resistor increases.

3. When the capacitor in your circuit is charging, why is the initial current through the resistor in series higher than the steady state current?

   Immediately after the switch is closed, current flows through the capacitor branch, as it has a lower resistance than the parallel resistor, resulting in a higher initial current.

4. When the capacitor is at a steady state, how does the potential difference across the capacitor compare to the potential difference across the resistor in series? In parallel? How do these values relate to the potential difference of the battery?

   The potential difference across the capacitor is greater than the potential difference across the resistor in series and equal to the potential difference across the resistor in parallel. The sum of the potential drops across the series resistor and the parallel resistor is equal to the potential gain of the battery.

5. When the capacitor is at a steady state, how does the current through the capacitor branch compare to the current through the resistors? Why is the current through each resistor the same?

   The current through the capacitor branch is equal to zero. The current through each resistor is the same since there is only one path for current to flow.

6. When the capacitor in your circuit is discharging, how is the potential difference across it changing? How is the potential difference across the series and parallel resistors changing during the same time? Why is the potential difference across the resistor in series zero?

   The potential difference across the capacitor is decreasing. The potential difference across the parallel resistor is also decreasing. The potential difference across the series resistor is zero due to the circuit being open.

7. When the capacitor in your circuit is discharging, how is the current through the capacitor branch changing? How is the current through the series and parallel resistors changing during the same time? Why is the current through the capacitor branch negative?

   The current through the capacitor branch is decreasing. The current through the series resistor is zero. The current through the parallel resistor is decreasing. The current through the capacitor branch is negative because the current is flowing in the opposite direction when discharging than it did when charging.

Synthesis Questions

❷ 1. Using the values and configuration in the circuit below, and those provided in Table 2, determine the voltage across and the current through each component after the switch has been closed for a long time. Write your results in Table 2 and explain how you arrived at your answers.

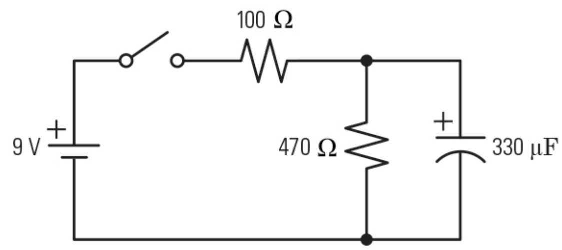


Table 2: Determine the remaining circuit component values

Component	Potential Difference (*V*)	Current (A)
330-μF Capacitor	7.42	0
100-Ω Resistor	1.58	0.0158
470-Ω Resistor	7.42	0.0158

No current flows through the capacitor, as it is fully charged. The current through the 470-ohm resistor is the same as the 100-ohm resistor since there is only one path for the current to travel. The voltage across the capacitor is the same as the voltage across the 470-ohm resistor since they are in parallel. The voltage across the 100-ohm resistor is determined by subtracting 7.42 V from 9 V.

❷ 2. For the circuit in the previous question, which values would change in Table 2 if

a. the 100-Ω and the 470-Ω resistors were switched? Explain.

Only the potential difference across the capacitor would change since it is now in parallel with a different resistor.

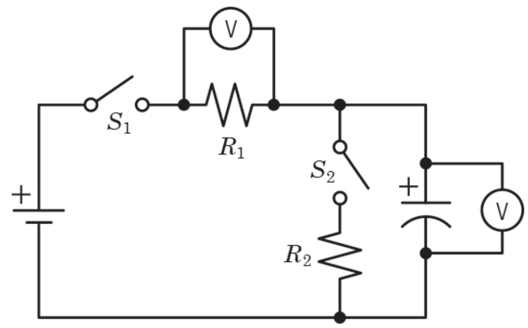
b. the capacitor was replaced with a 470-μF capacitor? Explain.

None of the values would change since the capacitance has no effect on the potential difference nor the current in the circuit.

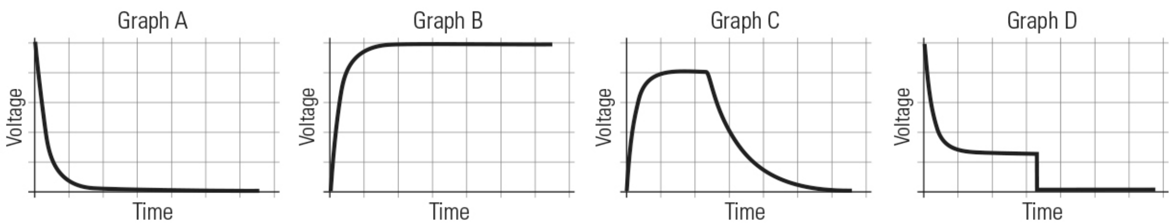
c. the 100-Ω resistor was replaced with a 1,000-Ω resistor? Explain.

The current through each resistor would change since the equivalent resistance of the circuit changed. The potential difference across each component would change since the 1000-Ω resistor would have a larger voltage drop than the 100-Ω resistor, which in turn causes the voltage drop to change across the other resistor.

❷ 3. The following circuit contains a battery, two resistors, and a capacitor. A switch is placed in series with each resistor and a voltmeter is placed in parallel across resistor R_1 and in parallel across the capacitor.



The following graphs represent four voltage measurements made by either voltmeter while switch S_1 was momentarily closed then reopened.



Match the description below with its corresponding graph. Explain how you determined each match.

a. Voltage measured across resistor R_1 while switch S_2 remains open.

Graph A. The voltage starts at a maximum value and decreases to zero as the capacitor charges.

b. Voltage measured across resistor R_1 while switch S_2 remains closed.

Graph D. Similar to the previous case, the voltage starts at a maximum value and decreases as the capacitor charges. However, in this case, the voltage does not decrease to zero since there is another resistor introduced in this circuit and current continues to flow through the resistors, but not through the capacitor branch. Once switch S_1 is reopened, the voltage will immediately drop to zero.

c. Voltage measured across the capacitor while switch S_2 remains open.

Graph B. The voltage starts at zero and increases as the capacitor charges.

d. Voltage measured across the capacitor while switch S_2 remains closed.

Graph C. Similar to the previous case, the voltage starts at zero and increases as the capacitor charges. However, in this case, instead of the voltage remaining constant, after switch S_1 is reopened, the voltage decreases to zero as the capacitor discharges through resistor R_2.

Extended Inquiry Suggestions

- Compare the energy stored in the capacitor to the energy dissipated by the parallel resistor. The energy stored in the capacitor can be calculated using the equation:

  $$U_{\text{C}} = \frac{1}{2}C\left(\Delta V\right)^2$$

  The energy dissipated by the resistor can be determined by measuring the area under the curve of a power versus time graph. The PASCO Voltage–Current sensor also measures power. If using different sensors, create a calculation in your data collection system to calculate power using the equation:

  $$P = I\Delta V$$

  Create a graph display of power versus time and use the area tool in your data collection system to determine the area under the curve.

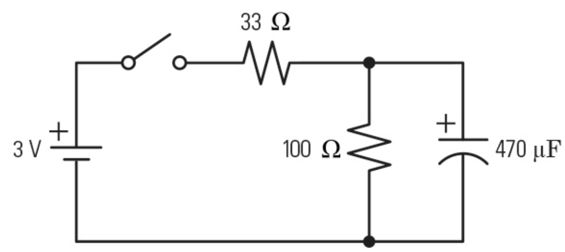
- Repeat the data measurements but with the 100-ohm resistor in series and the 33-ohm resistor in parallel and observe the changes in the steady state measurements.

15. RC CIRCUITS

STRUCTURED

Driving Question | Objective

How do the potential differences across the resistors and capacitor in a simple RC circuit differ when the capacitor is charging, discharging, and fully charged, and how do these differences affect the current through each component in the circuit? Build a circuit like the one shown below, and use it to answer this driving question and to understand the behavior of the potential differences and currents in an RC circuit.



Materials and Equipment

- Data collection system
- PASCO Voltage–Current Sensor[1]
- Banana plug patch cord[1] (2), 4-mm
- Banana plug patch cord alligator clip[1] (4), 4-mm
- PASCO AC/DC Electronics Laboratory[2]
- Resistor[2], 33-Ω
- Resistor[2], 100-Ω
- Capacitor[2], 470-µF
- Wire lead[2] (6)
- Battery (2), D-cell

[1]www.pasco.com/ap19

PASCO Voltage–Current Sensor

[2]www.pasco.com/ap04

PASCO AC/DC Electronics Laboratory

Background

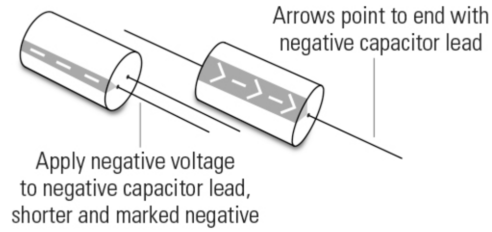
Resistors and capacitors are two basic components used to control the potential differences and currents in a circuit. When a voltage source is applied to a simple circuit containing a capacitor, current flows in the circuit and the capacitor begins to charge. The rate at which the capacitor charges and the time it takes the capacitor to fully charge, are dependent on the size of the resistors and capacitor in the circuit and how they are connected. As the capacitor charges, the potential differences and currents within the circuit change over time. When the capacitor is fully charged, the circuit is in a *steady-state* condition where the potential difference and current are no longer changing. When the voltage source is removed, the circuit once again experiences changes in potential difference and current as the capacitor discharges.

In this lab activity, you will wire a circuit containing two resistors, a capacitor, a battery, and a switch. The circuit positions one resistor in series with the capacitor and one in parallel with the capacitor. You will observe the changes in the potential difference and current for each component as the capacitor charges and discharges, and use your observations to answer questions about the magnitude and directions of the potential differences and currents in the circuit.

Safety

Follow these important safety precautions in addition to your regular classroom procedures:

- Do not connect the terminals of a battery without a load; this will cause a short circuit.
- If you are using polarized capacitors, be sure to always apply negative voltage to the marked side of the capacitor as shown.

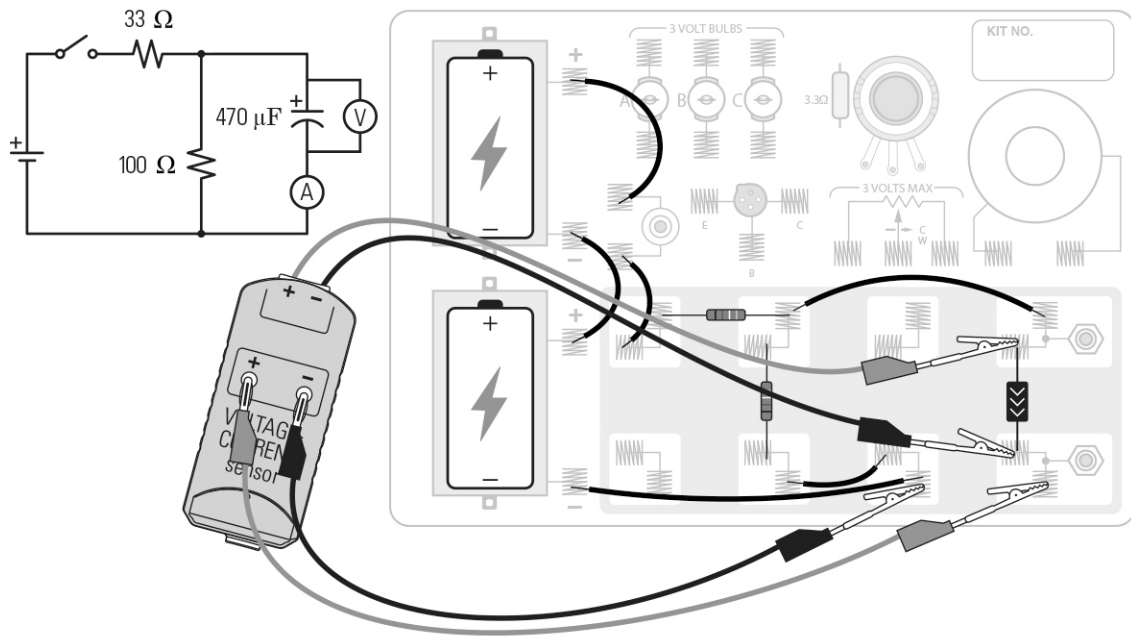


Procedure

Part 1 – Capacitor (470-µF)

SET UP

1. Assemble the circuit as shown using the two D-cell batteries. Connect one 33-Ω resistor in series with one 100-Ω resistor that is connected in parallel with one 470-µF capacitor.



2. Connect the voltage sensor leads in parallel across the 470-µF capacitor, and the current sensor leads in series with the capacitor.
3. Connect the voltage–current sensor to the data collection system and create two graph displays: 1) voltage versus time, and 2) current versus time.
4. Set the sample rate to 1,000 Hz.

COLLECT DATA

5. Begin recording data and then press and hold the push-button switch to close the circuit. Hold the switch down until the voltage and current measurements stabilize (capacitor has fully charged; steady-state), and then release the switch and allow the voltage and current to stabilize again (capacitor has fully discharged).
6. Once the voltage and current measurements stabilize, stop recording data.

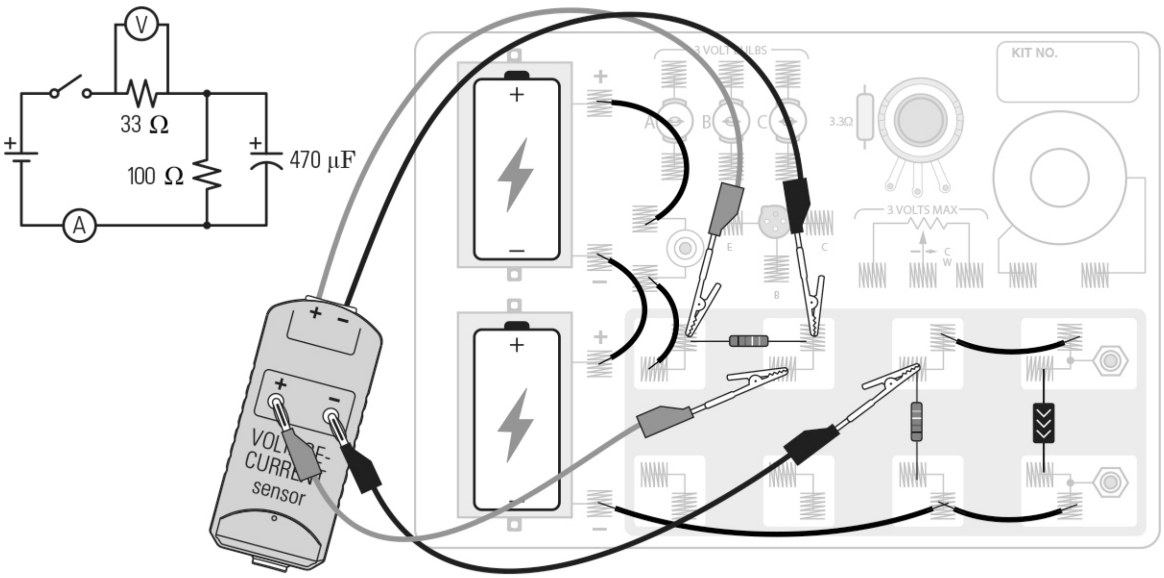
7. Sketch your voltage versus time and current versus time graphs in the blank Graph 1 axes in the Data Analysis section below. Label the time regions where the capacitor is charging, at a charged steady state, and discharging.

8. Use the tools on your data collection system to determine the potential difference across the 470-µF capacitor and the current in the capacitor branch when the circuit is at a steady-state. Record the values in Table 1 in the Data Analysis section.

Part 2 – Series Resistor (33-Ω)

SET UP

9. Revise the circuit components as shown below (the circuit does not change) and then connect the voltage sensor leads in parallel across the 33-Ω resistor, and the current sensor leads in series with the same resistor.



COLLECT DATA

10. Repeat the Part 1 data collection steps for the series 33-Ω resistor and then

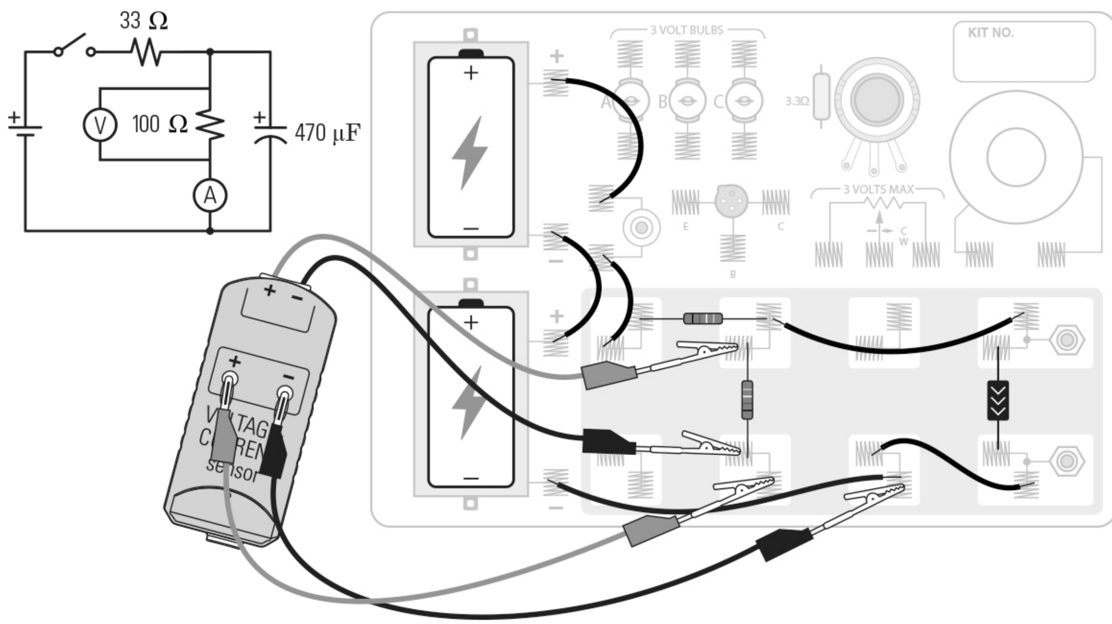
- Sketch your voltage versus time and current versus time graphs for Part 2 in the blank Graph 2 axes. Label the time regions where the capacitor is charging, at a charged steady state, and discharging.

- Record in Table 1 the potential difference across and current through the 33-Ω resistor when the circuit is at a steady state.

Part 3 – Parallel Resistor (100-Ω)

SET UP

11. Revise the circuit components as shown below (the circuit does not change) and then connect the voltage sensor leads in parallel across the 100-Ω resistor, and the current sensor leads in series with the same resistor.



COLLECT DATA

12. Repeat the Part 1 data collection steps for the parallel 100-Ω resistor and then

    - Sketch your voltage versus time and current versus time graphs for Part 3 in the blank Graph 3 axes. Label the time regions where the capacitor is charging, at a charged steady state, and discharging.

    - Record in Table 1 the potential difference across and current through the 100-Ω resistor when the circuit is at a steady-state.

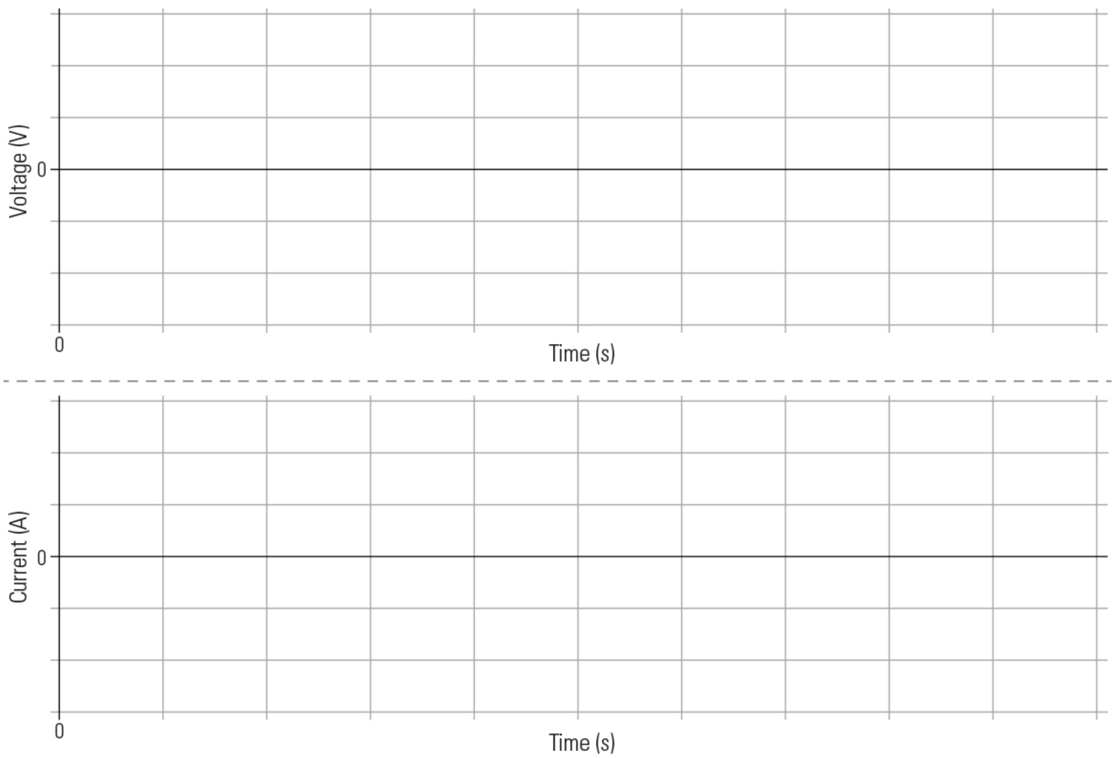
Data Analysis

Table 1: Potential difference and current in an RC circuit at a steady state

Component	Potential Difference (V)	Current (A)
Capacitor		
Resistor in Series		
Resistor in Parallel		

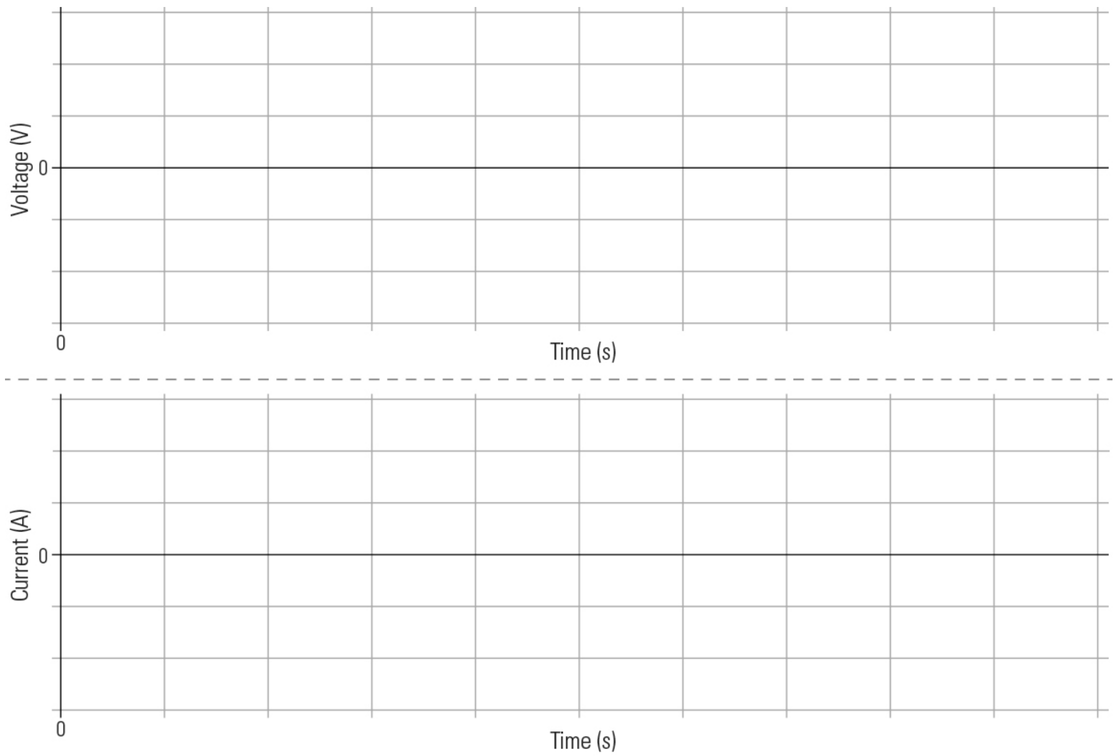
Part 1 – Capacitor (470-μF)

Graph 1: Voltage and current versus time for the 470-μF capacitor in an RC circuit during capacitor charge and discharge



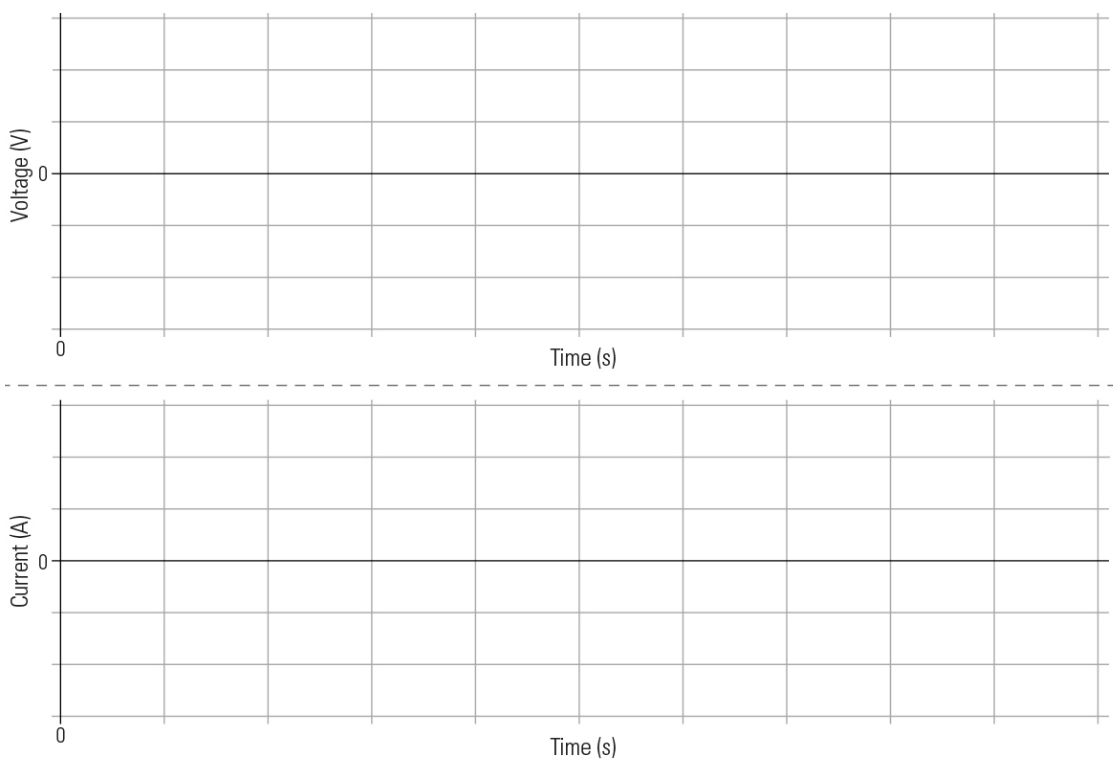
Part 2 – Series Resistor (33-Ω)

Graph 2: Voltage and current versus time for 33-Ω series resistor in an RC circuit during capacitor charge and discharge



Part 3 – Parallel Resistor (100-Ω)

Graph 3: Voltage and current versus time for 100-Ω parallel resistor in an RC circuit during capacitor charge and discharge



Analysis Questions

1. When the capacitor in your circuit is charging, how is the potential difference across it changing? How is the potential difference across the series and parallel resistors changing during the same time?

2. When the capacitor in your circuit is charging, how is the current through the capacitor branch changing? How is the current through the series and parallel resistors changing during the same time?

3. When the capacitor in your circuit is charging, why is the initial current through the resistor in series higher than the steady state current?

4. When the capacitor is at a steady state, how does the potential difference across the capacitor compare to the potential difference across the resistor in series? In parallel? How do these values relate to the potential difference of the battery?

5. When the capacitor is at a steady state, how does the current through the capacitor branch compare to the current through the resistors? Why is the current through each resistor the same?

6. When the capacitor in your circuit is discharging, how is the potential difference across it changing? How is the potential difference across the series and parallel resistors changing during the same time? Why is the potential difference across the resistor in series zero?

❓ 7. When the capacitor in your circuit is discharging, how is the current through the capacitor branch changing? How is the current through the series and parallel resistors changing during the same time? Why is the current through the capacitor branch negative?

Synthesis Questions

❓ 1. Using the values and configuration in the circuit below, and those provided in Table 2, determine the voltage across and the current through each component after the switch has been closed for a long time. Write your results in Table 2 and explain how you arrived at your answers.

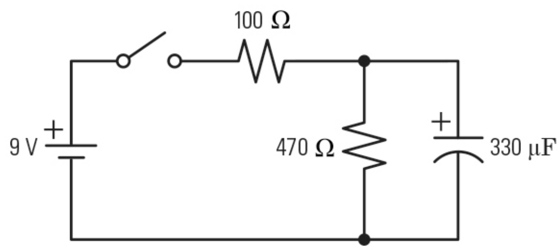


Table 2: Determine the remaining circuit component values

Component	Potential Difference (V)	Current (A)
330-μF Capacitor		
100-Ω Resistor		0.0158
470-Ω Resistor	7.42	

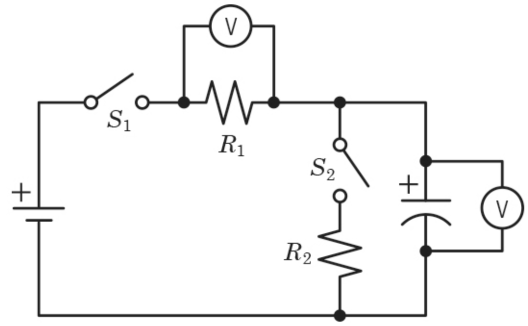
❓ 2. For the circuit in the previous question, which values would change in Table 2 if:

a. the 100-Ω and the 470-Ω resistors were switched? Explain.

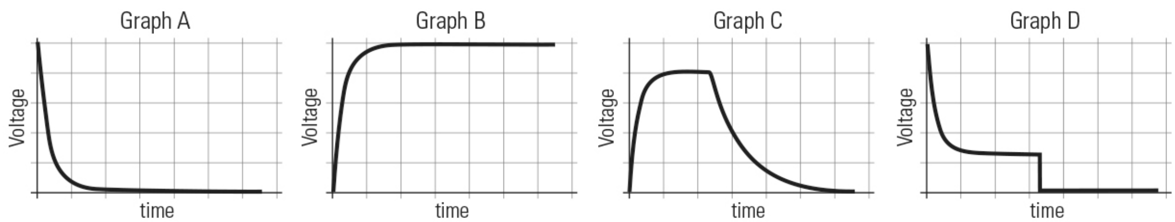
b. the capacitor was replaced with a 470-μF capacitor? Explain.

c. the 100-Ω resistor was replaced with a 1,000-Ω resistor? Explain.

3. The following circuit contains a battery, two resistors, and a capacitor. A switch is placed in series with each resistor and a voltmeter is placed in parallel across resistor R_1 and in parallel across the capacitor.



The following graphs represent four voltage measurements made by either voltmeter while switch S_1 was momentarily closed then reopened.



Match the description below with its corresponding graph. Explain how you determined each match.

a. Voltage measured across resistor R_1 while switch S_2 remains open.

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b. Voltage measured across resistor R_1 while switch S_2 remains closed.

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c. Voltage measured across the capacitor while switch S_2 remains open.

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d. Voltage measured across the capacitor while switch S_2 remains closed.

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16. PLANCK'S CONSTANT

Connections to the AP® Physics 2 Curriculum*

The lab activity correlates to the following pieces of the AP® Physics 2 framework:

Big Idea	6	Enduring Understanding	F	Essential Knowledge	3

Learning Objective 1: The student is able to support the photon model of radiant energy with evidence provided by the photoelectric effect.
Science Practices: 6.4

Big Idea	6	Enduring Understanding	F	Essential Knowledge	4

Learning Objective 1: The student is able to select a model of radiant energy that is appropriate to the spatial or temporal scale of an interaction with matter.
Science Practices: 6.4, 7.1

Time Requirement

Preparation Time: 10 minutes

Lab Activity: 45 minutes

Prerequisites

Students should be familiar with the following concepts:

- Energy transfer occurs when photons are absorbed or emitted, for example, by atoms or nuclei.
- Light can be characterized as photons that each carry discrete energy E_{photon} proportional to their frequency f, where $E_{\text{photon}} = hf$, and h is known as Planck's constant.

Driving Question | Objective

What is the value of Planck's constant and how can it be determined experimentally? Given the quantization of light energy and the relationship between photon energy and frequency, perform an experiment using the light emitted from monochromatic LEDs to determine the value of Planck's constant.

Procedural Overview

In the Structured version of this lab activity, students use a variable voltage source to power four LEDs, each with a different output frequency of light. Students identify the minimum applied voltage (the "turn-on" voltage) needed for each LED to just begin emitting light, and then plot a graph of turn-on voltage versus output frequency of each LED.

This graph displays a linear relationship with the slope equal to Planck's constant divided by the fundamental charge of an electron. Students apply a line of best fit to their data and use the slope of their best fit line to calculate an experimental value for Planck's constant.

Pre-Lab Discussion and Activity

Students should understand the structure and basic operation of a monochromatic LED before performing any version of this lab activity, as this may play an important part in their experimental design and data analysis strategy. Students performing the Structured version of the activity will find information regarding the physics of LED operation in the Background section in their student handouts. This information does not exist in either the Guided Inquiry or Student Designed handouts. It is advised that students performing the Guided Inquiry or Student Designed versions receive a brief introduction or lecture regarding the basics of LED construction, theory of operation, and practical application in a simple circuit. If you choose, the information in the Background section of the Structured student handout can serve as a guide.

Below is a question set that can be used as the basis for a pre-lab classroom discussion, or assigned as pre-lab questions that students can work on at home.

PRE-LAB QUESTIONS

❷ 1. If a microwave photon has a frequency of 25 GHz, what is its energy?

$$E = hf = \left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(25\times10^{9}\ \text{s}^{-1}\right) = 1.7\times10^{-23}\ \text{J}$$

❷ 2. What is the wavelength of a photon that has energy equal to 3.13×10^{-19} J?

$$E = hf \qquad f = \frac{c}{\lambda}$$

$$E = h\frac{c}{\lambda}$$

$$\lambda = h\frac{c}{E} = 6.63\times10^{-34}\ \text{J}\cdot\text{s}\left(\frac{3.00\times10^{8}\ \text{m/s}}{3.13\times10^{-19}\ \text{J}}\right) = 6.35\times10^{-7}\ \text{m} = 635\ \text{nm}$$

❷ 3. An energized electron in a hydrogen atom emits a photon as it transitions from its current energized state to its ground state (lowest energy state). If the wavelength of the emitted photon is 122 nm, what minimum amount of energy was required to move the electron from its ground state to its original energized state?

Because electron energy is quantized and the total energy is conserved, students can assume that the energy of the emitted photon E_{photon} is equal to the difference in energy between the electron's ground state and its original energized state. Conversely, for the electron to move from its ground state to its original energized state, it must absorb a minimum amount of energy $E_{excitation}$ equal to the difference between the two states. Therefore, the minimum amount of energy required to move the electron from its ground state to its original energized state is equivalent to the energy of a photon emitted as a result of the electron transition between the same two states:

$$E_{photon} = E_{excitation}$$

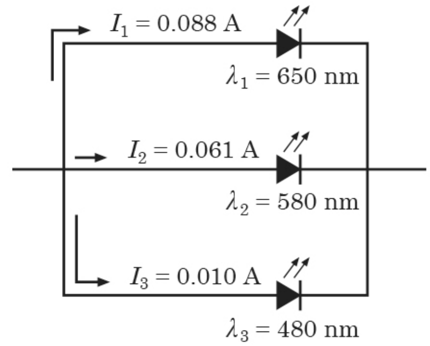
$$E_{excitation} = hf = h\frac{c}{\lambda}$$

$$E_{excitation} = 6.63\times10^{-34}\ \text{J}\cdot\text{s}\,\frac{3.00\times10^{8}\ \text{m/s}}{122\times10^{-9}\ \text{m}} = 1.63\times10^{-18}\ \text{J}$$

❷ 4. Two different color lasers, violet (λ = 405 nm) and red (λ = 680 nm), are used in an optical disk drive. If both lasers have an emission power rating of 13 mW, which laser emits more photons per unit time? Explain your reasoning.

Because the wavelength of violet light is shorter (higher frequency: $f = c/\lambda$) than that of red light, violet light has greater energy per photon than red light ($E_{photon} = hf$). If the emission power of each laser is the same, the energy emission rates are also the same. Because violet light has more energy per photon than the red light, the violet laser will emit fewer photons per unit time to have an equivalent energy emission rate as the red laser.

5. A constant 3.0 V potential difference is applied across three monochromatic LEDs arranged in parallel in a circuit. The wavelength of each LED and the constant current in each circuit branch is shown in the diagram to the right. Assuming that each LED converts all of the electrical energy supplied to it into photon energy, how many photons are emitted by each LED in a 10.0 second interval?



If each LED converts all of the electrical energy E_{electron} into photon energy E_γ, students can assume that the total electrical energy supplied to each LED over the 10.0 second interval will equal the total photon energy emitted by each LED:

$$E_{\text{electron total}} = E_{\gamma\ \text{total}}$$

$$E_{\text{electron total}} = V I \Delta t$$

$$E_{\gamma\ \text{total}} = h\frac{c}{\lambda} \times \text{number of photons}$$

$$h\frac{c}{\lambda} \times \text{number of photons} = V I \Delta t$$

$$\text{number of photons} = \lambda I \frac{V\Delta t}{hc}$$

$$\text{number of photons}_1 = \left(650 \times 10^{-9}\ \text{m}\right)\left(0.088\ \text{A}\right)\frac{(3.0\ \text{V})(10.0\ \text{s})}{\left(6.63 \times 10^{-34}\ \text{J} \cdot \text{s}\right)\left(3.00 \times 10^{8}\ \text{m/s}\right)} = 8.6 \times 10^{18}\ \text{photons}$$

$$\text{number of photons}_2 = \left(580 \times 10^{-9}\ \text{m}\right)\left(0.061\ \text{A}\right)\frac{(3.0\ \text{V})(10.0\ \text{s})}{\left(6.63 \times 10^{-34}\ \text{J} \cdot \text{s}\right)\left(3.00 \times 10^{8}\ \text{m/s}\right)} = 5.3 \times 10^{18}\ \text{photons}$$

$$\text{number of photons}_3 = \left(480 \times 10^{-9}\ \text{m}\right)\left(0.010\ \text{A}\right)\frac{(3.0\ \text{V})(10.0\ \text{s})}{\left(6.63 \times 10^{-34}\ \text{J} \cdot \text{s}\right)\left(3.00 \times 10^{8}\ \text{m/s}\right)} = 7.2 \times 10^{17}\ \text{photons}$$

Materials and Equipment

- Data collection system
- PASCO Voltage–Current Sensor[1]
- PASCO AC/DC Electronics Laboratory[2]
- Wire leads[2] (5)
- Resistor, 330-Ω[2]
- Battery, D-cell (2)
- LED, blue (450–500 nm)
- LED, green (501–565 nm)
- LED, yellow/amber (566–620 nm)
- LED, red (621–750 nm)
- Spectrometer (optional)

Probeware Resources

Below are web-link and QR codes that will direct you to instructional video resources for individual pieces of PASCO probeware, sensors, and other hardware used in the lab activity. These same links and codes are provided to students in their activity handouts.

[1]www.pasco.com/ap19

PASCO Voltage–Current Sensor

[2]www.pasco.com/ap04

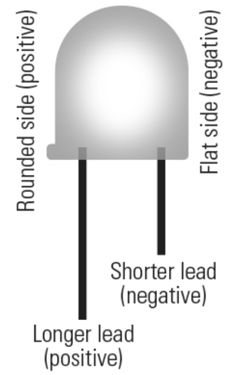
PASCO AC/DC Electronics Laboratory

Safety

Follow these important safety precautions in addition to your regular classroom procedures:

- Do not stare at the LEDs when they are fully lit as this may be harmful to your eyes.
- Do not use LEDs that emit ultraviolet light, which can cause permanent eye damage.
- Do not apply voltages to the LEDs above approximately 2.8 V as this can cause permanent damage to the LEDs.
- Voltage must only be applied to LEDs in the "forward-biased" orientation with current flowing from the positive lead to the negative lead. Connecting the LED incorrectly can damage it.

  Always connect the positive lead to positive voltage, and the negative lead to negative voltage. An LED's positive electrode lead is longer than the negative lead, and the negative lead has a flat spot on the side of the plastic LED housing.



Lab Preparation

Students performing the Structured version of this lab activity are required to use the light frequency of different monochromatic LEDs as the independent variable in their experiment. In their data analysis, students calculate LED frequency from the LED wavelength provided to them by the instructor or measured using an emission spectrometer (optional). If you choose to provide a spectrometer for student use, be certain that it has a range sufficient to measure the wavelength band of all LEDs being used, and that students use the wavelength value corresponding to the peak amplitude in their amplitude versus wavelength graph.

If you prefer to determine the LED wavelength values using an emission spectrometer, assemble the simple circuit outlined in the Set Up section of the Structured student handout (minus the voltage sensor, which is not needed). Place each LED, one at a time, in the circuit and adjust the potentiometer clockwise until the LED light becomes visible. Follow the instruction provided with your spectrometer to determine each LED wavelength. Again, choose the wavelength value corresponding to the peak amplitude in each amplitude versus wavelength graph.

If you wish to provide the wavelengths to students but an emission spectrometer is unavailable, output wavelength values for monochromatic LEDs are often published on the LED packaging within a general tolerance depending on the LED manufacturer.

Teacher Tips

Tip 1 – Using Third Party LEDs

- Monochromatic LEDs chosen for this lab activity are available from PASCO. However, LEDs purchased from a third party may also be used as long as the output wavelength of each falls within the wavelength bands specified in the Materials and Equipment section above. If you use LEDs sourced from a third party, be certain that the output light from each LED is in the visible spectrum only, as students performing the Structured version of this lab activity are required to look directly at each LED to identify when it begins to emit light. (Monochromatic LEDs generally emit several wavelengths of light simultaneously, including ultraviolet light which may cause permanent eye damage. Refer to the LED's manufacturer specifications to make certain only visible wavelengths are emitted.)

Tip 2 – Using a Variable-Voltage Power Supply

- The procedure outlined in the Structured version of this lab activity directs students to power their LEDs using a 3 volt variable-voltage power source constructed from two D-cell batteries and the 25-Ω potentiometer on the PASCO AC/DC Electronics Laboratory component board.

However, students can alternatively use a bench top variable-voltage power supply in place of the batteries and potentiometer. Students using a variable-voltage power supply must be careful not to drive the LEDs with too much current, as this could permanently damage the LEDs.

Current is generally controlled by placing a resistor in series with each LED and limiting the applied voltage. Typically, a 330-Ω resistor in series with each LED, and applied voltages under 3 VDC will preserve most monochromatic LEDs.

Tip 3 – Difficulty Observing LED Turn-On

- The procedure outlined in the Structured version of this lab activity requires students to look directly at each LED to identify when it just begins to emit light. This can be difficult to observe, as the intensity of the light is nearly zero at this point. To help, students should observe their LEDs in-line with the direction the LED focuses light. The construction of most LEDs includes a lens to help focus emitted light. LEDs with a hemispherical top will focus light through the hemispherical surface. These LEDs should be observed from above (through the hemispherical surface).
- Shrouding the LEDs will help to block ambient light that also makes observation of the LED turn-on point difficult. Students can cup their hands around the LED as they observe it, or look through a short piece of tubing or pipe (diameter > 3 cm) placed over the LED to block ambient light.

Sample Data

Below are sample data, acquired using the experimental setup and procedure outlined in the Structured version of the lab activity, and answers to questions in the Data Analysis section.

Data Analysis

Table 1: LED voltage and frequency data for determining Planck's constant

LED Color	Output Wavelength (nm)	Turn-On Voltage (V)	Output Frequency (Hz)
Blue	466.00	2.12	6.44×10^{14}
Green	525.22	1.79	5.71×10^{14}
Yellow	590.68	1.50	5.08×10^{14}
Red	625.11	1.38	4.80×10^{14}

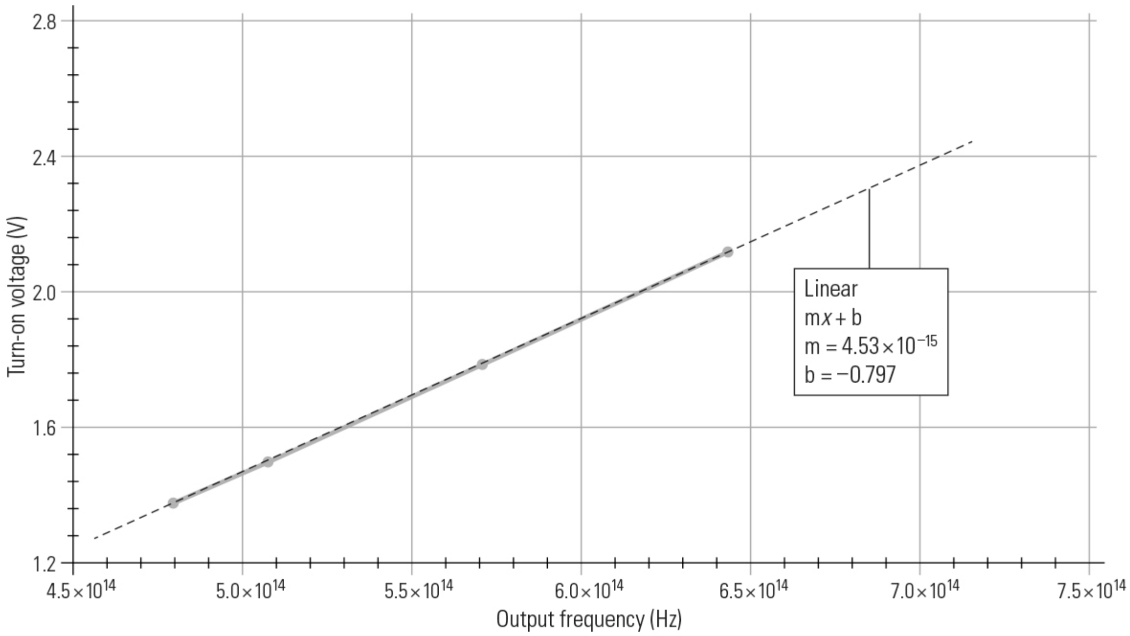
1. Calculate the output frequency of each LED using its recorded Output Wavelength value and Equation 2. Record each Output Frequency value in Table 1.

   Calculation using sample data for a blue LED:

$$f = \frac{c}{\lambda} = \frac{3.00 \times 10^{8}\ \text{m/s}}{466.00 \times 10^{-9}\ \text{m}} = 6.44 \times 10^{14}\ \text{s}^{-1}$$

2. Plot a graph of *turn-on voltage* versus *output frequency* in the blank Graph 1 axes below. Be sure to label both axes with the correct scale and units.

Graph 1: Turn-on voltage versus output frequency for various monochromatic LEDs



3. Draw a line of best fit through your data in Graph 1. Determine and record the equation of the line here:

Best fit line equation: $V = \left(4.53 \times 10^{-15}\ \text{V}\cdot\text{s}\right)f - 0.797\ \text{V}$

4. Use the slope from the best fit line to determine an experimental value for Planck's constant:

$$\text{slope} = \frac{h}{e}$$

Planck's constant h (J·s): 7.25×10^{-34} J·s

Calculation using sample data from Graph 1:

$$\text{slope} = \frac{h}{e}$$

$$h = e \times \text{slope} = \left(1.60 \times 10^{-19}\ \text{C}\right)\left(4.53 \times 10^{-15}\ \text{V}\cdot\text{s}\right) = 7.25 \times 10^{-34}\ \text{J}\cdot\text{s}$$

Guided Inquiry Questions

Below are sample responses to the Guiding Questions found in the Guided Inquiry version of this lab activity.

❷ 1. What is the equation that relates the energy of a photon to its frequency? Write the equation and identify each variable with correct units.

$E = hf$

where E is the energy of a photon in joules (J), h is Planck's constant ($h = 6.63 \times 10^{-34}$ J·s), and f is the frequency of the photon in inverse seconds (s^{-1}).

❷ 2. The semiconductor in a monochromatic LED will not emit light until the electrons within it are each given some threshold energy $E_{electron} = e\Delta V_0$, where ΔV_0 is the threshold (turn-on) voltage applied to the LED (the potential difference required for an LED to just begin emitting light), and e is the charge of an electron ($e = 1.60 \times 10^{-19}$ C). When the LED just begins to emit light, each electron loses that threshold energy in the form of a photon.

Assuming all of the threshold energy from each electron is converted into photon energy, what is the mathematical expression that relates the turn-on voltage ΔV_0 of a monochromatic LED to the frequency of a photon emitted by the LED?

If each electron converts all of the threshold energy $E_{electron} = e\Delta V_0$ into photon energy $E_{photon} = hf$, the two expressions are equal and can be combined into one expression relating turn-on voltage ΔV_0 to photon frequency f:

$$E_{electron} = E_{photon}$$

$$e\Delta V_0 = hf$$

❷ 3. In the expression established in your response to the previous question, which variables can be measured directly using tools available to you, and which variable must be measured indirectly? Identify the tools and techniques you would use to measure each variable.

Students should realize that the only variable that can be measured directly is the turn-on voltage using a voltage sensor, voltmeter, or other multimeter connected in parallel across the LED. Students should specify how they will determine when the turn-on voltage has been reached. In the Structured version of this lab activity, students look directly at each LED as the voltage is slowly increased until the LED just begins emitting light. The applied voltage at the point the LED just begins to emit light is the turn-on voltage.

Students may choose to configure their experiment using a current sensor so that the current through and voltage across the LED are measured simultaneously. Current through the LED will only flow when the LED is emitting light. The voltage at which the current first becomes non-zero is the turn-on voltage. Students using this technique should use a current sensor or other ammeter that has a resolution of 10 μA or better.

Photon frequency is the only other variable in the expression, but it cannot be measured directly. Students can measure LED photon frequency indirectly by measuring the output wavelength of the light emitted by the LED using an emission spectrometer, and then using the simple calculation, $f = c/\lambda$, to convert wavelength to frequency. If an emission spectrometer is not to be used, the output wavelength should be provided by the instructor.

❷ 4. The color of the light emitted by a monochromatic LED is rarely specified by its frequency, but rather, by its wavelength. What is the equation that relates the wavelength of a photon to its frequency? Write the equation and identify each variable with correct units. Why is this equation important in your experiment?

The equation that relates the wavelength of a photon to its frequency is

$$f = \frac{c}{\lambda}$$

where f is the frequency of the photon in inverse seconds (s^{-1}), c is the speed of light in a vacuum ($c = 3.00 \times 10^8$ m/s), and λ is the wavelength of the photon in meters (m).

Students will find that most monochromatic LEDs are classified by wavelength of output light, rather than frequency. In their data analysis, students should plot turn-on voltage versus the frequency of light output of each LED. To do this, students must calculate corresponding frequency values based on specified wavelength values.

❷ 5. Assuming you choose a graphical data analysis method based on the variables identified in the previous questions, what will the dependent (measured) variable and independent (manipulated) variable on your graph be? Explain why you chose the variables that you did.

Students should choose LED frequency as their independent (manipulated) variable and turn-on voltage as their dependent (measured) variable. Two important reasons for these choices are:

1) The turn-on voltage for monochromatic LEDs is fixed. Regardless of the applied voltage, the output light frequency will not change, thus making it impossible to "manipulate" turn-on voltage.
2) Plotting a graph of turn-on voltage for various frequencies of output light will generate a graph that shows a linear relationship whose slope equals h/e. Students can apply a best fit line to their data and then calculate Planck's constant h from the value of the slope and the known charge of an electron e.

6. How will you configure a simple LED circuit so that the independent and dependent variables can be manipulated and measured, as appropriate? Include in your description any voltmeters or other circuit measurement tools you may use.

   For each LED, students should wire a simple circuit with the LED connected in series to a variable voltage source so they may slowly adjust the applied voltage until the turn-on voltage is reached. Voltage sensors or other volt meters should be wired in parallel across the LED in the circuit(s) so that the voltage applied to the LED is measured directly. Any current sensors or other ammeters should be wired in series within the circuits. The LED should be positioned so that students can observe when the LED just begins to emit light. For an example of this simple circuit, see the Set Up section of the Structured version student handout.

   Students should add a resistor in series with the LEDs to prevent an overcurrent situation that may damage the LED. Typically, a 330-Ω resistor in series with each LED is enough resistance to prevent this.

Assessment Questions: Sample Responses

Sample responses to the Analysis and Synthesis questions found in each version of the lab activity:

Analysis Questions

1. What is your experimental value for Planck's constant, and how did you determine this value from your data?

   Based on the sample data and calculations in the Sample Data section above, $h = 7.25 \times 10^{-34}$ J·s

   Students following the procedure outlined in the Structured version of this lab activity measure the turn-on voltage needed for several different-color LEDs to just begin emitting light. They convert LED output wavelength to frequency using wavelength values that were either measured or specified by the instructor. After plotting a graph of turn-on voltage versus frequency of output light for their various LEDs and applying a best fit line to their data, students calculate Planck's constant h from their best fit line value for slope where $h = e \times$ slope.

2. What are factors that might have caused error in your measured value for Planck's constant? Explain how each factor you list could have been avoided or minimized.

   The list of factors that could cause error may include, but is not limited to:

   - Inaccurate determination of the voltage when each LED is just beginning to emit light. This error can be minimized using the techniques outlined in Teacher Tip 3 above.

     This error can be avoided if students configure their experiment using a current sensor so that the current through and voltage across the LED are measured simultaneously. Current through the LED will only flow when the LED is emitting light. The voltage at which the current first becomes non-zero is the turn-on voltage. Students using this technique should use a current sensor or other ammeter that has a resolution of 10 µA or better.

   - The peak output frequency and wavelength are not accurate. Although the LEDs are assumed to be monochromatic, each LED actually emits photons with varying frequencies over a very narrow span of frequencies. The manufacturer value for output frequency or wavelength of an LED may not represent the peak output frequency or wavelength within that band. Students can use an emission spectrometer to determine the actual peak output frequency or wavelength of each LED.

3. The actual value for Planck's constant is $h = 6.63 \times 10^{-34}$ J·s. Calculate the percent error between your experimental value and the actual value.

$$\text{Percent error} = \left| \frac{\text{Actual} - \text{Experimental}}{\text{Actual}} \right| \times 100$$

   Sample calculation:

$$\text{Percent error} = \left| \frac{6.63 \times 10^{-34}\ \text{J}\cdot\text{s} - 7.25 \times 10^{-34}\ \text{J}\cdot\text{s}}{6.63 \times 10^{-34}\ \text{J}\cdot\text{s}} \right| \times 100 = 9.35\%$$

Synthesis Questions

1. A solid state laser emits light at 532 nm from the laser LED within it. What minimum potential difference applied across the LED is required for light to be emitted from the laser diode?

$$e\Delta V_0 = h\frac{c}{\lambda}$$

$$\Delta V_0 = h\frac{c}{e\lambda} = 6.63 \times 10^{-34}\ \text{J}\cdot\text{s}\frac{3.00 \times 10^{8}\ \text{m/s}}{\left(1.60 \times 10^{-19}\ \text{C}\right)\left(532 \times 10^{-9}\ \text{m}\right)} = 2.34\ \text{J/C} = 2.34\ \text{V}$$

2. A blue LED (λ = 472.2 nm) emits 3.89×10^{16} photons per second. In a few sentences, explain the process by which you would determine the power output in watts.

Student responses should indicate that output power of the laser will be equal to the sum of the energies of individual photons emitted from the laser per unit time. Responses should also include that the energy of each photon is dependent on its wavelength. Below is a sample response:

The power output of a laser is a measure of the radiant energy output from the laser per unit time. Each photon output from the laser has a given amount of energy associated with it equal to $E_{photon} = hf = hc/\lambda$ where h is Planck's constant, *c is the speed of light in a vacuum,* and λ is the wavelength of the photon. If the energy per photon is equal to hc/λ and the laser outputs 3.89×10^{16} photons per second, then the power output is equal to the product of the energy per photon and the photon emission rate:

$$\text{Laser power} = \frac{\text{number of photons}}{\text{s}} \times \frac{hc}{\lambda} = 3.89 \times 10^{16}\frac{\text{photons}}{\text{s}} \times \frac{\left(6.63 \times 10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00 \times 10^{8}\ \text{m/s}\right)}{472.2 \times 10^{-9}\ \text{m}} = 0.0164\ \text{W}$$

3. The *photoelectric effect* is a process in which energized electrons are ejected from the surface of a metal when light strikes it. The maximum kinetic energy K_{max} of each ejected electron is equal to the difference between the energy of one photon E_{photon} incident on the metal and the minimum amount of energy needed to free the electron, known as *work function* φ:

$$K_{max} = E_{photon} - \varphi$$

Assuming that the energy of an ejected electron is always positive and non-zero, below what wavelength must the incident light be for electrons to be ejected from an aluminum surface? Assume $\varphi_{aluminum} = 6.54 \times 10^{-19}$ J. Show your work.

For electrons to be ejected from the aluminum surface, the photon energy must be greater than work function of the aluminum:

$$\varphi_{aluminum} < E_{photon}$$

$$\varphi_{aluminum} < h\frac{c}{\lambda}$$

$$\lambda < h\frac{c}{\varphi_{aluminum}}$$

$$\lambda < 6.63 \times 10^{-34}\ \text{J}\cdot\text{s}\frac{3.00 \times 10^{8}\ \text{m/s}}{6.54 \times 10^{-19}\ \text{J}}$$

$$\lambda < 3.04 \times 10^{-7}\ \text{m}$$

Extended Inquiry Suggestions

Extend this activity with a discussion of Robert Millikan's experiment in which he was able to demonstrate Einstein's photoelectric theory and experimentally determine Planck's constant within 0.5% of the theoretical value. Outline Millikan's important experimental results that showed that energized electrons are ejected from the surface of a metal only when the incident photons had sufficient energy based on their frequency, not the number of incident photons, or wave amplitude. Describe Millikan's experimental apparatus and procedure in which he used *stopping potential* to identify the maximum kinetic energy of ejected electrons, and a graphical method similar to that used in the Structured version of this lab activity to determine Planck's constant.

NAME PERIOD DATE

16. PLANCK'S CONSTANT

STRUCTURED

Driving Question | Objective

What is the value of Planck's constant and how can it be determined experimentally? Given the quantization of light energy and the relationship between photon energy and frequency, perform an experiment using the light emitted from monochromatic LEDs to determine the value of Planck's constant.

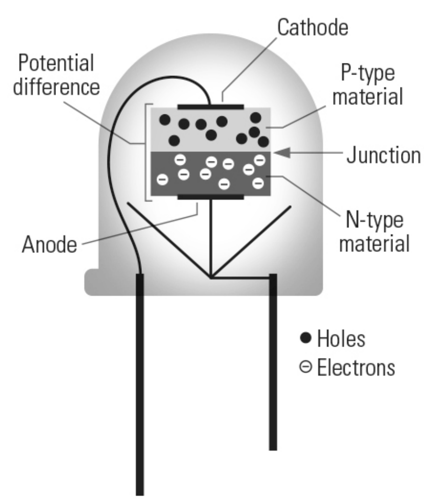
Materials and Equipment

- Data collection system
- PASCO Voltage–Current Sensor[1]
- PASCO AC/DC Electronics Laboratory[2]
- Wire leads[2] (5)
- Resistor, 330-Ω[2]
- Battery, D-cell (2)
- LED, blue (450–500 nm)
- LED, green (501–565 nm)
- LED, yellow/amber (566–620 nm)
- LED, red (621–750 nm)
- Spectrometer (optional)

[1]www.pasco.com/ap19
PASCO Voltage–Current Sensor

[2]www.pasco.com/ap04
PASCO AC/DC Electronics Laboratory

Background



A light-emitting diode (LED) is a simple *p–n junction* semiconductor device that converts electrical energy into light energy. LEDs consist of two semiconductor materials: *n-type*, where an excess of free electrons exists, and *p-type*, where a depletion of electrons, known as "holes," exists. The junction between the materials creates a *depletion region*.

The depletion region acts as an insulator blocking the flow of electrons across the junction. When a large enough potential difference (known as the "turn-on" voltage ΔV_0) is reached, electrons in the n-type material are given enough energy ($E_{electron} = e\Delta V_0$) to cross the p-n junction and join with the holes in the p-type material. At this point, current begins to flow through the LED.

The holes are at a lower energy state than the electrons. When an electron joins a hole, the electron moves to the lower energy state and one photon is emitted by the electron–hole pair. The energy lost by the electron $E_{electron}$ when it combines with a hole is equal to the energy of the emitted photon E_{photon}. Assuming that light behaves with particle nature and has quantized energy ($E_{photon} = hf$), *an equation describing both energies can be formulated:*

$$E_{electron} = E_{photon}$$

$$e\Delta V_0 = hf \qquad (1)$$

where e is the fundamental charge of an electron ($e = 1.60 \times 10^{-19}$ C), ΔV_0 is the potential difference (the "turn-on" voltage) required for an LED to just begin emitting light, f is the frequency of the emitted photon, and h is Planck's constant ($h = 6.63 \times 10^{-34}$ J·s).

Equation 1 states that the potential difference ΔV_0 (turn-on voltage) required for an LED to just begin emitting light is proportional to the frequency of the emitted light.

Additionally, frequency f is proportional to the speed of light in a vacuum ($c = 3.00 \times 10^8$ m/s), and inversely proportional to the photon's wavelength λ.

$$f = \frac{c}{\lambda} \qquad (2)$$

In this activity, you will use this relationship between potential difference (turn-on voltage) and photon frequency from an LED to determine an experimental value for Planck's constant.

RELEVANT EQUATIONS

$$e\Delta V_0 = hf \qquad (1)$$

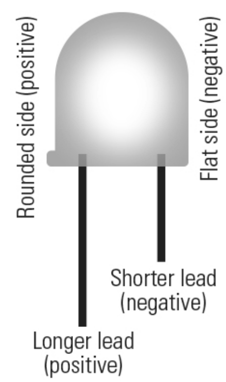
$$f = \frac{c}{\lambda} \qquad (2)$$

Safety

Follow these important safety precautions in addition to your regular classroom procedures:

- Do not stare at the LEDs when they are fully lit as this may be harmful to your eyes.
- Do not use LEDs that emit ultraviolet light, which can cause permanent eye damage.
- Do not apply voltages to the LEDs above approximately 2.8 V as this can cause permanent damage to the LEDs.
- Voltage must only be applied to LEDs in the "forward-biased" orientation with current flowing from the positive lead to the negative lead. Connecting the LED incorrectly can damage it.

  Always connect the positive lead to positive voltage, and the negative lead to negative voltage. An LED's positive electrode lead is longer than the negative lead, and the negative lead has a flat spot on the side of the plastic LED housing.

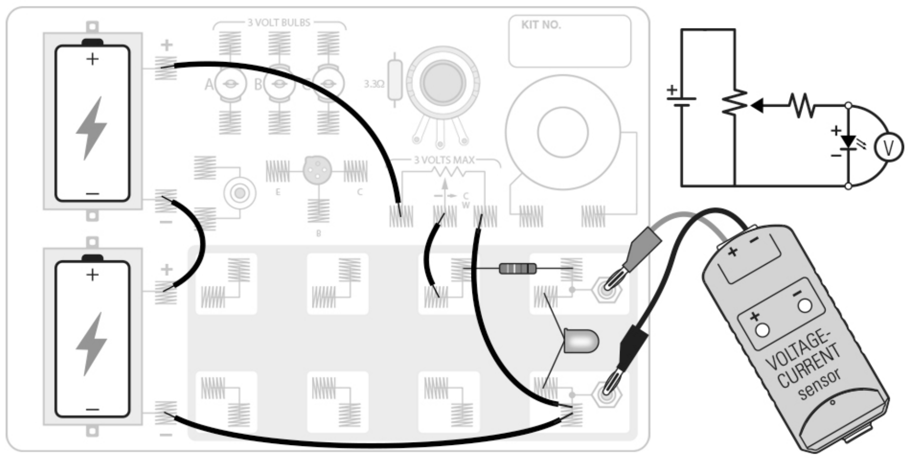


Procedure

Set Up

1. Assemble the circuit as shown, using two D-cell batteries, five wire leads, the on-board 25-Ω potentiometer, one 330-Ω resistor, and one of the LEDs. Be certain to connect the LED so it is forward-biased: the negative lead must be wired to the negative terminal on the battery.

NOTE: The potentiometer in your circuit is used to provide a variable voltage to the LED. Turning the dial clockwise increases the voltage to the LED; turning the dial anti-clockwise decreases the voltage.



2. Connect the voltage sensor leads in parallel across the LED in your circuit: red to the positive lead; black to the negative lead.

3. Connect the voltage sensor to the data collection system, and then create a digits display showing the voltage measured by the sensor.

4. Turn the dial on the potentiometer and observe the LED to make certain it emits light when the voltage is increased.

   NOTE: Do not stare at blue LEDs even when partially lit. If the LED does not emit light when the voltage is increased, check your circuit wiring and make sure all components are wired correctly and the LED is forward-biased.

Collect Data

5. Record the color and output wavelength of the LED in your circuit in Table 1 in the Data Analysis section below. To obtain this data:

   - If you are using a spectrometer: turn the dial on the potentiometer all the way clockwise, and then use the spectrometer to determine the color and peak output wavelength of the LED.

   - If you are NOT using the optional spectrometer: obtain the LED's color and output wavelength from your instructor.

6. Adjust the potentiometer by turning it all the way anti-clockwise.

7. Begin recording data, and then slowly turn the potentiometer clockwise until the LED just begins to emit light.

   NOTE: When the LED just begins to emit light, it will be very dim and difficult to see. You may need to shroud the LED with your hands to help. You may also need to adjust the voltage up and down to accurately find the point at which it just begins emitting light.

8. Record the voltage measurement on your data collection system as the turn-on voltage for the LED in your circuit in Table 1, and then stop recording data.

9. Repeat the same data collection steps three additional times using a different color LED each time. Record the turn-on voltage, color, and output wavelength for each LED in Table 1.

Data Analysis

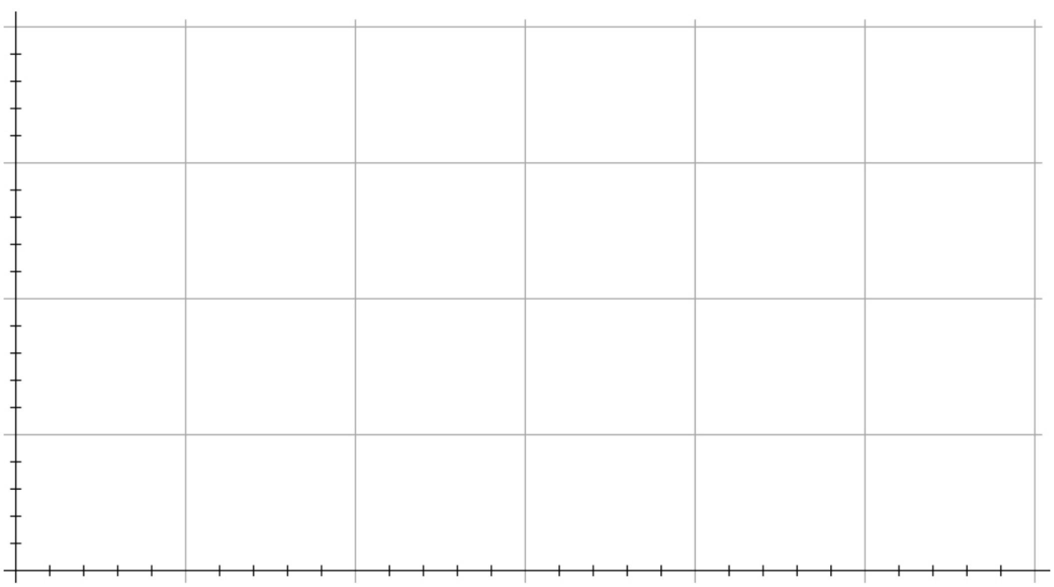
Table 1: LED voltage and frequency data for determining Planck's constant

LED Color	Output Wavelength (nm)	Turn-On Voltage (V)	Output Frequency (Hz)

1. Calculate the output frequency of each LED using its recorded Output Wavelength value and Equation 2. Record each Output Frequency value in Table 1.

2. Plot a graph of *turn-on voltage* versus *output frequency* in the blank Graph 1 axes below. Be sure to label both axes with the correct scale and units.

Graph 1: Turn-on voltage versus output frequency for various monochromatic LEDs



3. Draw a line of best fit through your data in Graph 1. Determine and record the equation of the line here:

   Best fit line equation: ____

4. Use the slope from the best fit line to determine an experimental value for Planck's constant:

   $$\text{slope} = \frac{h}{e}$$

   Planck's Constant h (J·s): ________________________________

Analysis Questions

1. What is your experimental value for Planck's constant, and how did you determine this value from your data?

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____

____

2. What are factors that might have caused error in your measured value for Planck's constant? Explain how each factor you list could have been avoided or minimized.

____

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3. The actual value for Planck's constant is $h = 6.63 \times 10^{-34}$ J·s. Calculate the percent error between your experimental value and the actual value.

   $$\text{Percent error} = \left|\frac{\text{Actual} - \text{Experimental}}{\text{Actual}}\right| \times 100$$

Synthesis Questions

1. A solid state laser emits light at 532 nm from the laser LED within it. What minimum potential difference applied across the LED is required for light to be emitted from the laser diode?

2. A blue LED (λ = 472.2 nm) emits 3.89×10^{16} photons per second. In a few sentences, explain the process by which you would determine the power output in watts.

3. The *photoelectric effect* is a process in which energized electrons are ejected from the surface of a metal when light strikes it. The maximum kinetic energy K_{max} of each ejected electron is equal to the difference between the energy of one photon E_{photon} incident on the metal and the minimum amount of energy needed to free the electron, known as *work function* φ:

$$K_{\text{max}} = E_{\text{photon}} - \varphi$$

Assuming that the energy of an ejected electron is always positive and non-zero, below what wavelength must the incident light be for electrons to be ejected from an aluminum surface? Assume $\varphi_{aluminum} = 6.54 \times 10^{-19}$ J. Show your work.