Advanced Physics 1 through Inquiry

Experiment Guide

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INTRODUCTION

PASCO scientific's *Advanced Physics 1 through Inquiry* manual includes investigations designed to move students from the low level task of memorization or confirmation of science facts, to higher-level tasks of experiment design, data analysis, concept construction, and application. For science to be learned at a deep level, it is essential to combine the teaching of abstract science concepts with "real-world" science investigations. Hands-on technology-based laboratory experiences serve to bridge the gap between the theoretical and the concrete, driving students toward a greater understanding of natural phenomena. Students also gain important science process skills that include: developing and using models, planning and carrying out independent investigations, interpreting data, and using mathematics.

The laboratory activities span the physics content as defined by the College Board® for AP® Physics 1. These activities introduce students to the tools of science and help develop conceptual understanding and link the academic content of college level physics to the experimental evidence that defines and supports this content. These activities have been selected and structured to support student achievement on the end-of-year AP® Physics 1 exam.

Three Levels of Scientific Inquiry

Fifteen laboratory activities cover topics in kinematics, dynamics (linear and rotational), statics, work and energy, impulse and momentum, periodic motion, waves, and DC circuits. Every activity is presented in three distinctly different formats, each with a varied level of inquiry-based content:

Structured: This traditional format provides students a concise background section and a formal step-by-step setup and procedure. The *Structured* format also includes a complete equipment list and data analysis procedure with prescribed data display forms and data manipulation techniques.

Guided Inquiry: With no prescribed setup or procedure, the *Guided Inquiry* format contains a series of questions designed to invoke inquiry in students that will guide them to a proper setup and execution. Students design their own setup and procedure while also deciding how they will present their data to properly fulfill the lab objective and correctly address the lab's driving question. This format does *not* include a background section.

Student Designed: This format includes simply a driving question, objective statement, and a suggested equipment list. Students are expected to design and execute their own setup and procedure with little or no guidance from the student handout. They choose how to present their data in a way that supports their answer to the driving question, while also fulfilling the lab objective. This format does *not* include a background section.

The three different formats for each lab activity support the instructional need to differentiate the level of scientific inquiry in the classroom. Teachers may choose first to provide activities in the *Structured* format where students receive full guidance while developing skills that include critical thinking (posing good questions, developing experimental strategies, finding and fixing operational errors), procedural expertise (calibrating equipment, collecting data), proficiency in design and construction (assembling apparatus, following safety procedures), and analytical skills (graphing, modeling, statistics).

Students can then progress to a more inquiry-based approach by carrying out subsequent activities in their *Guided Inquiry* formats. When students have formed the skills necessary to confidently design and build their own experiments without help from student handouts, the *Student Designed* format can be offered to provide students a nearly *open-inquiry* approach to the lab topics.

Each lab activity, regardless of the format, contains two assessment question sections—Analysis Questions and Synthesis Questions—that are identical and applicable to all of the three handout formats. These sections are explained in more detail in the Lab Activity Components section below. In addition to supporting the scientific inquiry process, the *Advanced Physics 1 through Inquiry* activities fulfill STEM education requirements by bringing together Science, Technology, Engineering, and Math in varying degrees in the lab activities. The use of sensors, data analysis and graphing tools, models and simulations, and work with instruments, all support scientific inquiry as implemented in a STEM-focused curriculum.

Manual Components

The *Advanced Physics 1 through Inquiry* lab manual offers five major components:

Student Lab Activity Handouts. Each of the lab activities has three independent student handouts, one for each of the three inquiry-based formats: *Structured*, *Guided Inquiry*, and *Student Designed*. All student handouts are available in Microsoft® Word format, allowing teachers to customize the labs for their curriculum, students, and equipment. All student handout files are available on the electronic storage device that comes with the printed *Advanced Physics 1 through Inquiry* manual. Refer to the Lab Activity Components section below for details on each handout.

Teacher Resources. Every lab activity has an accompanying *Teacher Resources* document that contains teacher-centered content specific to the activity, including alignments to the AP® Physics 1 Learning Objectives and Science Practices[1]; recommended time requirements for teacher preparation and student data collection; a procedural overview of the procedure in the *Structured* version of the lab activity; safety and lab preparation instructions (if applicable); teacher tips; sample data for the *Structured* version procedure; responses to the *Structured* version Data Analysis questions, *Guided Inquiry* version Guided Inquiry questions, and Analysis and Synthesis questions for all versions; and extended inquiry activity suggestions.

All *Teacher Resources* documents are available in PDF format on the electronic storage device that comes with every printed *Advanced Physics 1 through Inquiry* manual. Refer to the Lab Activity Components section below for further details on teacher resources.

Student Experiment Design Plan Handout. Students following the *Guided Inquiry* or *Student Designed* version of a lab activity can use this one-sheet handout to help design and implement their inquiry-based investigation. The handout provides students with a small amount of guidance and structure as they develop their own laboratory investigation, regardless of the lab topic. Students use this handout to identify important facets of their investigations: the objective of the lab activity; what variables should be part of their potential experiment; what variables should be manipulated and controlled; how these variables will be manipulated or controlled; and how they will structure their data analysis. This handout is available in PDF format on the electronic storage device that comes with the manual.

Probeware Resources Videos. Included in every lab activity handout are links (both URL and QR code) to short, equipment-specific videos that outline the functionality, specifications, and different use-cases of most of the PASCO hardware and probeware to be used. These information-rich videos will help students understand the functionality and applications of each piece of equipment before using it. The videos will also be a useful tool for students when they design their own inquiry-based investigations. These videos are hosted online and are also available on the electronic storage device that comes with the manual.

[1] From AP Physics 1 and 2 Course and Exam Description, Effective Fall 2014. Copyright © 2014 The College Board. Reproduced with permission. http://apcentral.collegeboard.com

PASCO Capstone™ and SPARKvue® Configuration Files. Although the *Structured* version of every lab activity in the manual includes instructions on how students should display and present their data (for example, in a table or a graph), students also have access to configuration files. Each SPARKvue and PASCO Capstone configuration file has been pre-configured to have the correct display type, sample rate, and other software components needed for each lab activity. With these configuration files, students can simply connect their sensors, open the corresponding configuration file, and begin recording data. These files are available on the electronic storage device that comes with the manual.

Lab Activity Components

Each lab activity consists of four documents: the *Teacher Resources* document and the three Student Handout formats: *Structured*, *Guided Inquiry*, and *Student Designed*.

Sections in each set of activity documents: Teacher Resources, Structured, Guided Inquiry, and Student Designed

SECTIONS	TEACHER RESOURCES	STRUCTURED FORMAT	GUIDED INQUIRY FORMAT	STUDENT DESIGNED FORMAT
Connections to the AP® Physics 1 Curriculum				
Time Requirement				
Prerequisites				
Driving Question | Objective				
Procedural Overview				
Pre-Lab Discussion and Activity				
Design and Conduct Your Experiment				
Materials and Equipment			**	**
Probeware Resources			**	**
Background				
Safety	*	*		
Lab Preparation				
Teacher Tips				
Procedure				
Sample Data				
Data Analysis				
Guided Inquiry/Guiding Questions				
Experimental Design				
Assessment Questions: Sample Responses				
Analysis Questions				
Synthesis Questions				
Extended Inquiry Suggestions				

* This section is present if safety considerations are needed.

** These materials, equipment, and probeware resources are recommended, not required.

Teacher Resources Document

This document contains all of the teacher-centered information regarding the lab activity: preparation instructions, sample data, and sample responses to the questions in all student versions of the lab activity. Each *Teacher Resources* document contains the following sections:

- **CONNECTIONS TO THE AP® PHYSICS 1 CURRICULUM** – Every lab is correlated to one or more Learning Objective identified in the AP® Physics 1 curriculum framework from the *AP® Physics 1 and 2 Course and Exam Description*, Effective Fall 2014. This section lists each Big Idea, Enduring Understanding, Essential Knowledge, Learning Objective, and Science Practice applicable to the lab activity.
- **TIME REQUIREMENTS** – Two time frames are defined: the length of time needed for teacher preparation, and the recommended time allotment for students to complete the procedure outlined in the *Structured* version of the lab activity. If there is no specific lab preparation needed, ten minutes is designated to take into account the time required for gathering the materials, making copies of the student handout, and any other normal preparations. Note that more or less time may be required for students to finish data collection when using the *Guided Inquiry* or *Student Designed* versions.
- **PREREQUISITES** – This section details the concepts students should know before doing the activity. Use this section to gauge when to include this activity in lesson plans, in assessing requirements for prior learning, and as an outline for a review or discussion before starting the lab activity.
- **DRIVING QUESTION | OBJECTIVE** – This is the driving question and lab objective that students address when performing their laboratory investigation. This section is the same for all three student handout formats as well as the *Teacher Resources* document.
- **PROCEDURAL OVERVIEW** – This section is a summary of the procedure students follow using the *Structured* version of the lab activity, how they present their data, and the results and conclusions to be drawn from that data.
- **PRE-LAB DISCUSSION AND ACTIVITY** – The pre-lab discussion, activity, or both, are designed to accomplish some or all of the following: engage student attention; access prior knowledge; identify misconceptions; model correct lab technique; model procedures for mathematical computations required in the activity; generate student questions. This section may include pre-lab homework questions that help prepare students to carry out the lab activity.
- **MATERIALS AND EQUIPMENT** – This section lists all student materials and equipment needed per student group to carry out the procedure in the *Structured* version of each lab activity. Items that need to be prepared by the teacher or created using additional materials are called out in the Lab Preparation section of this document. Items included as part of a kit or as part of another product are highlighted with a number connecting them to the information provided in the Probeware Resources section.
- **PROBEWARE RESOURCES** – URLs and QR codes link to equipment-specific videos that outline the functionality, specifications, and different use-cases of most of the hardware and probeware that appear in each activity. Videos are hosted online and are also available on the electronic storage media that comes with the printed manual. The URL links and QR codes in the *Teacher Resources* document are found in all three student handout formats.
- **SAFETY** – This section lists the pertinent safety procedures, if any are needed, for each lab activity beyond a classroom's normal laboratory safety procedures. The Safety section found in the *Teacher Resources* document may include additional safety considerations the teacher should be aware of. This section is also found in the *Structured* version of the lab.
- **LAB PREPARATION** – If applicable, this section includes teacher-directed lab preparation instructions that are either required or suggested to help minimize preparation time.

- **TEACHER TIPS** – Depending on the activity, this section includes any or all of the following: 1) common misconceptions that students have regarding the lab topic; 2) skill requirements for using equipment; 3) difficulties students may encounter executing the lab and how to avoid or correct them; and 4) strategies and techniques for substituting items in the Materials and Equipment list that students may not have access to.
- **SAMPLE DATA** – This section is identical to the Procedure and the Data Analysis sections in the *Structured* version, with the addition of sample sensor data, tables, and graphs, answers to the questions in the Data Analysis section, and sample calculations used to process or manipulate the data.
- **GUIDED INQUIRY QUESTIONS** – This section includes sample responses and teacher-information pertaining to the questions in the Guiding Questions section of the *Guided Inquiry* version of the lab activity.
- **ASSESSMENT QUESTIONS: SAMPLE RESPONSES** – This section includes sample or correct responses to the questions in the Analysis Questions and Synthesis Questions sections in all three versions of the student handouts.
- **EXTENDED INQUIRY SUGGESTIONS** – These suggestions are natural extensions of the activity and can be used for further student inquiry. They include ideas for further experimentation and hands-on exploration, classroom debates, field trips, or research papers.

Structured Format Student Handout

This is the traditional "cookbook" version of each lab activity containing the least amount of student-directed learning. Each *Structured* student handout contains the following sections:

- **DRIVING QUESTION | OBJECTIVE** – Each lab activity begins with a driving question and objective statement on which students will base their investigation of the scientific topic. This section is the same for all three student handout formats.
- **MATERIALS AND EQUIPMENT** – This section lists all student materials and equipment needed per student group to carry out the procedure outlined in the *Structured* version of the lab activity. In this section also are URL and QR code links to equipment-specific videos that outline the functionality, specifications, and different use-cases of the PASCO hardware and probeware used in the activity. Videos are hosted online and are also available on the electronic storage media that comes with the manual. The same Materials and Equipment list, URL links, and QR codes are found in all student handout formats.
- **BACKGROUND** – The Background section appears only in the *Structured* version of each lab activity and contains information related to the scientific topic being investigated. The information frames the activity for students in the context of related curriculum materials. For broader and deeper information on a topic, students should refer to textbooks or other reference materials.
- **SAFETY** – If applicable, this section lists the pertinent safety procedures for each lab activity beyond a classroom's normal laboratory safety procedures. The Safety section found in the *Teacher Resources* document may include additional safety considerations the teacher should be aware of.
- **PROCEDURE** – This section directs the student hands-on portion of each lab activity. Students follow numbered tasks to complete the procedure. Depending on the lab activity, the procedure may be divided into parts. Each part in a lab activity has a Set Up section in which students are given instructions on assembling laboratory equipment, including hardware, sensors (probeware), and data collection systems (see Using Data Collection Technology). Each part also contains a Collect Data section with instructions on how and when to collect data, and where to record data in the Data Analysis section.

- **DATA ANALYSIS** – In this section, students are instructed to analyze and present their data in ways specific to the lab activity, such as completing a data table, making calculations to manipulate or process data, plotting or sketching graphs of data, or identifying key parts of the data plots. In addition, several of the activities in this manual employ a data linearization technique found in AP® Physics 1 and 2 exams.
- **ANALYSIS QUESTIONS** – These questions help students understand their collected data as it pertains to the lab topic and driving question. Students make comparisons, summaries, arguments, and conclusions regarding the scientific concept using their data for verification. This section is the same for all student handout formats.
- **SYNTHESIS QUESTIONS** – These questions help students integrate information and concepts explored in the lab activity with information from other topics using real-world scenarios. Students develop a deeper understanding of concepts as they transfer knowledge learned in the lab to other situations. Some questions may require students to consult available resources, such as textbooks, reference books, resources on the Internet, and local experts. This section is the same for all student handout formats.

Guided Inquiry Format Student Handout

The *Guided Inquiry* format, compared to the traditional version, is a more student-directed approach. The step-by-step Procedure and Data Analysis sections found in the *Structured* format are replaced with a set of guiding questions intended to help students design their own procedure and analysis strategies. The *Guided Inquiry* student handout contains the following sections:

- **DRIVING QUESTION | OBJECTIVE** – Each lab activity begins with a driving question and objective statement on which students will base their investigation of the scientific topic. This section is the same for all student handout formats.
- **DESIGN AND CONDUCT YOUR EXPERIMENT** – Instead of following a step-by-step procedure, students are directed to design an experiment that fulfills the lab objective, and whose data will support their answer to the driving question. Although the same materials and equipment list found in the *Structured* version also appears in this section, it is presented as suggested equipment. Students using the *Guided Inquiry* or *Student Designed* formats will have the freedom to choose any reasonable equipment at their disposal. This section is the same in the *Guided Inquiry* and *Student Designed* formats.
- **GUIDING QUESTIONS** – This section contains a series of questions designed to stimulate inquiry in students that will guide them to determine their experiment design. Although these questions will vary depending on the lab activity, most questions help students: identify variables that will be part of their experiment; define which variables to manipulate and which variables to control; determine how each variable can be measured, how data should be collected and in what order, and how to manipulate, process, and present data to isolate values of interest and identify unknowns. Along with each set of guiding questions, you may also choose to provide students with the *Experiment Design Plan* handout (refer to the Manual Components) to help facilitate the experimental design process.
- **EXPERIMENTAL DESIGN** – This section is divided into the Setup, Procedure, and Collect Data subsections in which students document the experimental setup and procedure they have chosen, and present any data that is part of their experiment. This section is the same in both the *Guided Inquiry* and *Student Designed* formats.
- **ANALYSIS QUESTIONS** – See the corresponding section in the *Structured Format Student Handout* above. This section is the same for all three student handout formats.
- **SYNTHESIS QUESTIONS** – See the corresponding section in the *Structured Format Student Handout* above. This section is the same for all three student handout formats.

Student Designed Format Student Handout

This format is the most student-directed version of a lab activity, containing the least amount of instruction and assistance. Students are responsible for designing and executing their own experimental setup and procedure with little or no guidance from the student handout. Students choose how to present their data in a way that supports their answer to the driving question, while also fulfilling the lab objective.

Each *Student Designed* handout contains the same sections found in the *Guided Inquiry* format except it has no Guiding Questions section.

Conducting Successful Inquiry-Based Lab Activities

Establish the Foundation

Preparing students to conduct their own scientific inquiry activities takes time and intention. Students need a foundation in conceptual development, laboratory bench skills, using electronic data collection and display equipment, and interpreting data. The following strategies help students build this foundation:

- Work with students to complete tutorials for equipment and software they will be using.
- Demonstrate the first few activities using the *Structured* or *Guided Inquiry* formats so you model the correct use of equipment and materials.
- Work with students to complete all sections of several lab activities until they understand your expectations.
- Create teams, giving defined responsibilities to members. (A key behavioral component of a STEM curriculum experience is that students work in teams and successfully solve problems as a team.) Devise a method to track the roles each student carries out, such as a team leader, a recorder, and a technician. Make sure each student has multiple opportunities to perform each role.
- Create opportunities for students to repeat activities that seemed beyond their grasp the first time through—perhaps with student-suggested modifications. You will see substantial improvement as students are given increased opportunities to work with the equipment and analyze the data.

Foster Inquiry Skills

Foster the growth and development of inquiry skills. Provide multiple opportunities for students to work with the equipment, analyze data, and communicate and discuss conclusions. The following strategies support development of laboratory and data analysis skills:

- Model the more complex technical tasks, such as mathematical computations.
- Provide multiple and varied opportunities for practice with hands-on activities using the data collection tools.
- Compile and compare class data whenever possible. Discuss the sources of variation in data and the best interpretation of the data.
- Challenge students to identify applications of the concept just studied.
- Have students brainstorm related questions they would like to explore in their own investigations.

Cultivate Student-Directed Inquiry

At the heart of an effective STEM curriculum is the cultivation of inquiry skills in students. As students complete instructor-directed activities in this manual (using the *Structured* and *Guided Inquiry* formats), their interest may be stimulated regarding one or more issues. Watch for these moments and provide students with assistance for generating their own driving questions and related objectives. For either students' own questions or for those provided in the *Student Designed* format of the lab activities, use the following strategies:

- Require a written plan with procedures. Review these plans and guide students accordingly. Make sure students define projects that are practical under the conditions of your classroom environment.
- Provide plenty of time, material, and equipment resources.
- Incorporate check points to assess progress.
- To guide the students, ask questions such as those in the *Guided Inquiry* version of each lab.

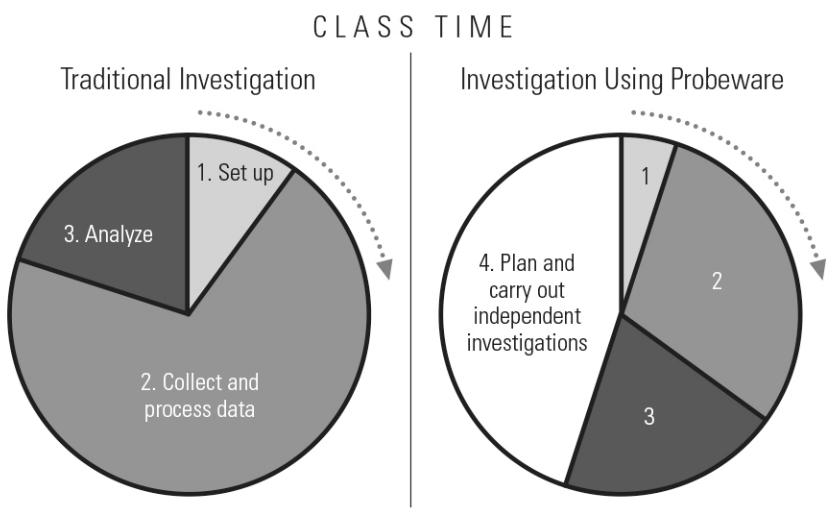
Communicate the Results of Student-Directed Inquiry

Provide opportunities for students to communicate the results of student-directed inquiry. Strategies include:

- Formal research papers, PowerPoint® presentations, video productions, and poster presentations are ways for students to share what they have learned.
- Student-directed inquiries related to community resources may be of interest to area news or conservation groups. Have students report on their findings in a community venue such as the school website or newspaper, local newspapers, or other publications.

Using Data Collection Technology

The use of electronic sensors (probeware) in investigations greatly reduces the class time required for set up and data collection, increases the accuracy of results, allows for richer analysis of data, and provides more time in the classroom for independent investigations.



Additionally, using electronic-sensor data collection, display, and analysis devices allows students to focus not on the tedium of collecting data, but rather on the trends, patterns, and relationships which become immediately discernible when gathering real-time data.

The Data Collection System

In this manual, *data collection system* refers to the system students use to record, visualize, and analyze sensor data during their experiments. The system consists of all components necessary to connect a sensor to a device containing the software that detects the sensor measurement and collects, records, and displays the data.

Some systems, such as the Xplorer GLX® or SPARK Science Learning System™, are stand-alone systems. These contain built-in software applications and students simply attach a sensor and begin collecting data. Other systems use a computer or tablet with downloaded software applications. In these systems, a USB or Bluetooth® interface is used to connect a sensor to the device. Software options for these include SPARKvue and PASCO Capstone software.

The activities are designed so that any PASCO scientific data collection system can be used to carry out the procedures.

Getting Started with Your Data Collection System

To become familiar with the many features of your data collection system, start with the tutorials and instructional videos available in the video library on the PASCO scientific website (www.pasco.com). Also, each system's software has a built-in help system.

There are free SPARKlab™ activities included in the SPARKvue software. Performing one of these activities can be a good starting place for students to familiarize themselves with connecting a sensor, viewing data, saving their work, and other tasks related to probeware use.

PASCO scientific also has a terrific technical and teacher support team. They pride themselves on providing timely and comprehensive help to teachers and students using PASCO scientific products.

Phone: 1-800-772-8700

Email: support@pasco.com

Web: www.pasco.com/support

Inside the Printed Manual

The printed *Advanced Physics 1 through Inquiry* lab manual includes the following documents:

- *Table of Contents*
- *Introduction*
- *Master Materials and Equipment List*
- *Experiment Design Plan* handout
- *Teacher Resources* for each of the lab activities
- *Structured* format student handout for each of the lab activities

Documents *not* printed but available on the accompanying electronic storage device are:

- *Guided Inquiry* format student handout for each of the lab activities
- *Student Designed* format student handout for each of the lab activities

Electronic Materials

The electronic storage device accompanying this manual contains the following:

- Complete *Advanced Physics 1 through Inquiry* manual in PDF format (Acrobat™ compatible)
- Each lab activity's *Teacher Resources* document in PDF format.
- Student handout versions of each laboratory activity in all three formats (*Structured*, *Guided Inquiry*, and *Student Designed*) in an editable Microsoft Word format. PASCO scientific provides editable files of the student lab activities so that teachers can customize activities to their needs.
- Student *Experiment Design Plan* handout in PDF format.
- A complete set of the Probeware Resources Videos used in the *Advanced Physics 1 through Inquiry manual*. Although these videos are hosted online, PASCO scientific provides them in MP4 format for those who may not have a reliable Internet connection, or cannot access videos due to internal system or website restrictions.
- PASCO Capstone and SPARKvue Configuration Files for every lab activity.

AP® Physics 1 Correlations

Below is a list of the 15 lab activities in the *Advanced Physics 1 through Inquiry* lab manual. In the rightmost column of the table are the correlations for each lab activity to the AP® Concept Outline found in the *AP Physics 1 and 2 Course and Exam Description*, Effective Fall 2014 curriculum framework published by the College Board®.

Each reference number indicates the Big Idea, Enduring Understanding, Essential Knowledge, and Learning Objective to which the activity is correlated. For example "3.B.1.2" indicates that the activity is correlated to Learning Objective 2 found within the first Essential Knowledge statement, which is within Enduring Understanding B, and in turn part of Big Idea 3. Shown in brackets next to the title of each lab are the applicable Science Practices identified by the College Board® based on each correlated Learning Objective.

AP® Physics 1 correlations to the activities in this manual

Activity Number	Activity Name and Description	AP® Physics 1 Correlations
1	Graphical Analysis: Motion [1.5, 2.1, 2.2, 4.2, 5.1] *Students use a motion sensor to measure the position and velocity of a cart on a track to determine the graphical relationship between position, velocity, and acceleration versus time graphs.*	3.A.1.1 3.A.1.2 3.A.1.3
2	Newton's Second Law [1.1, 1.4, 1.5, 2.2, 4.2, 5.1, 6.4, 7.2] *Students use a motion sensor to determine the relationship between a system's mass, acceleration, and the net force being applied to the system.*	3.B.1.1 3.B.1.2 3.B.1.3 3.B.2.1
3	Atwood's Machine [4.2, 5.1, 6.4, 7.2] *Students use a photogate and pulley system to determine the mathematical relationship between the acceleration of an Atwood's machine, the difference between its two masses, and the sum of those two masses.*	3.B.1.1 3.B.1.2
4	Coefficients of Friction [6.1, 6.2] *Students use a motion sensor and a force sensor to determine the static and kinetic friction coefficients between two contacting surfaces.*	3.C.4.1 3.C.4.2

Activity Number	Activity Name and Description	AP® Physics 1 Correlations
5	Two Dimensional Motion: Projectiles [1.4, 2.2] *Students use a photogate and mini launcher to measure the variables that affect the two-dimensional motion of a projectile launched horizontally, and then use those variables to accurately predict and test the projectile's horizontal range.*	3.E.1.3 3.E.1.4
6	Conservation of Mechanical Energy [1.4, 2.1, 2.2, 6.4, 7.2] *Students use a photogate and dynamics system to explore how a cart's kinetic energy, gravitational potential energy, and total mechanical energy changes as it rolls down an inclined track.*	5.B.4.1 5.B.4.2
7	Work and Kinetic Energy [1.4, 2.2, 6.4, 7.2] *Students use a photogate and dynamics system to investigate the relationship between the change in kinetic energy of an object experiencing a non zero net conservative force and the work done by that net force on the object, and then use their data to establish a measurement-based relationship between work and kinetic energy.*	4.C.2.1 4.C.2.2
8	Conservation of Momentum [2.1, 2.2, 4.1, 4.2, 4.4, 5.1, 5.3] *Students use a motion sensor and a dynamics system to demonstrate that linear momentum and kinetic energy are conserved in an elastic collision, and linear momentum is conserved but kinetic energy is not conserved in an inelastic collision.*	5.D.1.3 5.D.2.2 5.D.2.4
9	Momentum and Impulse [4.2, 5.1] *Students use a motion sensor, force sensor, and dynamics system to investigate the relationship between the change in momentum of a cart undergoing a collision and the impulse imparted to the cart to change its momentum, and then use their data to establish a measurement-based relationship between change in momentum and impulse.*	3.D.2.3 3.D.2.4
10	Rotational Dynamics [4.1, 4.2, 5.1, 6.4] *Students use a rotary motion sensor to determine the mathematical relationship between torque, rotational inertia, and angular acceleration of a rotating object.*	3.F.2.1 3.F.2.2 3.A.1.3
11	Rotational Statics [1.4, 2.2, 2.3, 4.1, 4.2, 5.1] *Students use a force sensor and tension protractor to demonstrate that the sum of the forces acting on an object in static translational equilibrium is equal to zero, and the sum of the torques acting on an object in static rotational equilibrium is equal to zero.*	3.F.1.1 3.F.1.2 3.F.1.3 3.F.1.4 3.F.1.5
12	Periodic Motion: Mass and Spring [2.2, 4.2, 5.1, 6.2, 6.4, 7.2] *Students use a motion sensor to determine the physical properties of a hanging mass and spring system that affect its period of oscillation, and then use their data to support a mathematical model relating period, mass, and spring constant.*	3.B.3.1 3.B.3.2 3.B.3.3 3.B.3.4
13	Simple Pendulum [2.2, 4.2, 5.1, 6.4, 7.2] *Students use a photogate and pendulum to determine the physical properties of a simple pendulum that affect its period, and then use their data to support a mathematical model relating period to pendulum arm length.*	3.B.3.1 3.B.3.2 3.B.3.3
14	Resonance and Standing Waves [1.2, 1.5, 2.2, 6.1] *Students use a resonance air column, tuning forks, and the principles of resonance and standing waves for a pipe with one closed end to experimentally determine a value for the speed of sound in air.*	6.D.3.4 6.D.4.1 6.D.4.2
15	DC Circuits [2.2, 4.1, 4.2, 5.1, 6.4, 7.2] *Students use a voltage–current sensor and an AC/DC electronics laboratory to construct simple resistor circuits with resistors in series or in parallel, or both (with at most one parallel loop of resistors), to demonstrate the validity of Kirchhoff's loop rule (conservation of energy), and Kirchhoff's junction rule (conservation of charge).*	1.B.1.2 5.B.9.2 5.B.9.3 5.C.3.1

International Baccalaureate Organization (IBO) Support

The International Baccalaureate Organization (IBO) uses a specific science curriculum model that includes both theory and practical investigative work. While this lab guide was not produced by the IBO and does not include references to the IB internal assessment rubrics, the lab activities can be adapted easily to the IB classroom. The labs in this manual correlate closely to core and optional topics of the IB Physics standard level and higher level programs: mechanics, oscillations and waves, electric currents, forces, energy, and others. These correlations are listed in the table below.

By the end of the IB Diploma Program, students are expected to have completed a set number of practical investigative hours and are assessed using the specified internal assessment criteria. Students should be able to design a lab based on an original idea, carry out the procedure, draw conclusions, and evaluate their results. These scientific processes require an understanding of laboratory techniques and equipment as well as a high level of thinking, skills that are developed and sharpened by completing the investigations in this manual.

IB Physics core and optional topic correlations to the activities in this manual

Activity Number	Activity Name and Description	IB Physics Correlations
1	Graphical Analysis: Motion *Students use a motion sensor to measure the position and velocity of a cart on a track to determine the graphical relationship between position, velocity, and acceleration versus time graphs.*	2.1
2	Newton's Second Law *Students use a motion sensor to determine the relationship between a system's mass, acceleration, and the net force being applied to the system.*	2.2
3	Atwood's Machine *Students use a photogate and pulley system to determine the mathematical relationship between the acceleration of an Atwood's machine, the difference between its two masses, and the sum of those two masses.*	2.2
4	Coefficients of Friction *Students use a motion sensor and a force sensor to determine the static and kinetic friction coefficients between two contacting surfaces.*	2.2
5	Two Dimensional Motion: Projectiles *Students use a photogate and mini launcher to measure the variables that affect the two-dimensional motion of a projectile launched horizontally, and then use those variables to accurately predict and test the projectile's horizontal range.*	1.3 2.1
6	Conservation of Mechanical Energy *Students use a photogate and dynamics system to explore how a cart's kinetic energy, gravitational potential energy, and total mechanical energy changes as it rolls down an inclined track.*	2.3
7	Work and Kinetic Energy *Students use a photogate and dynamics system to investigate the relationship between the change in kinetic energy of an object experiencing a non-zero net conservative force and the work done by that net force on the object, and then use their data to establish a measurement-based relationship between work and kinetic energy.*	2.3
8	Conservation of Momentum *Students use a motion sensor and a dynamics system to demonstrate that linear momentum and kinetic energy are conserved in an elastic collision, and linear momentum is conserved but kinetic energy is not conserved in an inelastic collision.*	2.4

Activity Number	Activity Name and Description	IB Physics Correlations
9	Momentum and Impulse *Students use a motion sensor, force sensor, and dynamics system to investigate the relationship between the change in momentum of a cart undergoing a collision and the impulse imparted to the cart to change its momentum, and then use their data to establish a measurement-based relationship between change in momentum and impulse.*	2.4
10	Rotational Dynamics *Students use a rotary motion sensor to determine the mathematical relationship between torque, rotational inertia, and angular acceleration of a rotating object.*	B.1
11	Rotational Statics *Students use a force sensor and tension protractor to demonstrate that the sum of the forces acting on an object in static translational equilibrium is equal to zero, and the sum of the torques acting on an object in static rotational equilibrium is equal to zero.*	B.1
12	Periodic Motion: Mass and Spring *Students use a motion sensor to determine the physical properties of a hanging mass and spring system that affect its period of oscillation, and then use their data to support a mathematical model relating period, mass, and spring constant.*	4.1 9.1
13	Simple Pendulum *Students use a photogate and pendulum to determine the physical properties of a simple pendulum that affect its period, and then use their data to support a mathematical model relating period to pendulum arm length.*	4.1 9.1
14	Resonance and Standing Waves *Students use a resonance air column, tuning forks, and the principles of resonance and standing waves for a pipe with one closed end to experimentally determine a value for the speed of sound in air.*	B.4
15	DC Circuits *Students use a voltage–current sensor and an AC/DC electronics laboratory to construct simple resistor circuits with resistors in series or in parallel, or both (with at most one parallel loop of resistors), to demonstrate the validity of Kirchhoff's loop rule (conservation of energy), and Kirchhoff's junction rule (conservation of charge).*	4.1 4.2 4.5

MASTER MATERIALS AND EQUIPMENT LIST

This Master Materials and Equipment List shows the equipment required to perform the *Structured* version of each lab activity from the *Advanced Physics 1 through Inquiry* lab manual. Italicized entries indicate items not available from PASCO. The quantity indicated is per student or group.

Teachers can conduct some lab activities with sensors and probes other than those listed here. For assistance with substituting compatible sensors and probes for a lab activity, contact PASCO Teacher Support (800-772-8700 inside the United States or http://www.pasco.com/support).

Lab	Title	Materials and Equipment	PASCO Part Number	Qty
1	GRAPHICAL ANALYSIS: MOTION Students use a motion sensor to measure the position and velocity of a cart on a track to determine the graphical relationship between position, velocity, and acceleration versus time graphs.	**FOR EACH STUDENT STATION** Data Collection System PASPORT Motion Sensor PASCO PAStrack PASCO PAScar Four Scale Meter Stick *Thick Text Book*	 PS-2103A ME-6960 ME-6950 SE-8695	 1 1 1 1 1 1
2	NEWTON'S SECOND LAW Students use a motion sensor to determine the relationship between a system's mass, acceleration, and the net force being applied to the system.	**FOR EACH STUDENT STATION** Data Collection System PASPORT Motion Sensor PASCO PAStrack PASCO PAScar PASCO Dynamics Track End Stop PASCO Super Pulley with Clamp* PASCO 250-g Compact Cart Mass PASCO Mass and Hanger Set Thread **FOR THE ENTIRE CLASS** Ohaus Scout Pro Balance 2,000-g	 PS-2103A ME-6960 ME-6950 ME-8971 w/ME-9433 ME-6755 ME-8979 ME-9875 SE-8757A	 1 1 1 1 1 1 2 1 1 m 1
3	ATWOOD'S MACHINE Students use a photogate and pulley system to determine the mathematical relationship between the acceleration of an Atwood's machine, the difference between its two masses, and the sum of those two masses.	**FOR EACH STUDENT STATION** Data Collection System PASCO Smart Gate PASCO Super Pulley with Mounting Rod* PASCO Mass and Hanger Set PASCO Aluminum Table Clamp 60-cm Stainless Steel Rod Right Angle Clamp Thread *Scissors*	 PS-2180 w/ME-9433 ME-8979 ME-8995 ME-8977 SE-9444 ME-9875	 1 1 1 1 1 1 1 1 m 1

Lab	Title	Materials and Equipment	PASCO Part Number	Qty
4	COEFFICIENTS OF FRICTION	**FOR EACH STUDENT STATION**		
	Students use a motion sensor and a force sensor to determine the static and kinetic friction coefficients between two contacting surfaces.	Data Collection System		1
		PASPORT Motion Sensor	PS-2103A	1
		PASPORT High Resolution Force Sensor w/hook	PS-2189	1
		PASCO Discover Friction Accessory tray	ME-8574	1
		PASCO 250-g Cart Mass*	w/ME-6950	5
		Thread	ME-9875	1 m
		FOR THE ENTIRE CLASS		
		Ohaus Scout Pro Balance 2,000-g	SE-8757A	1
5	TWO DIMENSIONAL MOTION: PROJECTILES	**FOR EACH STUDENT STATION**		
	Students use a photogate and mini launcher to measure the variables that affect the two-dimensional motion of a projectile launched horizontally, and then use those variables to accurately predict and test the projectile's horizontal range.	Data Collection System		1
		PASCO Smart Gate	PS-2180	1
		PASCO Photogate Mounting Bracket	ME-6821A	1
		PASCO Mini Launcher w/bracket	ME-6825A	1
		Mini launcher loading rod*	w/ME-6825A	1
		Steel ball, 1.6-cm diameter*	w/ME-6825A	1
		Large C Clamp	SE-7285	1
		Four Scale Meter Stick	SE-8695	1
		Carbon Paper	SE-8693	3 sheets
		White Paper, sheet		1 sheet
		Cardboard, 10"x10" Square		1
6	CONSERVATION OF MECHANICAL ENERGY	**FOR EACH STUDENT STATION**		
	Students use a photogate and dynamics system to explore how a cart's kinetic energy, gravitational potential energy, and total mechanical energy changes as it rolls down an inclined track.	Data Collection System		1
		PASCO Smart Gate	PS-2180	1
		PASCO Photogate Bracket-IDS	ME-9806	1
		PASCO PAStrack	ME-6960	1
		PASCO PAScar	ME-6950	1
		PASCO Dynamics Track Rod Clamp	ME-9836	1
		PASCO Cart Picket Fence-IDS	ME-9804	1
		PASCO Angle Indicator	ME-9495A	1
		PASCO Dynamics Track End Stop	ME-8971	1
		PASCO Aluminum Table Clamp	ME-8995	1
		Rod, 45-cm	ME-8736	1
		Four Scale Meter Stick	SE-8695	1
		FOR THE ENTIRE CLASS		
		Ohaus Scout Pro Balance 2,000-g	SE-8757A	1

Lab	Title	Materials and Equipment	PASCO Part Number	Qty
7	WORK AND KINETIC ENERGY	**FOR EACH STUDENT STATION**		
	Students use a photogate and dynamics system to investigate the relationship between the change in kinetic energy of an object experiencing a non zero net conservative force and the work done by that net force on the object, and then use their data to establish a measurement-based relationship between work and kinetic energy.	Data Collection System		1
		PASCO Smart Gate	PS-2180	1
		PASCO Photogate Bracket-IDS	ME-9806	1
		PASCO PAStrack	ME-6960	1
		PASCO PAScar	ME-6950	1
		PASCO Dynamics Track Rod Clamp	ME-9836	1
		PASCO Cart Picket Fence-IDS	ME-9804	1
		PASCO Angle Indicator	ME-9495A	1
		PASCO Dynamics Track End Stop	ME-8971	1
		PASCO Aluminum Table Clamp	ME-8995	1
		Rod, 45-cm	ME-8736	1
		Four Scale Meter Stick	SE-8695	1
		FOR THE ENTIRE CLASS		
		Ohaus Scout Pro Balance 2,000-g	SE-8757A	1
8	CONSERVATION OF MOMENTUM	**FOR EACH STUDENT STATION**		
	Students use a motion sensor and a dynamics system to demonstrate that linear momentum and kinetic energy are conserved in an elastic collision, and linear momentum is conserved but kinetic energy is not conserved in an inelastic collision.	Data Collection System		1
		PASPORT Motion Sensor	PS-2103A	2
		PASCO PAStrack	ME-6960	1
		PASCO PAScar	ME-6950	2
		PASCO 250-g Cart Mass*	w/ME-6950	2
		FOR THE ENTIRE CLASS		
		Ohaus Scout Pro Balance 2,000-g	SE-8757A	1
9	MOMENTUM AND IMPULSE	**FOR EACH STUDENT STATION**		
	Students use a motion sensor, force sensor, and dynamics system to investigate the relationship between the change in momentum of a cart undergoing a collision and the impulse imparted to the cart to change its momentum, and then use their data to establish a measurement-based relationship between change in momentum and impulse.	Data Collection System		1
		PASPORT Motion Sensor	PS-2103A	1
		PASPORT High Resolution Force Sensor	PS-2189	1
		PASCO PAScar	ME-6950	1
		PASCO PAStrack	ME-6960	1
		PASCO Dynamics Track Rod Clamp	ME-9836	1
		PASCO Discover Collision Bracket	ME-8973	1
		PASCO Aluminum Table Clamp	ME-8995	1
		Rod, 45-cm	ME-8736	1
		FOR THE ENTIRE CLASS		
		Ohaus Scout Pro Balance 2,000-g	SE-8757A	1

Lab	Title	Materials and Equipment	PASCO Part Number	Qty
10	ROTATIONAL DYNAMICS Students use a rotary motion sensor to determine the mathematical relationship between torque, rotational inertia, and angular acceleration of a rotating object.	**FOR EACH STUDENT STATION**		
		Data Collection System		1
		PASPORT Rotary Motion Sensor	PS-2120	1
		PASCO Pendulum Accessory	ME-8969	1
		PASCO Super Pulley with Clamp*	w/ME-9433	1
		PASCO Mass and Hanger Set	ME-8979	1
		PASCO Aluminum Table Clamp	ME-8995	1
		60-cm Stainless Steel Rod	ME-8977	1
		Four Scale Meter Stick	SE-8695	1
		Thread	ME-9875	2 m
		Stainless Steel Calipers	SF-8711	1
		Scissors		1
		FOR THE ENTIRE CLASS		
		Ohaus Scout Pro Balance 2,000-g	SE-8757A	1
11	ROTATIONAL STATICS Students use a force sensor and tension protractor to demonstrate that the sum of the forces acting on an object in static translational equilibrium is equal to zero, and the sum of the torques acting on an object in static rotational equilibrium is equal to zero.	**FOR EACH STUDENT STATION**		
		Data Collection System		1
		PASPORT High Resolution Force Sensor w/rubber bumper	PS-2189	1
		PASCO Tension Protractor	ME-6855	2
		PASCO Aluminum Table Clamp	ME-8995	2
		90-cm Stainless Steel Rod	ME-8738	1
		60-cm Stainless Steel Rod	ME-8977	2
		Right Angle Clamp	SE-9444	2
		Hooked Mass Set	SE-8759	1
		Four Scale Meter Stick	SE-8695	1
		Thread	ME-9875	2 m
		Tape		1 roll
		AA-cell Battery or similar cylindrical object		1
		Scissors		
12	PERIODIC MOTION: MASS AND SPRING Students use a motion sensor to determine the physical properties of a hanging mass and spring system that affect its period of oscillation, and then use their data to support a mathematical model relating period, mass, and spring constant.	**FOR EACH STUDENT STATION**		
		Data Collection System		1
		PASPORT Motion Sensor	PS-2103A	1
		PASCO Demonstration Spring Set	ME-9866	1
		PASCO Aluminum Table Clamp	ME-8995	1
		90-cm Stainless Steel Rod	ME-8738	1
		Rod, 45-cm	ME-8736	1
		Right Angle Clamp	SE-9444	1
		Hooked Mass Set	SE-8759	1
		Four Scale Meter Stick	SE-8695	1
		Tape		1 roll

Lab	Title	Materials and Equipment	PASCO Part Number	Qty
13	SIMPLE PENDULUM Students use a photogate and pendulum to determine the physical properties of a simple pendulum that affect its period, and then use their data to support a mathematical model relating period to pendulum arm length.	**FOR EACH STUDENT STATION** Data Collection System PASCO Smart Gate PASCO Photogate Pendulum Set PASCO Pendulum Clamp PASCO Aluminum Table Clamp 90-cm Stainless Steel Rod Four Scale Meter Stick Thread *Scissors* **FOR THE ENTIRE CLASS** Ohaus Scout Pro Balance 2,000-g	 PS-2180 ME-8752 ME-9506 ME-8995 ME-8738 SE-8695 ME-9875 SE-8757A	 1 1 1 1 1 1 1 2 m 1 1
14	RESONANCE AND STANDING WAVES Students use a resonance air column, tuning forks, and the principles of resonance and standing waves for a pipe with one closed end to experimentally determine a value for the speed of sound in air.	**FOR EACH STUDENT STATION** PASCO Resonance Air Column Tuning Fork Set Four Scale Meter Stick	 WA-9606 SE-7342 SE-8695	 1 1 1
15	DC CIRCUITS Students use a voltage–current sensor and an AC/DC electronics laboratory to construct simple resistor circuits with resistors in series or in parallel, or both (with at most one parallel loop of resistors), to demonstrate the validity of Kirchhoff's loop rule (conservation of energy), and Kirchhoff's junction rule (conservation of charge).	**FOR EACH STUDENT STATION** Data Collection System PASPORT Voltage Current sensor PASCO AC/DC Electronics Lab Kit 4-mm Banana Plug Patch Cord* 4-mm Banana Plug Alligator Clip* Resistor, 4.7-Ω* Resistor, 33-Ω* Resistor, 10-Ω* *D-cell Battery*	 PS-2115 EM-8656 w/PS-2115 w/PS-2115 w/EM-8656 w/EM-8656 w/EM-8656	 1 1 1 2 4 1 1 1 1

* These items are included with the specific kit, apparatus, or sensor used in the experiment.

ACTIVITY BY PASCO ITEM

This table indicates which lab activities use the PASCO scientific sensors or special equipment listed. The quantities shown indicate the number of each item required to complete all the activities that require the specified item.

Items Available from PASCO	PASCO Part Number	Qty	Activity Where Used
PASCO SENSORS			
PASPORT High Resolution Force Sensor w/hook	PS-2189	1	4, 9, 11
PASPORT Motion Sensor	PS-2103A	2	1, 2, 4, 8, 9, 12
PASPORT Rotary Motion Sensor	PS-2120	1	10
PASCO Smart Gate	PS-2180	1	3, 5, 6, 7, 13
PASPORT Voltage Current sensor	PS-2115	1	15
PASCO LABWARE			
PASCO 250-g Cart Mass*	w/ME-6950	5	4, 8
PASCO 250-g Compact Cart Mass	ME-6755	2	2
PASCO AC/DC Electronics Lab Kit	EM-8656	1	15
PASCO Angle Indicator	ME-9495A	1	6, 7
PASCO Aluminum Table Clamp	ME-8995	2	3, 6, 7, 9, 10, 11, 12, 13
PASCO Cart Picket Fence-IDS	ME-9804	1	6, 7
PASCO Demonstration Spring Set	ME-9866	1	12
PASCO Discover Collision Bracket	ME-8973	1	9
PASCO Discover Friction Accessory tray	ME-8574	1	4
PASCO Dynamics Track End Stop	ME-8971	1	2, 6, 7
PASCO Mass and Hanger Set	ME-8979	1	2, 3, 10
PASCO Mini Launcher w/bracket	ME-6825A	1	5
PASCO PAScar	ME-6950	2	1, 2, 6, 7, 8, 9
PASCO PAStrack	ME-6960	1	1, 2, 6, 7, 8, 9
PASCO Pendulum Accessory	ME-8969	1	10
PASCO Pendulum Clamp	ME-9506	1	13
PASCO Photogate Bracket-IDS	ME-9806	1	6, 7
PASCO Photogate Mounting Bracket	ME-6821A	1	5
PASCO Photogate Pendulum Set	ME-8752	1	13
PASCO Dynamics Track Rod Clamp	ME-9836	1	6, 7, 9
PASCO Resonance Air Column	WA-9606	1	14
PASCO Super Pulley Kit	ME-9433	1	2, 3, 10
PASCO Tension Protractor	ME-6855	2	11
OTHER LABWARE			
45-cm Rod	ME-8736	1	6, 7, 9, 11, 12
60-cm Stainless Steel Rod	ME-8977	2	3, 10, 11
90-cm Stainless Steel Rod	ME-8738	1	11, 12, 13
Carbon Paper	SE-8693	3 sheets	5

Items Available from PASCO	PASCO Part Number	Qty	Activity Where Used
Four Scale Meter Stick	SE-8695	1	1, 5, 6, 7, 10, 11, 12, 13, 14
Hooked Mass Set	SE-8759	1	11, 12
Large C Clamp	SE-7285	1	5
Ohaus Scout Pro Balance 2,000-g	SE-8757A	1	2, 4, 6, 7, 8, 9, 10, 13
Right Angle Clamp	SE-9444	2	3, 11, 12
Stainless Steel Calipers	SF-8711	1	10
Thread	ME-9875	9 m	2, 3, 4, 10, 11
Tuning Fork Set	SE-7342	1	14

* These items are included with the specific kit, apparatus, or other sensor.

EXPERIMENT DESIGN PLAN HANDOUT

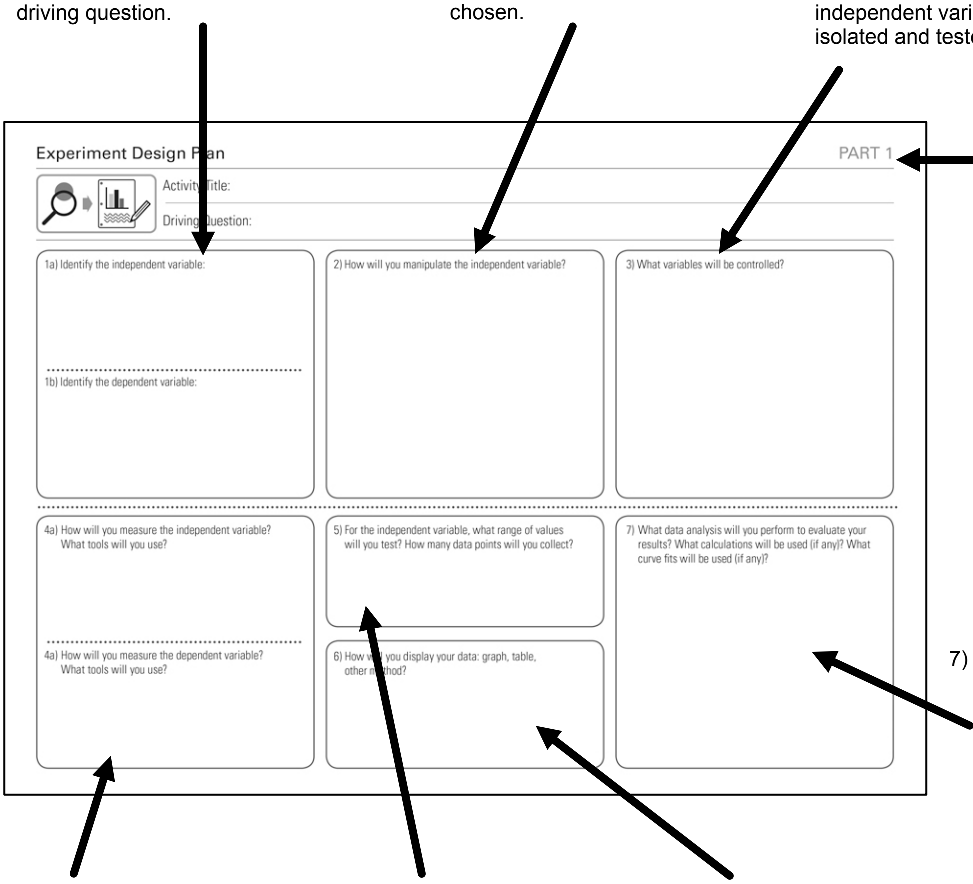
Students following the *Guided Inquiry* or *Student Designed* version of a lab activity can use the one-sheet handout on the following page to help design and implement their inquiry-based investigation. The handout provides students with a small amount of guidance and structure as they develop their own laboratory investigation, regardless of the lab topic. The different sections and their roles are described below:

Components of the Handout

1) Students first identify the independent and dependent variables in their proposed experiment, based on the driving question.

2) Students then determine how to physically manipulate the independent variable they've chosen.

3) Students identify other variables in their proposed experiment that must be controlled so the independent variable can be isolated and tested.



The same content is repeated on the document's backside for students whose proposed experiment requires more than one configuration.

7) Students determine how they will analyze their data: Will there be any additional data manipulation (this includes linearization), calculations, curve fits, smoothing, et cetera.

4) Students determine how they will measure the independent and dependent variables in their proposed experiment, what sensors they will use, and what steps they will take to make the measurements.

5) Students determine the values for testing their independent variable. Results of too small of a range of values may prove inconclusive; using too large of a range may exceed the capabilities of their measurement tools.

6) Students then determine how best to display the data (such as a graph or table) they will collect in their proposed experiment. Students who choose to plot graphs should identify which variable to plot on the *x*-axis, and which to plot on the *y*-axis.

Experiment Design Plan

Activity Title:

Driving Question:

1a) Identify the independent variable:

1b) Identify the dependent variable:

2) How will you manipulate the independent variable?

3) What variables will be controlled?

4a) How will you measure the independent variable? What tools will you use?

4a) How will you measure the dependent variable? What tools will you use?

5) For the independent variable, what range of values will you test? How many data points will you collect?

6) How will you display your data: graph, table, other method?

7) What data analysis will you perform to evaluate your results? What calculations will be used (if any)? What curve fits will be used (if any)?

Experiment Design Plan

1a) Identify the independent variable:

1b) Identify the dependent variable:

2) How will you manipulate the independent variable?

3) What variables will be controlled?

4a) How will you measure the independent variable? What tools will you use?

4a) How will you measure the dependent variable? What tools will you use?

5) For the independent variable, what range of values will you test? How many data points will you collect?

6) How will you display your data: graph, table, other method?

7) What data analysis will you perform to evaluate your results? What calculations will be used (if any)? What curve fits will be used (if any)?

1. GRAPHICAL ANALYSIS: MOTION

Connections to the AP® Physics 1 Curriculum*

The lab activity correlates to the following pieces of the AP® Physics 1 framework:

Big Idea	3	Enduring Understanding	A	Essential Knowledge	1

Learning Objective 1: The student is able to express the motion of an object using narrative, mathematical, and graphical representations.
Science Practices: 1.5, 2.1, 2.2

Learning Objective 2: The student is able to design an experimental investigation of the motion of an object.
Science Practices: 4.2

Learning Objective 3: The student is able to analyze experimental data describing the motion of an object and is able to express the results of the analysis using narrative, mathematical, and graphical representations.
Science Practices: 5.1

Time Requirement

Preparation Time: 10 minutes

Lab Activity: 50 minutes

Prerequisites

Students should be familiar with the following concepts:

- Displacement, position, velocity, and acceleration, including specific definitions of average velocity, average acceleration, instantaneous velocity, and instantaneous acceleration.

Driving Question | Objective

How are the graphs of position versus time, velocity versus time, and acceleration versus time of an object undergoing constant acceleration related? Experimentally determine the relationships between the graphs of position, velocity, and acceleration versus time for an object undergoing constant acceleration, both positive and negative.

Procedural Overview

The Structured version of this lab activity is divided into three parts:

Part 1 – Students use a motion sensor to measure the position and velocity of a cart on a level track as they push the cart down the track with constant velocity.

Part 2 – Students use a motion sensor to measure the position and velocity of a cart on an inclined track as they release the cart, which travels down the track with constant positive acceleration.

Part 3 – Students use a motion sensor to measure the position and velocity of a cart on an inclined track as they push the cart up the track with constant negative acceleration.

Based on their data from all three parts, students establish a graphical relationship between the curves of position, velocity, and acceleration versus time graphs for a cart in motion. Students are then given the shape of one of the motion graphs and using this relationship, they predict the shape of the curves of the other two graphs.

Pre-Lab Discussion and Activity

This is a pre-lab suggestion and activity.

Students should already be comfortable with position, velocity, and acceleration, and should have some understanding of how to graph the motion of an object as a function of time if given data. A simple discussion of content similar to that in the Background section of the Structured lab will help students understand the concepts covered in this lab.

Position, Velocity, and Acceleration Graphs

A brief review of constant velocity and acceleration will help students carry out this lab. Linking the position versus time graph to the velocity versus time graph via the slope is a critical step. You may want to examine the slope at each of three points on the position versus time graph and show that the velocity at each of those points on the velocity versus time graph is exactly equal to the slope at the corresponding time. Similarly, the slope of the velocity versus time graph equals the acceleration on the acceleration versus time graph at each point in time. You can do this for each type of motion (constant velocity, and positive and negative acceleration) if you feel it is necessary.

Finally, point out that the area under the curve of a graph often has significance. Start with the area under the curve of a position versus time graph. Whatever the numerical value of this area, the units would be meters · seconds. Is this unit meaningful? Probably not. Then look at the area under the curve of a velocity versus time graph. Ignore the numerical value, but look at the units: meters/second · seconds = meters. What is measured in meters? Distance traveled. Hence, students should see that the area under the velocity versus time graph equals the distance traveled after time t. Be careful to point out that not all objects start moving from the origin. What if the object started 10 meters away from the origin (or the "starting line")? Then the area under the velocity versus time graph would equal the distance traveled from that point, so the total distance from the origin would equal the area + 10 meters.

Continue this discussion by asking students if the area under the acceleration versus time graph has significance. Since $(m/s^2) \cdot (s) = m/s$, students should conclude that this equals the change in the object's velocity over that interval. Again, point out that acceleration is the change in velocity, so any initial nonzero velocity must be considered (such conditions are often referred to as *boundary conditions*).

Negative and Positive Areas under a Curve

Finally, discuss whether areas under the curve of a graph matter if the curve goes below the x-axis. Do areas cancel? Should a *negative* area be subtracted from a *positive* area? Students should quickly reach the conclusion that the signs do matter. The teacher can provide several simple examples showing that a constant positive velocity for 4 seconds followed by an equal but negative constant velocity for 4 seconds will return the moving object to its original position. This may help illustrate that the signs of the area do matter and must be considered in any calculations.

PRE-LAB QUESTIONS

1. How can you use a cart on a track to demonstrate each of the three types of basic motion: constant velocity, positive acceleration, and negative acceleration? Assume the track can be moved if needed.

   Constant velocity: give the cart a gentle push and watch it roll smoothly with very little change in speed.

   Positive Acceleration: tilt the track with the motion sensor at the top and let the cart accelerate down the ramp.

   Negative Acceleration: put the sensor at the bottom of the track, and push the cart uphill away from the sensor, decelerating as the cart moves toward the top of the track.

2. Describe what you expect the shape of the position versus time graph will look like for each of the three types of motion in this lab.

   When the cart moves at a constant velocity, the curve should be a straight line with a constant slope. When the cart moves with positive acceleration, the curve should be a line with an increasing slope. With negative acceleration, the result should be a curved line of decreasing slope.

3. Describe what you expect the shape of the velocity versus time graph will look like for each of the three types of motion in this lab.

   A graph of constant velocity should be a straight line of zero slope at the exact speed of the cart (line may actually slope slightly downward due to friction). A graph showing positive acceleration should be a straight line with a positive slope as the cart accelerates downhill. With negative acceleration, the graph should show a straight line with a negative slope, since the cart is decelerating as it moves up the hill.

Materials and Equipment

- Data collection system
- PASCO Motion Sensor[1]
- PASCO Dynamics track[2]
- PASCO Dynamics cart[3]
- Meter stick
- Thick text book

Probeware Resources

Below are web-link and QR codes that will direct you to instructional video resources for individual pieces of PASCO probeware, sensors, and other hardware used in the lab activity. These same links and codes are provided to students in their activity handouts.

[1]www.pasco.com/ap18 — PASCO Motion Sensor

[2]www.pasco.com/ap08 — PASCO PAStrack

[3]www.pasco.com/ap07 — PASCO PAScar

Teacher Tips

Tip 1 – Common Misconceptions

- While it may not apply directly to this lab, further discussion could include negative velocities and accelerations. Although negative velocities are simply motion in a negative direction, a negative acceleration should be handled carefully: deceleration can indicate slowing down while moving in a positive direction, but it can also indicate increasing velocity in a negative direction.

Tip 2 – Aiming the Motion Sensor

- Be sure motion sensors are positioned to read in a 1-dimensional path.

Tip 3 – Difficulties Students May Encounter

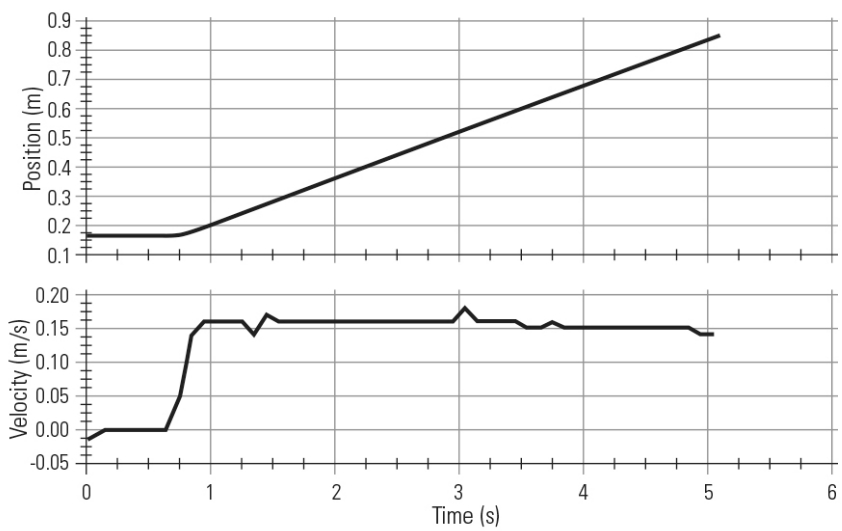
- The motion sensor can sometimes return noisy data if the target passes in and out of the beam path, so the experiment may need to be repeated after adjusting the sensor. Be sure students understand what graphs look like if the motion sensor signal is missing its target.

Sample Data

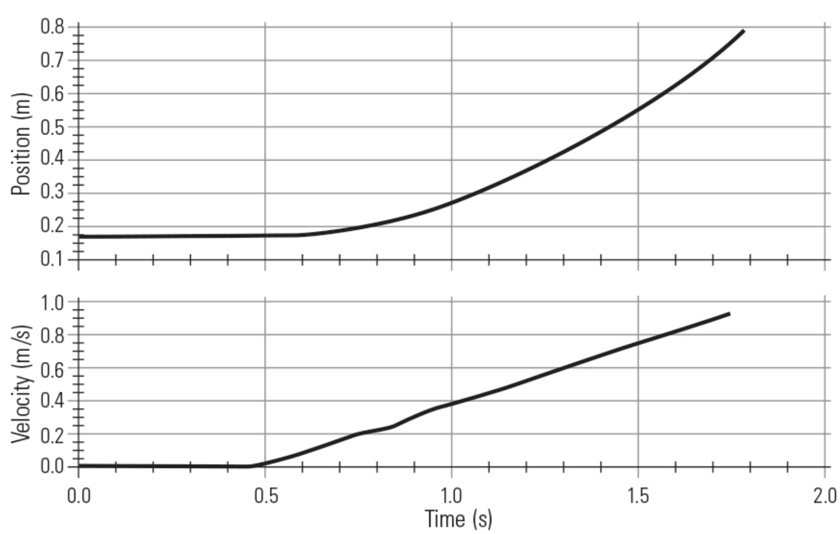
Below are sample data, acquired using the experimental setup and procedure outlined in the Structured version of the lab activity, and answers to questions in the Data Analysis section.

Data Analysis

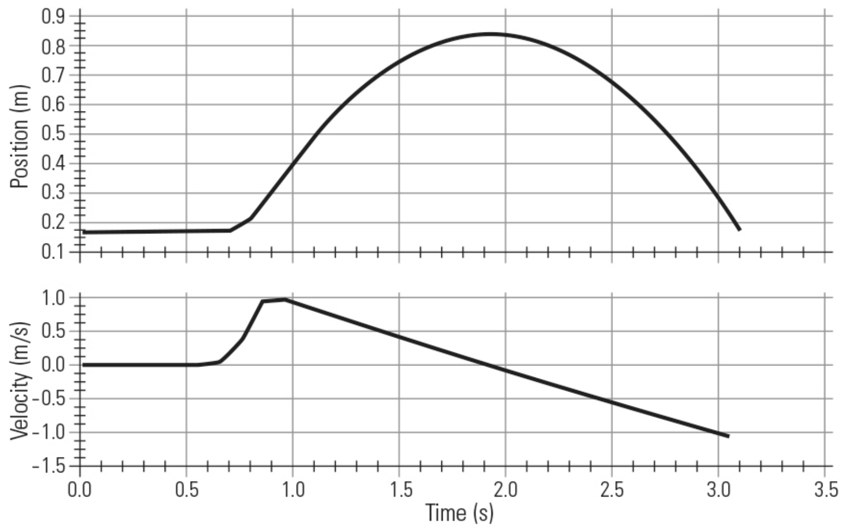
Student Part 1 graphs of position versus time and velocity versus time while pushing the cart down the track with constant velocity will look similar to:



Student Part 2 graphs of position versus time and velocity versus time while the cart accelerates down an inclined track will look similar to:



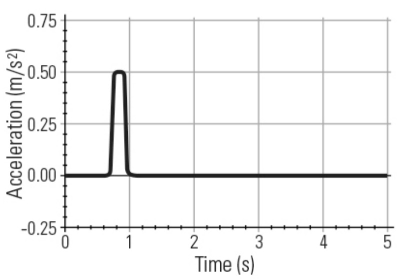
Student Part 3 graphs of position versus time and velocity versus time while the cart decelerates up and accelerates down an inclined track will look similar to:

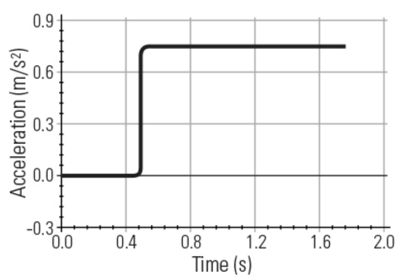


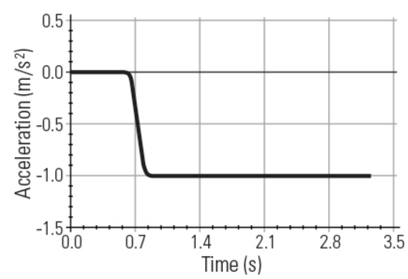
1. Explain the meaning of the slope from both the position versus time and velocity versus time graphs.

   The slope of the position versus time graph equals the velocity at the corresponding point in time. The slope of the velocity versus time graph equals the acceleration at the corresponding point in time.

2. Without looking at the acceleration versus time graphs on your data collection system, sketch the acceleration versus time graphs for each type of motion from their corresponding velocity versus time graphs. Be sure to label both axes with the correct scale and units.







Guided Inquiry Questions

Below are sample responses to the Guiding Questions found in the Guided Inquiry version of this lab activity.

1. What equipment will you need to accurately produce a constant-velocity graph? Describe your setup for gathering this data. Be sure you consider the equipment made available to you.

   Equipment will include a dynamics cart and track, a motion sensor and data collection system, and a meter stick. Attach the motion sensor to the track. Tilt the track to produce a ramp or incline for the cart.

2. How could you use a dynamics cart and track to set up a situation that produces a constant (positive) acceleration graph?

   For positive acceleration: tilt the track with the motion sensor at the top and let the cart accelerate down the ramp.

3. Describe a setup that will demonstrate negative acceleration and produce an accurate graph of this motion.

   For negative acceleration: put the sensor at the bottom of the track and push the cart uphill away from the sensor at a rate that decelerates as it moves uphill.

4. If the change in position of the cart with respect to time is equal to the cart's velocity, what does the slope of the position versus time graph represent? Similarly, what does the slope of the velocity versus time graph represent?

   The slope of the position versus time graph represents the cart's velocity. The slope of the velocity versus time graph represents the cart's acceleration.

5. Given a graph of position versus time, describe how you would sketch the remaining two graphs (velocity versus time and acceleration versus time).

   Position versus time graph →velocity versus time graph →acceleration versus time graph: First, look at the initial slope at the origin: does the object start at time zero with zero speed or is it already moving? Then analyze the slope of the position versus time graph at several points, and plot the velocity at each point on a velocity versus time graph. Notice any trends; connect the points to get the velocity versus time graph. If there are any abrupt changes in slopes or directions, check each portion of the graph to ensure it makes sense. Then repeat the process by analyzing the velocity versus time graph at several points to derive the acceleration versus time graph. Be sure to check and compare each segment of each graph to the other two graphs.

Assessment Questions: Sample Responses

Sample responses to the Analysis and Synthesis questions found in each version of the lab activity:

Analysis Questions

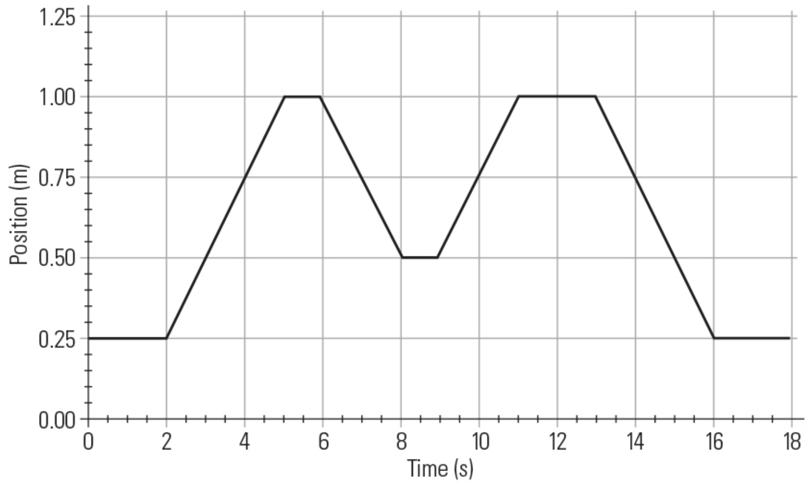
1. One of the three types of motion you explored was constant velocity, but was the velocity of the cart totally constant during your trials or did it slow slightly? What are some factors that may have caused the velocity of the cart not to be constant?

   If the track is not level, the cart may slow or accelerate slightly. Friction in the cart bearings and perhaps air resistance may have also caused the velocity of the cart to slow.

2. Another type of motion you explored was constant acceleration. What caused the constant acceleration of the cart? How could you have increased this acceleration, and how would your graphs of position and velocity versus time be different if the acceleration was greater?

   The acceleration was caused by gravity. We could have increased the acceleration by increasing the incline of the track. If the acceleration was greater, the position versus time graph would still be parabolic, but the parabolic opening would be narrower. Our velocity versus time graph would still be a straight line, but the slope would be greater.

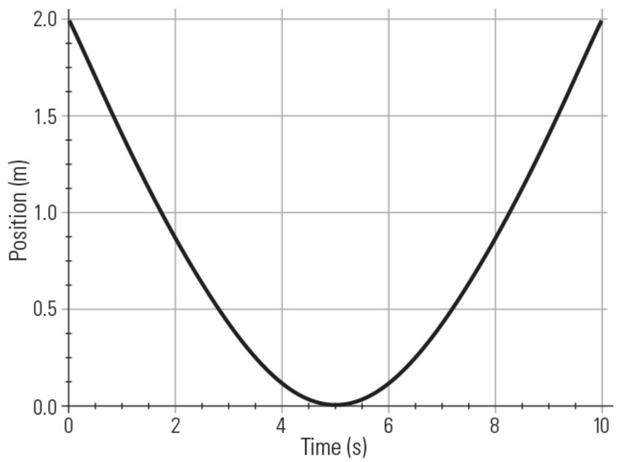
❓ 3. Imagine you were standing in front of a motion sensor which was measuring your position as a function of time as you moved toward or away from the sensor. Assuming the graph below shows this data, describe your motion during data collection in terms of your speed and direction.

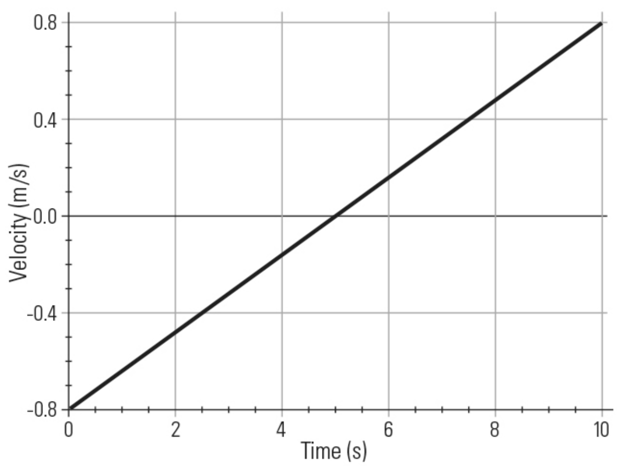


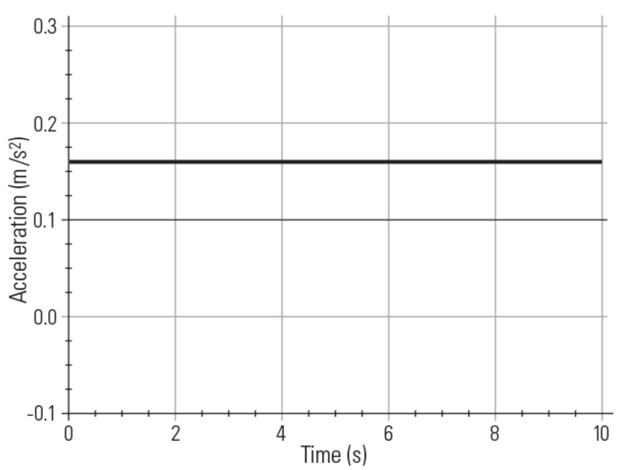
For the first two seconds I was stationary at 0.25 m from the sensor. Between seconds two and five I traveled at a constant speed away from the sensor, stopped at a distance of 1 m and stayed stationary for one second. At second six, I traveled at the same constant speed toward the sensor for two seconds, stopping at a distance of 0.5 m from the sensor. At second nine, I traveled at the same constant speed away from the sensor for two seconds stopping again at a distance of 1 m from the sensor. After waiting two seconds, I traveled towards the sensor at the same constant speed for three seconds and stopped at the point at which I started.

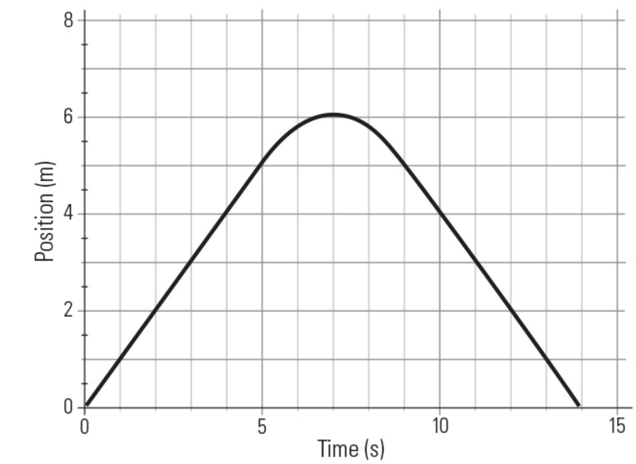
Synthesis Questions

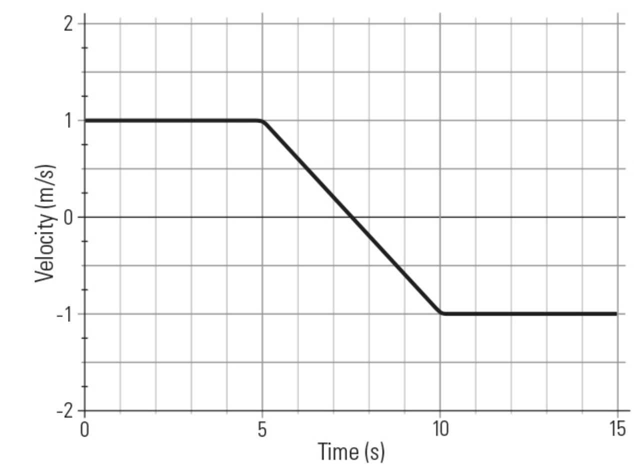
1. For each of the two *position versus time* graphs below, sketch the corresponding velocity and acceleration versus time graphs in the blank axes below each graph. Be sure to label both axes of each graph with the correct scale.

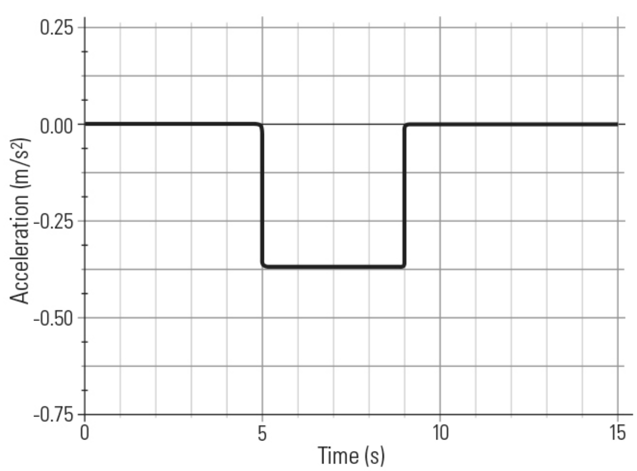




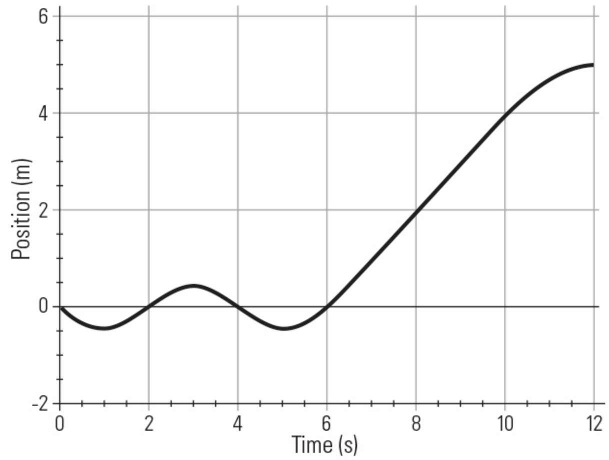


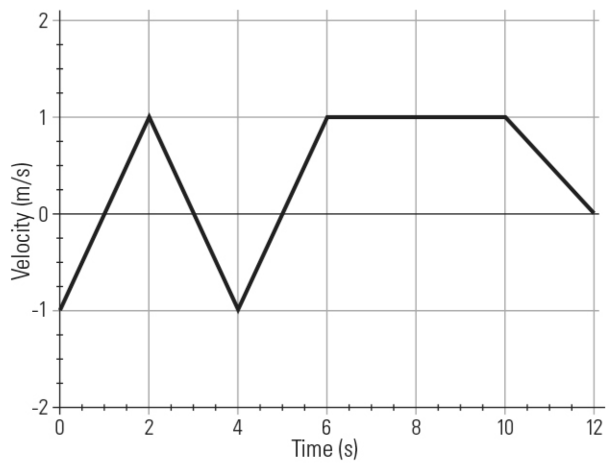


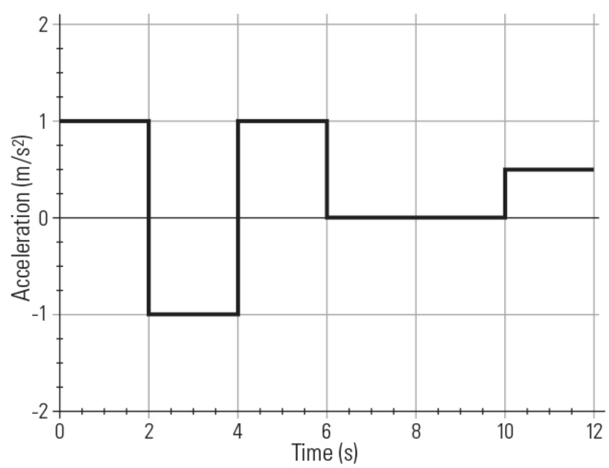




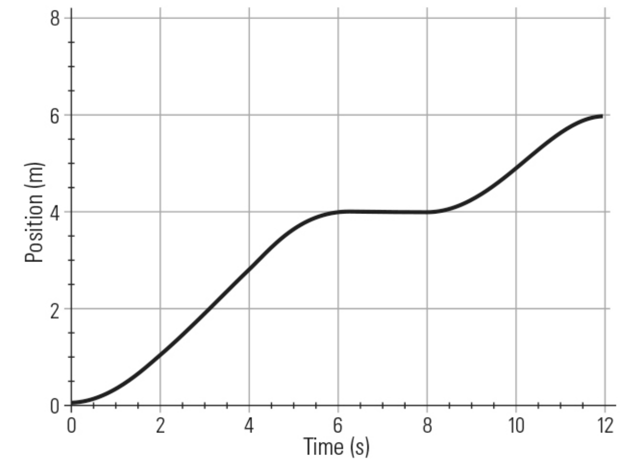
2. Given the velocity versus time graph shown below, sketch the corresponding *position versus time* and *acceleration versus time* graphs. (Assume the object starts at $x = 0$ m at time $t = 0$ s). Be sure to label both axes of each graph with the correct scale.

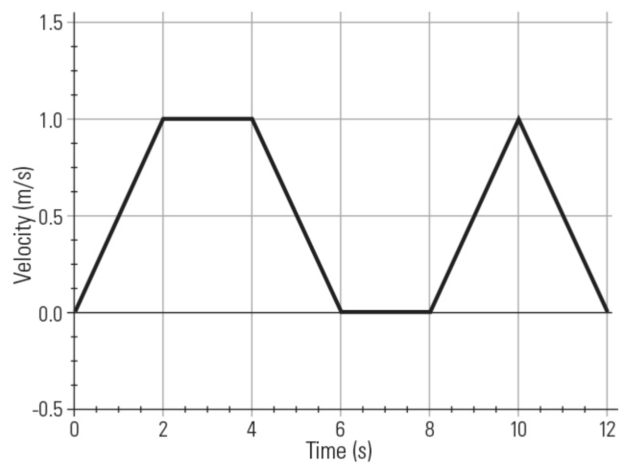


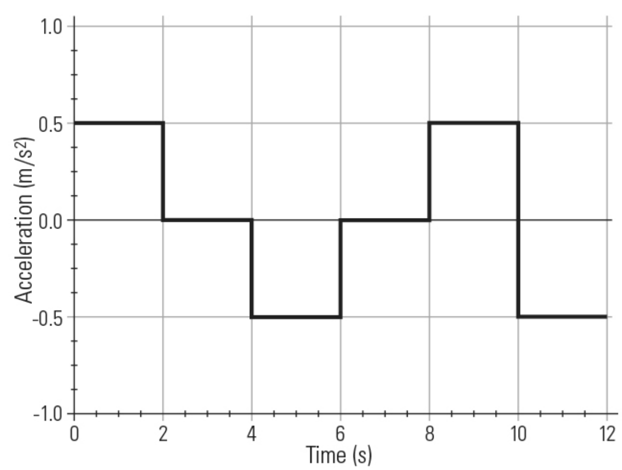




3. Given the *acceleration versus time* graph shown below, sketch the corresponding *velocity versus time* and *position versus time* graphs. Assume that initial velocity and position at $t = 0$ s is equal to 0 m/s and 0 m respectively. Be sure to label both axes of each graph with the correct scale.







Extended Inquiry Suggestions

You may challenge students further with difficult graphical questions or *match graph* challenges. Match graph activities become significantly more difficult if students are given the velocity versus time graph and a starting position (or the a–t graph, a starting point, and a starting velocity). Instructors can really stretch students' skills by starting with the a–t graph and having students find the v–t graph and the x–t graph.

Another great activity is the graphical analysis of a bouncing ball. Students can suspend the motion sensor so it points vertically downward, drop a basketball, and track its position for several bounces. Rich discussion can arise from looking at the v–t graphs and the a–t graphs, especially regarding the motion of the ball when it is subject to gravity compared to the brief time interval where the ball bounces. Analysis of the bouncing ball is covered in detail in many common physics textbooks. Connections to calculus can be made for AP Physics C (Mechanics) classes by discussing the derivative of the v–t graph at each bounce.

NAME PERIOD DATE

1. GRAPHICAL ANALYSIS: MOTION

STRUCTURED

Driving Question | Objective

How are the graphs of position versus time, velocity versus time, and acceleration versus time of an object undergoing constant acceleration related? Experimentally determine the relationships between the graphs of position, velocity, and acceleration versus time for an object undergoing constant acceleration, both positive and negative.

Materials and Equipment

- Data collection system
- PASCO Motion Sensor[1]
- PASCO Dynamics track[2]
- PASCO Dynamics cart[3]
- Meter stick
- Thick text book

[1]www.pasco.com/ap18 PASCO Motion Sensor

[2]www.pasco.com/ap08 PASCO PAStrack

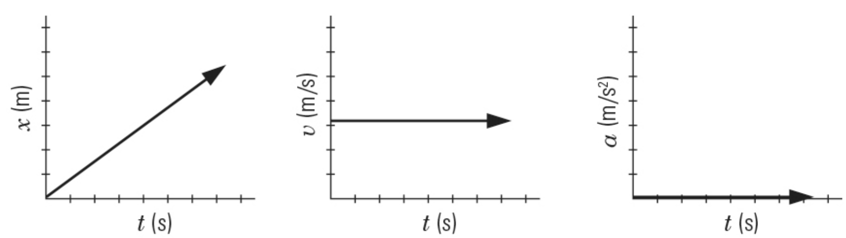
[3]www.pasco.com/ap07 PASCO PAScar

Background

Imagine a car moving down your street at a constant speed of 11 m/s (about 25 mph). What does a graph of the car's position x with respect to time t look like? Since it's moving at a constant speed, the graph is a straight line. Its slope is equal to the car's velocity v.

So what does a graph of velocity versus time look like? At a constant 11 m/s, velocity is a straight, flat line. What is the slope of a flat line? Zero. And what does this example show? It shows that since the car's velocity is not changing, the car is not accelerating, so the slope of a velocity versus time graph is equal to acceleration a.

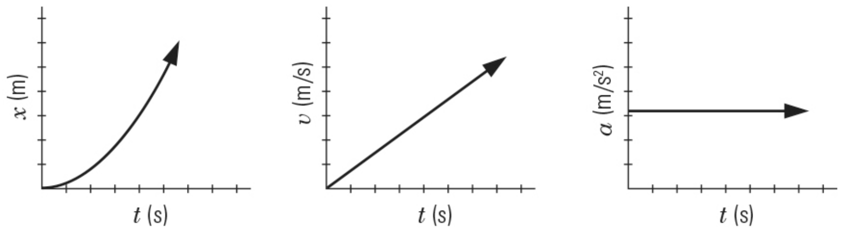
Position, velocity, and acceleration versus time graphs for a car moving with constant velocity:



If the same car accelerates from rest at a constant rate, the position versus time graph is no longer a straight line. The graph curves upward like a parabola because the car is covering a greater distance each second.

Following the same convention to determine the accelerating car's velocity, the slope of the parabolic curve is equal to the car's velocity: a non-zero straight line. Constant acceleration means that the car's velocity is increasing at a constant rate which agrees with the constantly increasing slope of the car's position versus time graph. The slope of the velocity versus time graph shows how quickly the car accelerates (a steeper slope means greater acceleration).

Position, velocity, and acceleration versus time graphs for a car moving with constant non-zero acceleration:



Finally, for this example, the acceleration versus time graph is a straight, flat line whose constant value is equal to the slope of the velocity versus time graph.

Procedure

Part 1 – Constant Velocity

SET UP

1. Place the dynamics track on the lab table and set the cart in the center of the track.

2. Adjust the level of the track by raising or lowering the track feet until the cart sits stationary in the center of the track without rolling.

3. Mount the motion sensor to one end of the dynamics track with the face of the sensor pointing down the length of the track.

   NOTE: Ensure that the switch on the top of the sensor is set to the cart icon.

4. Connect the motion sensor to the data collection system.

5. Display two graphs simultaneously. On one graph, display position on the y-axis and time on the x-axis. On the second graph, display velocity on the y-axis and time on the x-axis.

6. Test the motion sensor by recording a short run of data while moving your hand toward and away from the front of the sensor. Do the graphs show that the sensor detected your hand's motion? Is the data consistent with the motion of your hand? If so, continue to the next step. If not, readjust the motion sensor so it reads properly. Contact your teacher for further help if necessary.

COLLECT DATA

7. Place the cart stationary on the track about 0.20 m from the sensor.

8. Begin recording data, and then gently push the cart with your hand so that it glides smoothly away from the sensor.

9. Stop the cart before it leaves the track, and then stop data recording.

   NOTE: On your position versus time graph, you should see a flat line briefly before you pushed the cart, then a smooth, sloped line showing the cart's increasing position as time passes. Finally, you should see the sloped line abruptly become horizontal at the point where the cart stops. If these trends are not present, or do not make sense to you, repeat the previous steps until your data is more representative of the motion of constant velocity.

Part 2 – Constant Acceleration

COLLECT DATA

10. Put a textbook beneath the track near the motion sensor so the cart will accelerate down the hill (away from the sensor) when released.

11. Hold the cart stationary at the top of the track about 0.20 m from the sensor.

12. Begin data recording and release the cart, catching it at the bottom of the ramp.

13. When the cart reaches the bottom of the ramp, stop data recording.

Part 3 – Constant Negative Acceleration

COLLECT DATA

14. Move the textbook to the other end of the track so the motion sensor is at the bottom of the incline.

15. Hold the cart stationary at the bottom of the track about 0.20 m from the sensor.

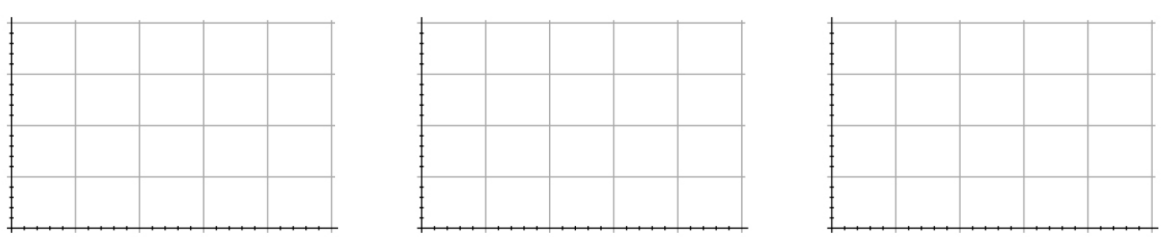
16. Begin data recording and then give the cart a gentle push so it moves uphill on its own (away from the sensor). It will decelerate as it moves to the top of its path. Do not let the cart leave the track at the top of the hill. Then let the cart go back down the track. Keep recording data as the cart descends. Do not let the cart hit the sensor at the bottom (catch the cart before it reaches the sensor).

17. Stop data recording.

Data Analysis

1. Explain the meaning of the slope from both the position versus time and velocity versus time graphs.

2. Without looking at the acceleration versus time graphs on your data collection system, sketch the acceleration versus time graphs for each type of motion from their corresponding velocity versus time graphs. Be sure to label both axes with the correct scale and units.

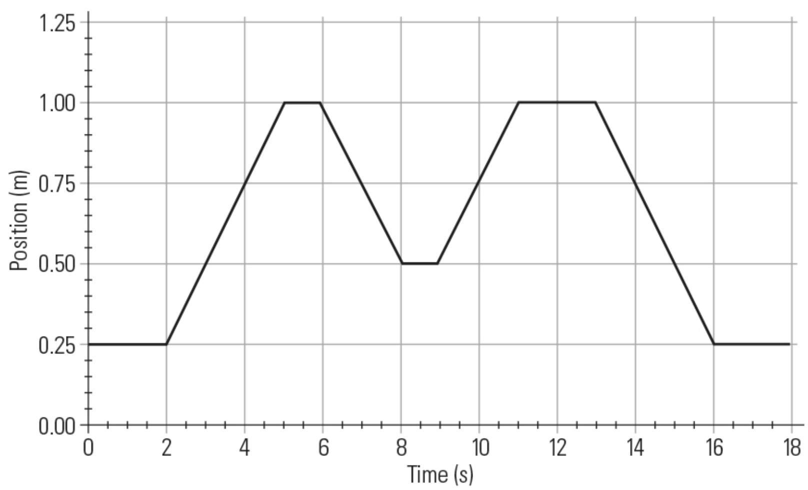


Analysis Questions

1. One of the three types of motion you explored was constant velocity, but was the velocity of the cart totally constant during your trials or did it slow slightly? What are some factors that may have caused the velocity of the cart not to be constant?

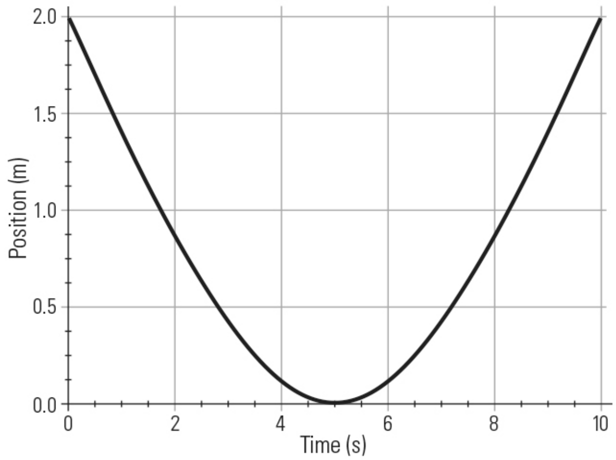
2. Another type of motion you explored was constant acceleration. What caused the constant acceleration of the cart? How could you have increased this acceleration, and how would your graphs of position and velocity versus time be different if the acceleration was greater?

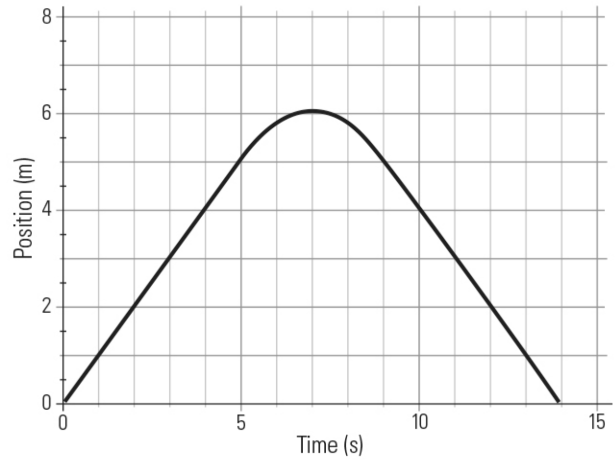
3. Imagine you were standing in front of a motion sensor which was measuring your position as a function of time as you moved toward or away from the sensor. Assuming the graph below shows this data, describe your motion during data collection in terms of your speed and direction.

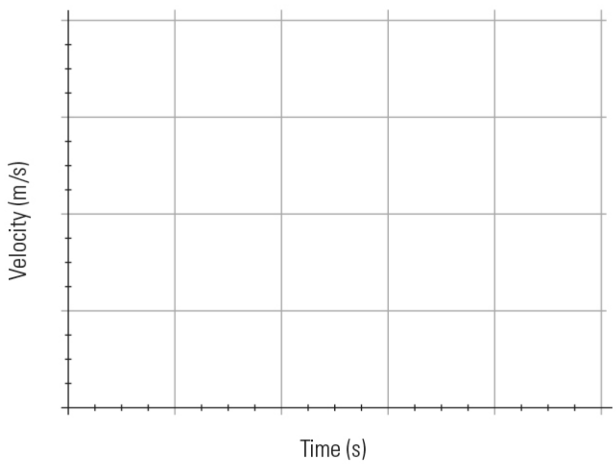


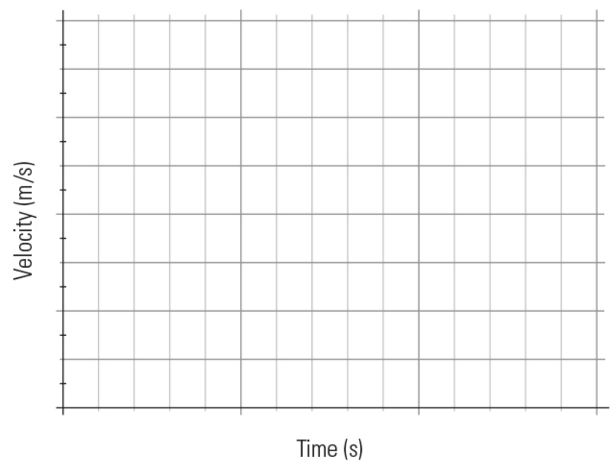
Synthesis Questions

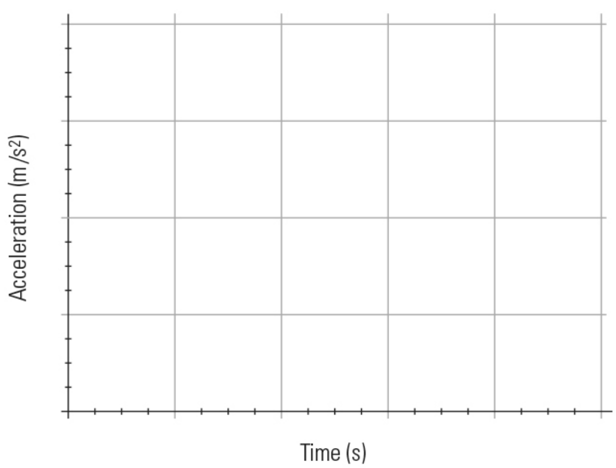
1. For each of the two position versus time graphs below, sketch the corresponding velocity and acceleration versus time graphs in the blank axes below each graph. Be sure to label both axes of each graph with the correct scale.

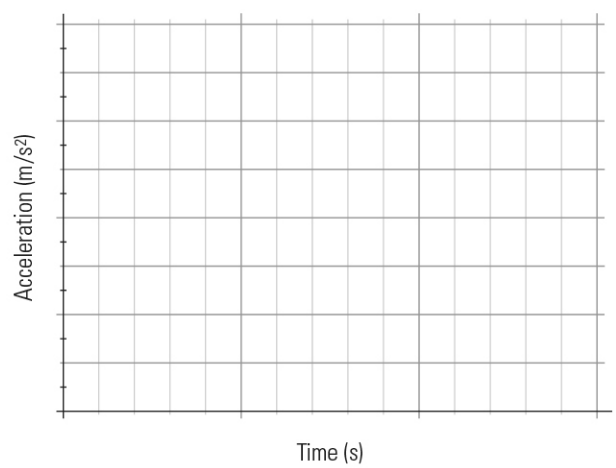




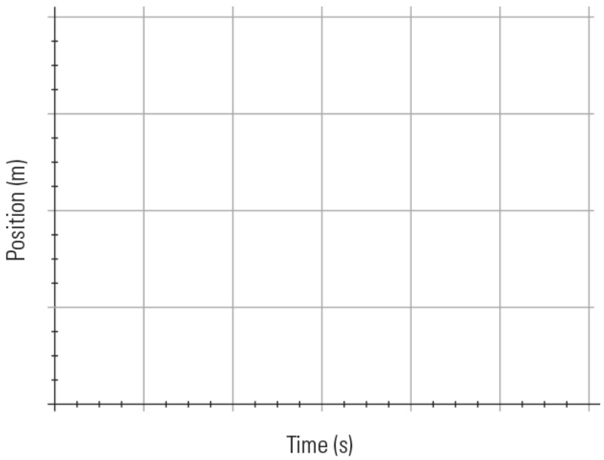


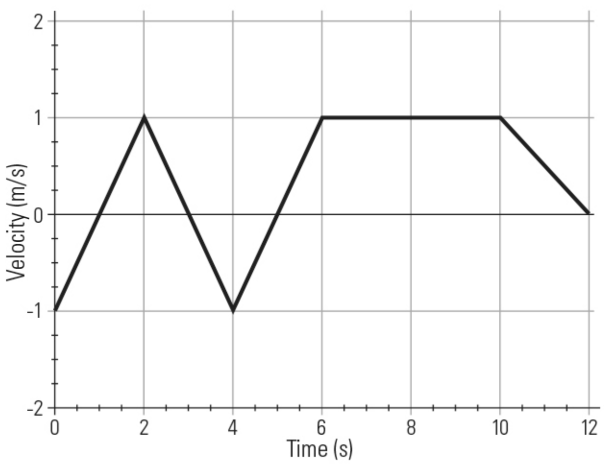


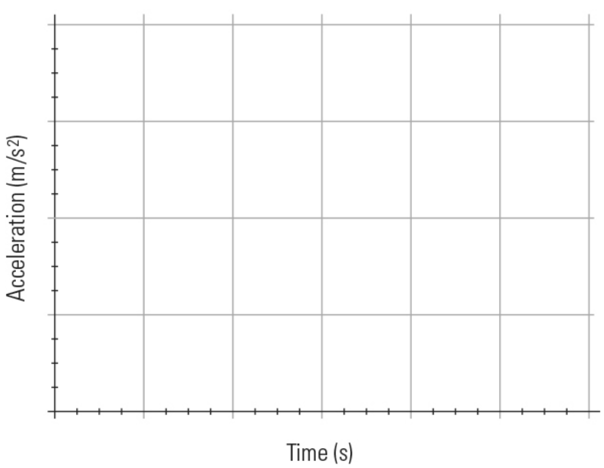




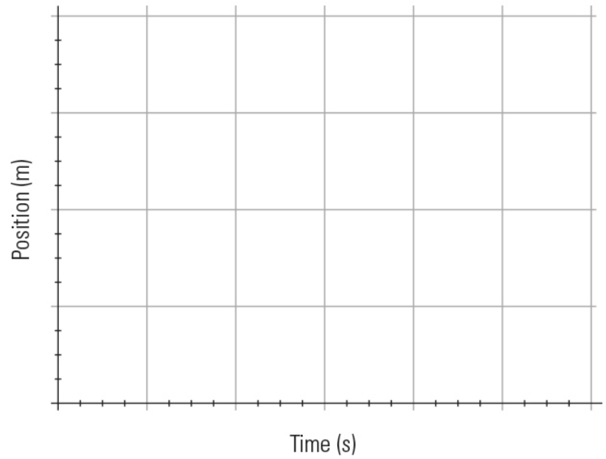
❷ 2. Given the velocity versus time graph shown below, sketch the corresponding *position versus time* and *acceleration versus time* graphs. (Assume the object starts at $x = 0$ m at time $t = 0$ s). Be sure to label both axes of each graph with the correct scale.

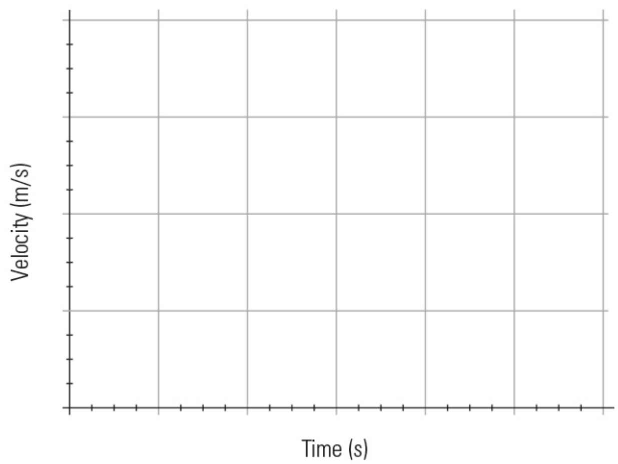


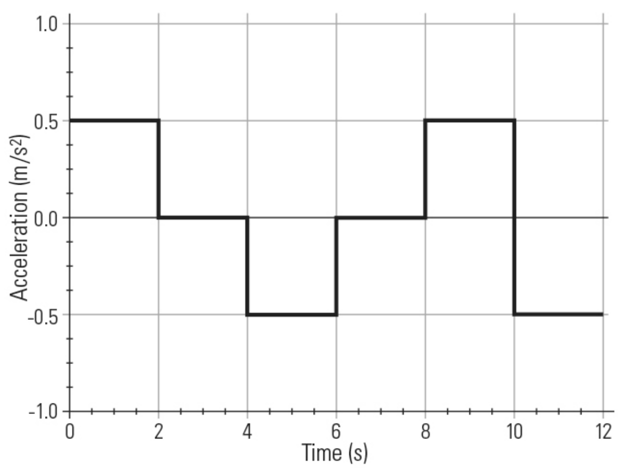




❷ 3. Given the *acceleration versus time* graph shown below, sketch the corresponding *velocity versus time* and *position versus time* graphs. Assume that initial velocity and position at $t = 0$ s is equal to 0 m/s and 0 m respectively. Be sure to label both axes of each graph with the correct scale.







2. NEWTON'S SECOND LAW

Connections to the AP® Physics 1 Curriculum*

The lab activity correlates to the following pieces of the AP® Physics 1 framework:

Big Idea	3	Enduring Understanding	B	Essential Knowledge	1

Learning Objective 1: The student is able to predict the motion of an object subject to forces exerted by several objects using an application of Newton's second law in a variety of physical situations with acceleration in one dimension.
Science Practices: 6.4, 7.2

Learning Objective 2: The student is able to design a plan to collect and analyze data for motion (static, constant, or accelerating) from force measurements and carry out an analysis to determine the relationship between the net force and the vector sum of the individual forces.
Science Practices: 4.2, 5.1

Learning Objective 3: The student is able to reexpress a free-body diagram representation into a mathematical representation and solve the mathematical representation for the acceleration of the object.
Science Practices: 1.5, 2.2

Big Idea	3	Enduring Understanding	B	Essential Knowledge	2

Learning Objective 1: The student is able to create and use free-body diagrams to analyze physical situations to solve problems with motion qualitatively and quantitatively.
Science Practices: 1.1, 1.4, 2.2

Time Requirement

Preparation Time: 10 minutes

Lab Activity: 50 minutes

Prerequisites

Students should be familiar with the following concepts:

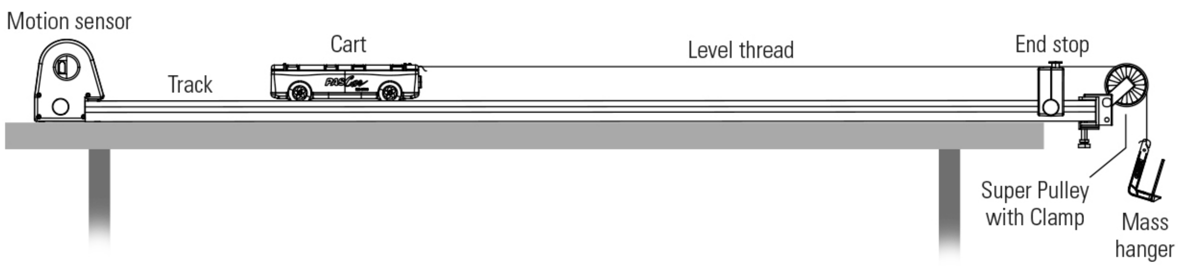
- *Net force* is the vector sum of the forces acting on an object or system.
- A *system* is a collection of two or more objects that directly or indirectly affect each other by force and or change in momentum. For the purpose of this lab activity, a system is defined as two masses mechanically connected to each other by a thread, upon which a net force will cause changes in the motion of both objects acting together.

Driving Question | Objective

What factors affect the acceleration of an object or system? Experimentally determine the relationship between an object's or system's mass, acceleration, and the net force being applied to the object or system.

Procedural Overview

The Structured version of this activity is divided into two parts. In both parts, students use a dynamics track and cart and set up the system so a mass hangs over a pulley at the end of the level track, accelerating the cart as the mass falls to the floor.



Using a motion sensor mounted at the end of the track opposite the pulley, students measure the cart's velocity as it accelerates, and then use the slope of the velocity versus time graph to determine the cart's acceleration.

Part 1 – Students keep the amount of hanging mass constant (constant net force) while varying the mass added to the top of the cart (varying system mass), and observe how this affects the acceleration of the system. Student data will show a linear relationship between acceleration and 1/mass.

Part 2 – Students keep the system mass constant but change the net force acting on the system by moving mass from the top of the cart to the mass hanger, and observe how this change affects the acceleration of the system. Student data will show that the acceleration of the cart is proportional to the net force acting on the cart.

Combining these two proportionalities, students experimentally derive the equation that summarizes Newton's Second Law:

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m}$$

Pre-Lab Discussion and Activity

Students may choose to address the lab objective using a solitary object or a system of objects. In the Structured version of this lab activity, a modified Atwood's machine, which may be unfamiliar to your students, is used. In the case of any system chosen, it is important that students understand how to determine the net force acting on the system so they can explore its effects on the acceleration of the system. The following pre-lab discussion addresses the technique for determining the net force acting on a modified Atwood's machine:

Draw the Figure 1 free-body diagram on the board with vectors, and explain: two objects with masses M and m, where $M > m$, are mechanically connected using a small piece of massless inextensible thread. When the two objects are at rest atop a flat frictionless surface, the net force on the two-object system is zero, as any gravitational force acting on the masses is counteracted by the normal force from the surface.

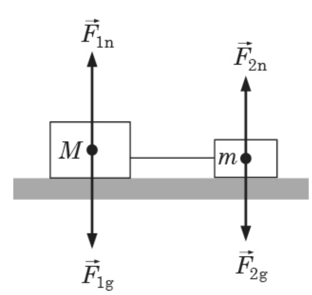


Figure 1

Draw the Figure 2 free-body diagram without force vectors on the board and explain: however, when the smaller object is hung off the edge of the surface, strung over a massless frictionless pulley, the system now experiences a non-zero net force as gravity acts on the hanging mass.

Ask students to identify the forces acting on each object in the system, and remind students that the thread is massless and inextensible, the pulley is massless and frictionless, and the surface that *M* rests on is frictionless. As students identify the forces acting on the objects, draw them as vectors in the Figure 2 free-body diagram on the board. There are five force vectors to be drawn on the diagram as shown in Figure 2.

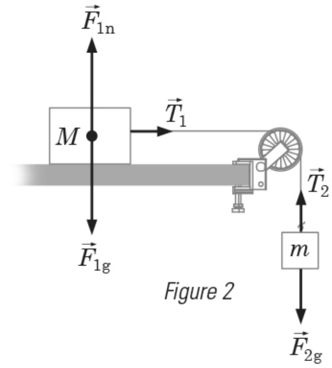


Figure 2

Finally, have students sum the vectors to determine the net force acting on the two-object system. The sum of the five vectors will equal only the downward-acting force vector due to earth's gravitational force, making the net force on the system equal to the weight of the hanging mass:

$$\vec{F}_{\text{net}} = \sum \vec{F} = m\vec{g}$$

It will be clear to students that the vertical force vectors acting on *M* are equal and opposite, but it may not be clear that the two tension vectors ($\vec{T}_1$ acting on *M*, and $\vec{T}_2$ acting on *m*) are equal and opposite. Draw the system linearly (Figure 3) to help students better visualize this, so they see that the only vector not counteracted is the force acting on *m* from earth's gravitational force.

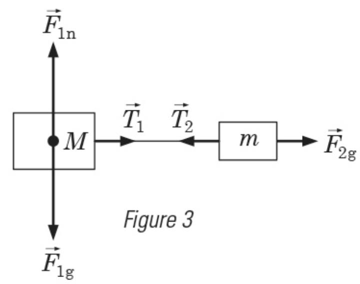


Figure 3

Students should be encouraged to follow the same general procedure when determining the net force acting on their object or system: establish a free-body diagram of their object or system; identify all of the forces acting on the object or system; draw the force vectors in the free-body diagram; sum the vectors to determine the net force acting on the object or system.

Materials and Equipment

- Data collection system
- PASCO Motion Sensor[1]
- PASCO Dynamics Track[2]
- PASCO Dynamics Cart[3]
- PASCO Dynamics Track End Stop[4]
- PASCO Super Pulley with Clamp[5]
- PASCO Compact Cart Mass (2), 250-g
- PASCO Mass and Hanger Set
- Thread
- Balance, 0.1-g resolution, 2,000-g capacity (1 per class)

Probeware Resources

Below are web-link and QR codes that will direct you to instructional video resources for individual pieces of PASCO probeware, sensors, and other hardware used in the lab activity. These same links and codes are provided to students in their activity handouts.

[1]www.pasco.com/ap18	[2]www.pasco.com/ap08	[3]www.pasco.com/ap07	[4]www.pasco.com/ap11
PASCO Motion Sensor	PASCO PAStrack	PASCO PAScar	PASCO Dynamics Track End Stop

[5]www.pasco.com/ap13

PASCO Super Pulley with Clamp

Teacher Tips

Tip 1 – Leveling the Track

- Remind students that they can verify that the track is level by placing a cart on the track and seeing if it rolls. If it does roll, they can adjust the track accordingly until the cart remains at rest in the middle of the track.

Tip 2 – Adjusting the Super Pulley

- The string connecting the hanging mass to the force sensor should be as level as possible. To level the string, the angle of the Super Pulley can be adjusted by loosening the thumb screws that mount the Super Pulley to the clamp. Once set to the proper angle, tighten the thumb screws.

Tip 3 – Aiming the Motion Sensors

- Students following the setup and procedure outlined in the Structured version of this lab activity, as well as any students using motion sensors to make measurements, should be careful to aim the motion sensor directly at its target. In the case of the PASCO Dynamics Cart and track, having the motion sensor aimed at a very slight upward angle from the motion sensor to the cart can help to return better data and avoid false echoes from the track surface.

Tip 4 – Using Magnetic Bumpers

- Students following the setup and procedure outlined in the Structured version of this lab activity, as well as any students using a cart and track as part of their investigation, should attach a PASCO Dynamics Track End Stop just in front of the Super Pulley to prevent the cart from travelling off the end of the track and potentially damaging it. Every PASCO Dynamics Track End Stop comes with a built-in magnetic bumper that matches the bumpers found in any PASCO Dynamics Cart. When set up to face each other, the magnetic bumpers in the end stop and cart will repel each other. Students should use this configuration to minimize the impact to any carts with the end stop.

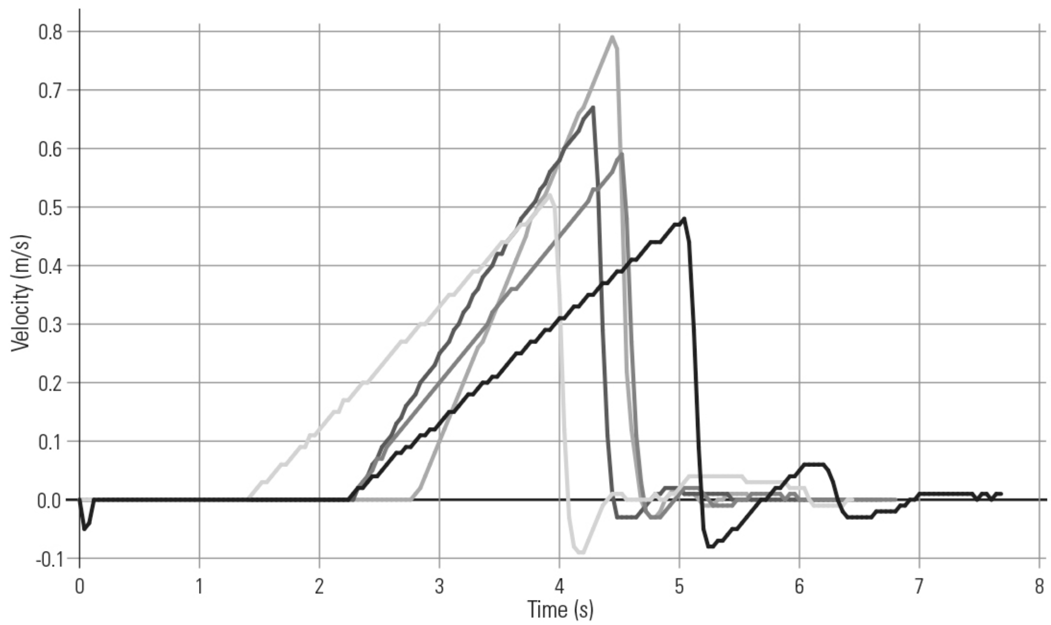
Sample Data

Below are sample data, acquired using the experimental setup and procedure outlined in the Structured version of the lab activity, and answers to questions found in the Data Analysis section.

Data Analysis

PART 1 – CONSTANT NET FORCE, VARYING SYSTEM MASS

Student Part 1 sample graphs of velocity versus time with varying system mass:



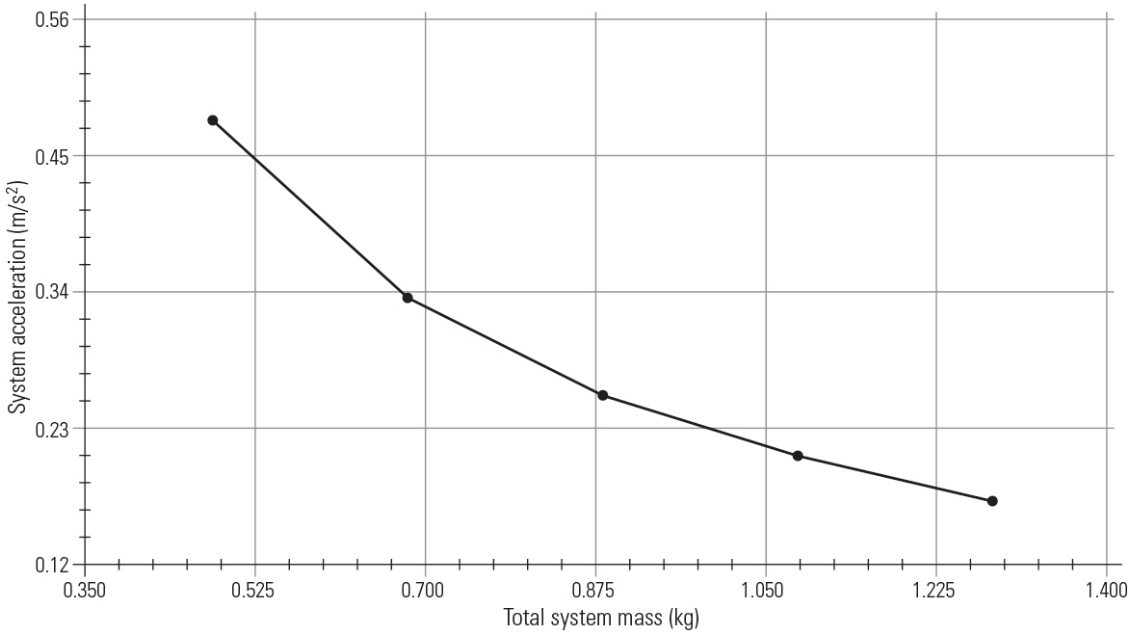
Mass of cart (kg): 0.256

Table 1: Acceleration of a system with varying mass and constant net force

Trial	Total System Mass (kg)	System Acceleration (m/s^2)	1/Mass (kg^{-1})
1	1.281	0.171	0.781
2	1.081	0.207	0.925
3	0.881	0.256	1.14
4	0.681	0.335	1.47
5	0.481	0.478	2.08

1. Plot a graph of *system acceleration* versus *total system mass* in the blank Graph 1 axes. Be sure to label both axes with the correct scale and units.

Graph 1: Acceleration versus mass for a system experiencing constant net force



2. *Linearize* your System Acceleration versus Total System Mass data:

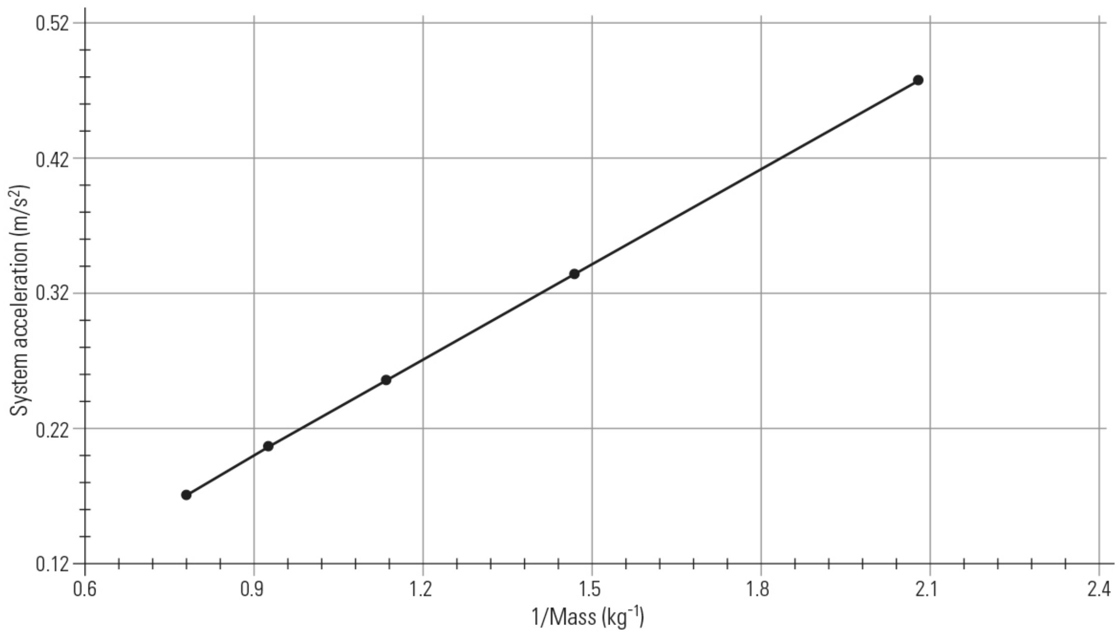
   a. Calculate 1/Total System Mass for each system mass value in Table 1. Record the results into Table 1 (1/Mass).

      Calculation using sample data for Trial 1:

      1/1.281 kg = 0.781 kg^{-1}

   b. Plot a graph of *system acceleration* versus *1/mass* in the blank Graph 2 axes. Be sure to label both axes with the correct scale and units and give the graph a title.

Graph 2: Acceleration versus 1/mass for a system experiencing constant net force



❷ 3. What does the slope of a best fit line on your Acceleration versus 1/Mass graph represent? *Hint: the units for slope are* $kg \cdot m/s^2$.

The slope of a best fit line applied to a graph of System Acceleration versus 1/Mass is equal to the net force being applied to the system. The slope has units of kg·m/s^2, which is equal to N, the units of force.

PART 2 – CONSTANT SYSTEM MASS, VARYING NET FORCE

Student Part 2 sample graphs of velocity versus time with varying net force:

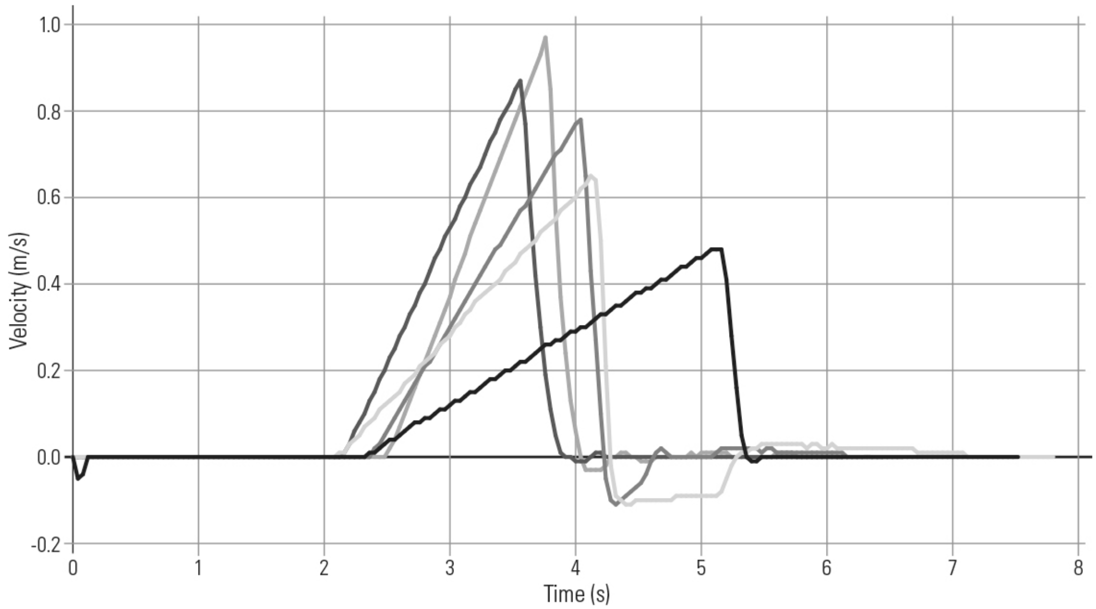


Table 2: Acceleration of a system with varying net force and constant mass

Trial	Hanging Mass (kg)	System Acceleration (m/s^2)	Net Force (N)
1	0.025	0.171	0.25
2	0.045	0.321	0.44
3	0.065	0.471	0.64
4	0.085	0.620	0.83
5	0.105	0.773	1.03

4. Calculate the magnitude of the net force $\left|\vec{F}_{\text{net}}\right|$ acting on the system in each trial:

$$\left|\vec{F}_{\text{net}}\right| = mg$$

where m is the amount of hanging mass in each trial and g is earth's gravitational constant (g = 9.8 m/s^2). Record your results in Table 2.

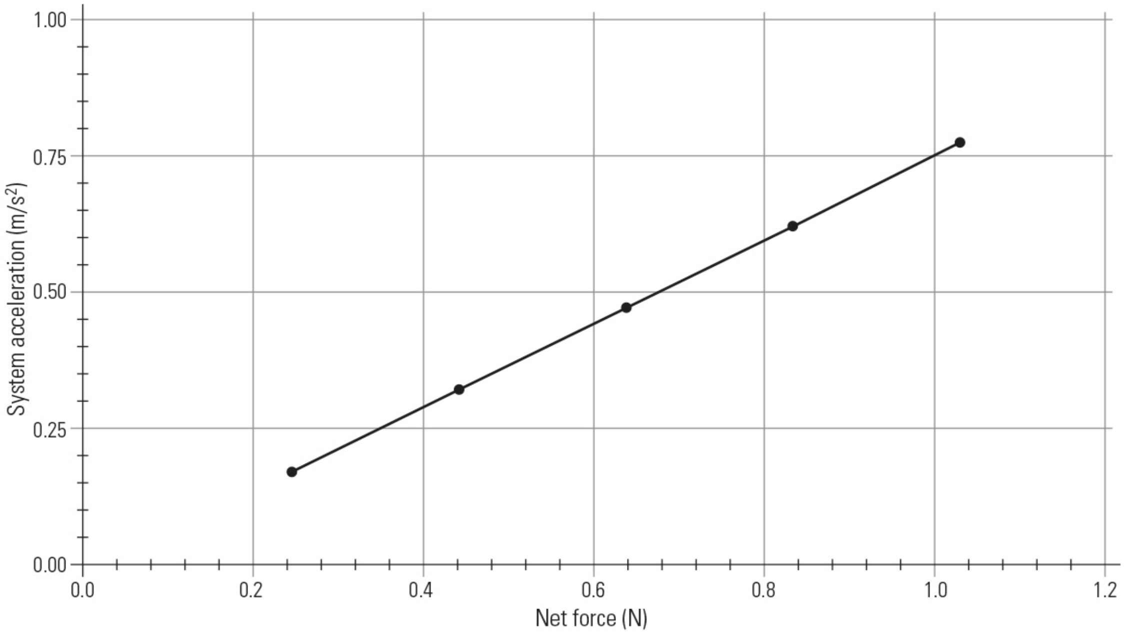
Calculation using sample data for Trial 1:

$$\left|\vec{F}_{\text{net}}\right| = mg$$

$$\left|\vec{F}_{\text{net}}\right| = (0.025\ \text{kg})\left(9.8\ \text{m/s}^2\right) = 0.25\ \text{N}$$

5. Plot a graph of *system acceleration* versus *net force* in the blank Graph 3 axes. Be sure to label both axes with the correct scale and units.

Graph 3: Acceleration versus net force for a system with constant mass



Guided Inquiry Questions

Below are sample responses to the Guiding Questions found in the Guided Inquiry version of this lab activity.

1. Based on the lab objective, what variables will you measure and what piece of equipment will you use to measure each variable?

   Students should measure (directly or indirectly) the net force applied to their object or system, the total mass of their object or system, and the acceleration of their system. Students following the procedure outlined in the Structured version of this lab activity measure: net force using the unbalanced weight of a hanging mass attached to their modified Atwood's machine system; total mass using a balance and accurately labeled masses; acceleration using a motion sensor.

2. How will you display your measurements so they can be analyzed to determine the relationship between the variables?

   Students may plot acceleration versus mass, but then should realize that the curve needs to be linearized by plotting acceleration versus 1/mass to yield a linear relationship whose slope has physical meaning. Students may also choose to plot acceleration versus force, or force versus acceleration, to yield a linear relationship whose slope represents the mass of the accelerated object or system.

3. What variables will you change to get at least 5 different data points for the acceleration of your object or system? What variables must remain constant in order to get a consistent graph of acceleration?

   Students should be incrementally changing the mass of their object or system while holding the net force constant, and incrementally changing the net force applied to their object or system while holding the mass of the system constant.

4. Describe some potential errors that could prevent you from gathering accurate consistent data?

   Students using thread or string may find error in their measurements due to the unaccounted mass from the thread or string in their system. Students using a modified Atwood's machine may find error in the unaccounted mass (rotational inertia) of the pulley used. Students using a cart and track system may find that the wheels in their carts are not frictionless, but add a small amount of unaccounted applied force to their object or system.

Assessment Questions: Sample Responses

Sample responses to the Analysis and Synthesis questions found in each version of the lab activity:

Analysis Questions

❓ 1. Qualitatively, what effect did your object's or system's mass have on its acceleration? Support your answer with data.

Student data will show that the acceleration of an object or system will increase as the mass of the system decreases, given that the net force on the object or system remains constant.

❓ 2. What is the relationship (inverse, proportional, equal, squared, et cetera) between the mass of your object or system and its acceleration? How do you know?

The relationship between an object or system's mass and its acceleration is inversely proportional. Students using the Structured version of this lab activity show this by plotting acceleration versus inverse mass, which is a linear relationship.

❓ 3. Qualitatively, what was the effect on your object's or system's acceleration as the net force acting on it increased? Support your answer with data.

Student data will show that the acceleration of an object or system will increase as the net force acting on the object or system increases, given that the mass of the object or system remains constant.

❓ 4. What is the relationship (inverse, proportional, equal, squared, et cetera) between your object's or system's acceleration and the net force acting on it? How do you know?

The relationship between an object's or system's acceleration and the net force acting on the system is directly proportional. Students using the Structured version of this lab activity show this by plotting acceleration versus net force, which is a linear relationship.

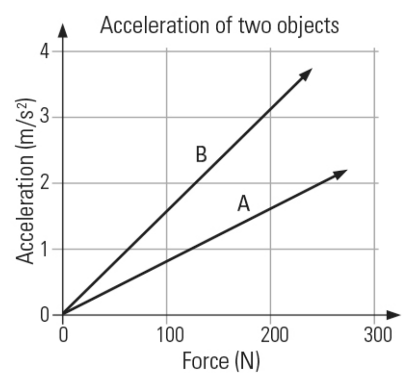
❓ 5. There are two common mathematical expressions for Newton's Second Law. One of these expressions is given below. How does your data support this mathematical relationship?

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m}$$

Student data will show that the acceleration of an object is inversely proportional to the mass of the object or system and directly proportional to the net force acting on the system, which agrees with this mathematical relationship.

Synthesis Questions

❓ 1. Two different carts are accelerated by a net force. The graph shows their respective accelerations as a function of this net force. What can you conclude about the mass of cart A compared to the mass of cart B? How do you know?



The slope of the line for both plots has units of (m/s^2)/N, which is the inverse of mass, kg^{-1}. Because the slope represents the inverse of the carts' masses, one can conclude that cart A is more massive than cart B.

❓ 2. We know from experience that the harder we throw a ball (apply more force), the faster it will be moving (greater initial velocity resulting from acceleration). If you throw a 1 kg softball as hard as you can, and it is traveling at 20 m/s when it leaves your hand, how fast do you think a 5 kg shot put would travel with the same throw?

Assuming the "same throw" means that the applied force is equal in both cases, the shot put should be traveling at 4 m/s, or 1/5 the final speed of the softball.

❷ 3. If we launch a rocket that has been designed to produce a constant force, will the acceleration at initial launch be the same as the acceleration just before the fuel is completely expended? Explain your answer.

Because fuel has mass, the acceleration will be greater when most of the fuel is consumed (after which, the total mass of the system is less).

❷ 4. A 1,000.0 kg rocket is traveling straight up with its engine producing a force of 39,240 N. If the rocket experiences a retarding force from air resistance equal to –1,227 N, what is its acceleration?

$$\sum \vec{F} = 39{,}240\ \text{N} - 1{,}227\ \text{N} = 38{,}013\ \text{N upward}$$

$$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{38{,}013\ \text{N}}{1{,}000.0\ \text{kg}}$$

$$\vec{a} = 38.013\ \text{m/s}^2\ \text{upward}$$

❷ 5. A teacher challenges her students to find the mass of their physics book using the system shown at right and their understanding of Newton's Second Law. A motion sensor measures the cart's acceleration due to three different hanging masses: 0.020 kg, 0.040 kg, and 0.060 kg. The acceleration and force data are provided in the table. The mass of the cart is 0.300 kg. Use the provided information to find the mass of the physics book. Show all of your work and explain your reasoning and process for deriving the book's mass.

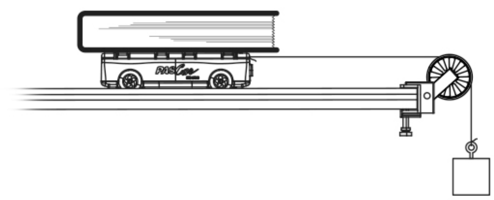


Table: Acceleration of a cart with varying net force and constant mass

Trial	Net Force Acting on the Cart (N)	Acceleration of the Cart (m/s^2)
1	0.196	0.131
2	0.392	0.261
3	0.588	0.392

Students should explain that with the physics book as the added mass on the cart, and the mass held constant, the cart will accelerate in proportion to the net force applied to it.

$$\vec{a} = \frac{\sum \vec{F}}{m}$$

Trial 1: $m = \frac{\vec{F}}{\vec{a}} = \frac{0.196\ \text{N}}{0.131\ \text{m/s}^2}$

Trial 1: $m = 1.50$ kg

Trial 2: $m = 1.50$ kg

Trial 3: $m = 1.50$ kg

This 1.50 kg represents the total mass, of which the cart is 0.300 kg. Therefore, the physics book has a mass of:

1.50 kg – 0.300 kg = 1.20 kg

Some students may choose to plot the data and analyze it graphically. Their graph should show a linear relationship whose slope is equal to the mass or its inverse. The total mass represents the sum of the physics text book and the cart.

Extended Inquiry Suggestions

An extended inquiry demonstration (if you want an opportunity to get your students outside and your school permits) is to load as many students as you can into a vehicle. Then have three or four strong students push the vehicle with the engine off and the gears disengaged. Try it first with an empty vehicle (with a driver, of course, to steer and brake the vehicle). Notice the acceleration. Then repeat with the vehicle filled with students. It is obvious (and memorable) that more massive objects accelerate much less than a less massive object. You can also scale this down to a PASCO Kinesthetic Cart or a skateboard (or better still, a student-built hovercraft) and a smooth level floor using students of different mass.

NAME PERIOD DATE

2. NEWTON'S SECOND LAW

STRUCTURED

Driving Question | Objective

What factors affect the acceleration of an object or system? Experimentally determine the relationship between an object's or system's mass, acceleration, and the net force being applied to the object or system.

Materials and Equipment

- Data collection system
- PASCO Motion Sensor[1]
- PASCO Dynamics Track[2]
- PASCO Dynamics Cart[3]
- PASCO Dynamics Track End Stop[4]
- PASCO Super Pulley with Clamp[5]
- PASCO Compact Cart Mass (2), 250-g
- PASCO Mass and Hanger Set
- Thread
- Balance, 0.1-g resolution, 2,000-g capacity (1 per class)

[1]www.pasco.com/ap18 PASCO Motion Sensor

[2]www.pasco.com/ap08 PASCO PAStrack

[3]www.pasco.com/ap07 PASCO PAScar

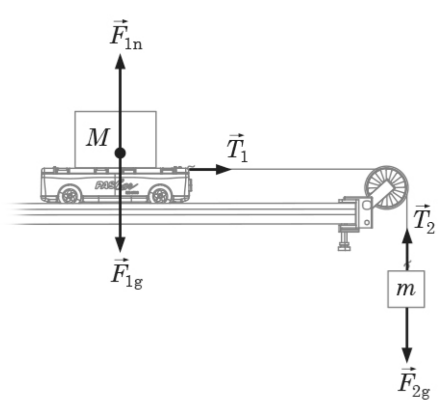
[4]www.pasco.com/ap11 PASCO Dynamics Track End Stop

[5]www.pasco.com/ap13 PASCO Super Pulley with Clamp

Background

Often, several forces act on an object simultaneously. In such cases, it is the *net force*, or the vector sum of all the forces acting, that is important. Newton's First Law of Motion states that if no net force acts on an object, the velocity of the object remains unchanged. If the velocity is not changing, the object is not accelerating. Newton's Second Law relates to the effect of unbalanced forces acting on an object. If forces are unbalanced, there is a net force and the object accelerates.

Like Newton, you will observe a simple system to look for a relationship between net force, mass, and acceleration. The components of the system are shown in the diagram. The system consists of a cart attached by thread to a falling mass. The falling mass applies the force of gravity to the thread which is then translated through thread tension to the cart. Although the gravitational force on the cart is counteracted by the normal force from the track, the applied force from the falling mass has no counteraction (assuming frictional force in the cart's wheels is zero), resulting in a non-zero net force acting on the cart in the direction of the thread.



In this exploration you will investigate how this net force and the mass of the system are related to the system's acceleration.

RELEVANT EQUATIONS

$$\vec{a}_{\text{ave}} = \frac{\Delta \vec{v}}{\Delta t} \tag{1}$$

This equation states that the average acceleration of an object is equal to the change in the object's velocity $\Delta \vec{v}$ divided by the elapsed time Δt. If the object experiences constant acceleration (similar to acceleration from gravity), the linear slope of the object's velocity time graph will equal the object's constant acceleration.

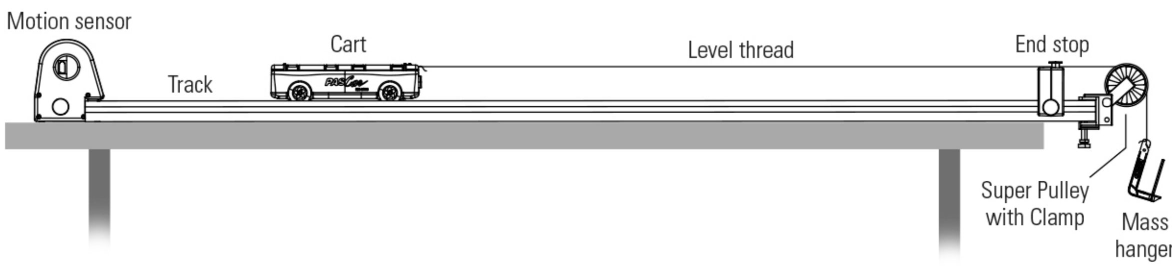
Procedure

Part 1 – Constant Net Force, Varying System Mass

SET UP

1. Measure the mass of the cart and record this value in kilograms into Table 1 in the Data Analysis section below.

2. Set up the equipment, as shown in the diagram, using a dynamics track and the Super Pulley with Clamp. Follow the guidelines below as you set up this system:



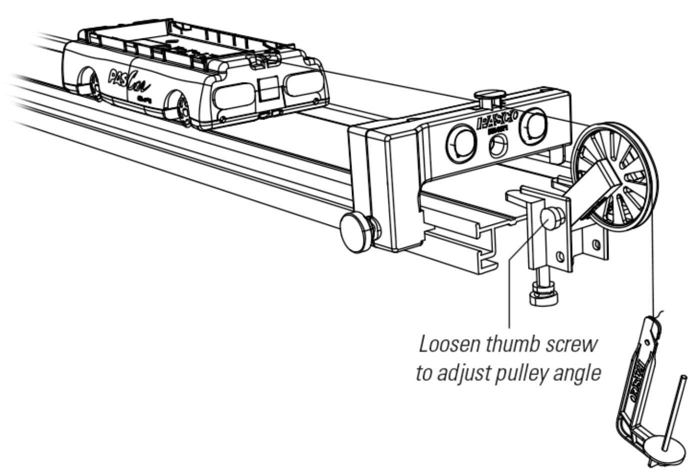
   a. Be sure the track is level, and mount the end stop to the track just in front of the pulley.

   b. Determine the length of thread needed to have the mass hanger be near the floor when the cart reaches the end stop of the track. One end of the thread should be tied to the hook on the front of the cart, and the other tied to the mass hanger.

   c. Adjust the angle of the pulley so that the thread is parallel to the track, as pictured.

   NOTE: The angle of the pulley can be adjusted by loosening the thumb screws that mount it to the clamp. Once adjusted to the proper angle, tighten the thumb screws. With the pulley at the proper angle, the thread can be run through the gap on the top of the end stop to avoid rubbing.

   d. Secure a motion sensor to the track on the end opposite the pulley with the front of the motion sensor aimed down the length of the track.



3. Place the two 250-g cart masses into the cart, and then add additional masses from the mass set until there is a total of 1,000 g in the cart.

4. Hang 20 g of mass on the 5-g mass hanger, for a total hanging mass of 25 g.

5. Connect the motion sensors to your data collection system, and then position the switch on the top of the motion sensor to the cart icon.

6. On the data collection system, create a graph display of velocity versus time, and then adjust the sample rate to 25 Hz.

COLLECT DATA

7. In Table 1, record the total mass of the system for Trial 1:

   Total mass of system = mass of cart + mass in the cart + hanging mass

8. Pull the cart away from the end stop, toward the motion sensor, until the mass hanger hangs just below the pulley.

9. Begin recording data. Wait about two seconds and then release the cart. It should move smoothly down the track.

10. When the cart reaches the end stop, stop recording data.

11. Remove 200 g of mass from the cart, and then follow the same steps to record a second run of data. Record the total system mass for Trial 2 into Table 1.

12. Repeat the same data collection steps three additional times, decreasing the amount of mass in the cart by 200 g in each trial. Record the total system mass for each trial into Table 1.

13. Use the tools on your data collection system to determine the acceleration of the system after it was released in each trial: use a line of best fit applied to the velocity data only when the system was in motion. The slope of the best fit line is equal to the acceleration of the system. Record this data into Table 1 for each trial.

Part 2 – Constant System Mass, Varying Net Force

SET UP

14. Use the same Part 1 setup: hang 20 g of mass on the 5-g mass hanger, for a total hanging mass of 25 g; place the two 250-g cart masses, and additional masses, back into the cart until there is a total of 1,000 g in the cart.

COLLECT DATA

15. In Table 2, record the current amount of hanging mass for Trial 1 in the Part 2 Data Analysis section below.

16. Pull the cart away from the end stop, toward the motion sensor, until the mass hanger hangs just below the pulley.

17. Begin recording data. Wait several seconds, and then release the cart. It should move smoothly down the track.

18. When the cart reaches the end stop, stop recording data.

19. Take 20 g of mass out of the cart and add the mass you just removed from the cart to the mass hanger, for a total of 45 g of hanging mass. This technique keeps the total mass of the cart-masses-mass-hanger-system constant.

20. In Table 2, record the current amount of hanging mass for Trial 2.

21. Pull the cart away from the end stop until the mass hanger hangs just below the pulley, and then record another run of data as you release the cart.

22. Repeat the same data collection steps three additional times, removing 20 g of mass from the cart and adding it to the mass hanger in each trial. Record the hanging mass in each trial in Table 2.

23. Use the tools on your data collection system to determine the acceleration of the system after it was released in each trial. Record this data into Table 2.

Data Analysis

Part 1 – Constant Net Force, Varying System Mass

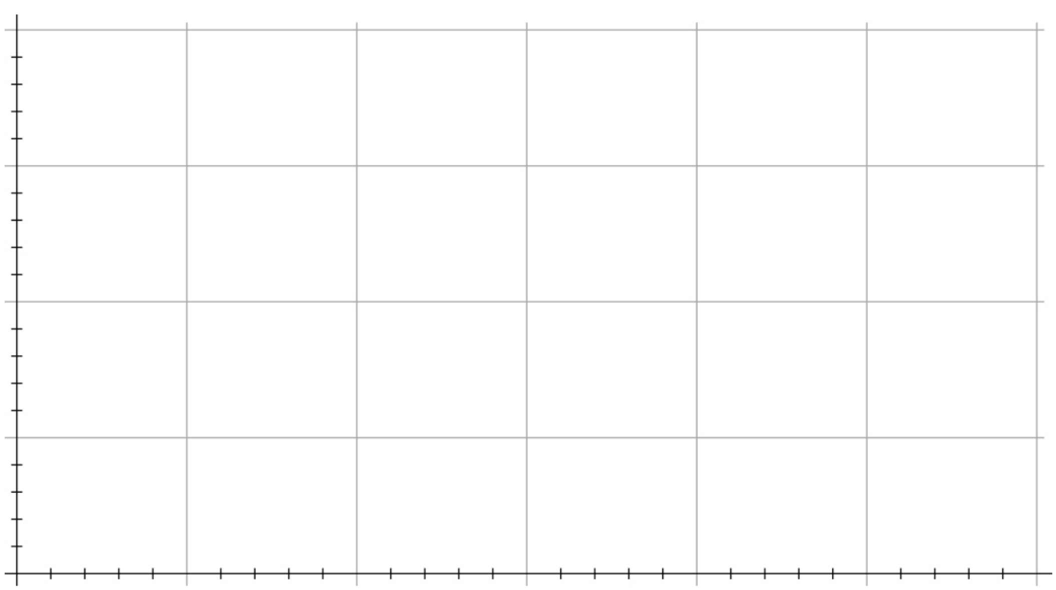
Mass of Cart (kg): ________________________________

Table 1: Acceleration of a system with varying mass and constant net force

Trial	Total System Mass (kg)	System Acceleration (m/s^2)	1/Mass (kg^{-1})
1			
2			
3			
4			
5			

1. Plot a graph of *system acceleration* versus *total system mass* in the blank Graph 1 axes. Be sure to label both axes with the correct scale and units.

Graph 1: Acceleration versus mass for a system experiencing constant net force

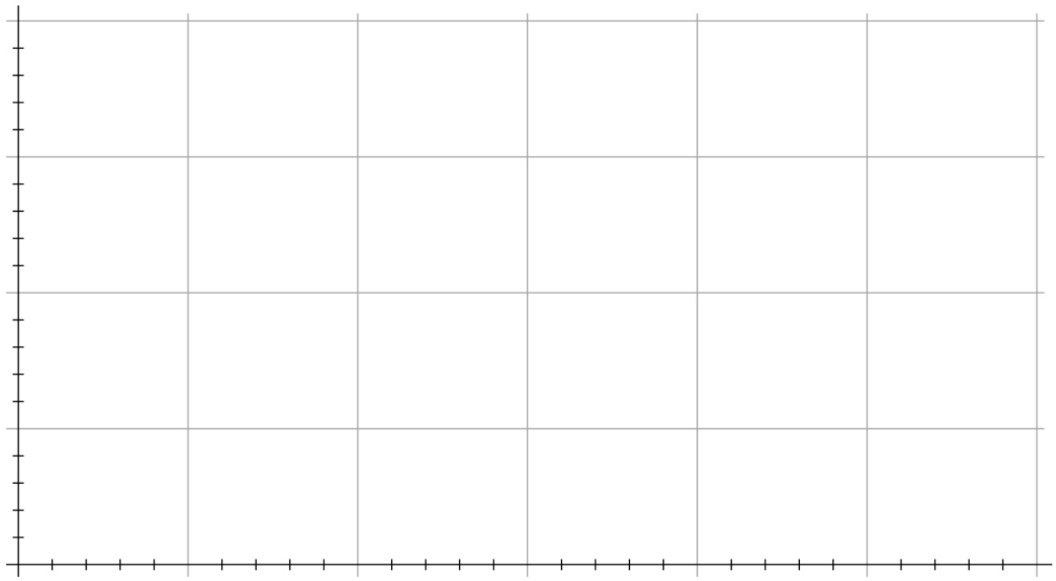


2. *Linearize* your System Acceleration versus Total System Mass data:

   a. Calculate 1/Total System Mass for each system mass value in Table 1. Record the results into Table 1 (1/Mass).

   b. Plot a graph of *system acceleration* versus *1/mass* in the blank Graph 2 axes. Be sure to label both axes with the correct scale and units and give the graph a title.

Graph 2:



3. What does the slope of a best fit line on your Acceleration versus 1/Mass graph represent? *Hint: the units for slope are* $kg \cdot m/s^2$.

____

____

____

Part 2 – Constant System Mass, Varying Net Force

Table 2: Acceleration of a system with varying net force and constant mass

Trial	Hanging Mass (kg)	System Acceleration (m/s^2)	Net Force (N)
1			
2			
3			
4			
5			

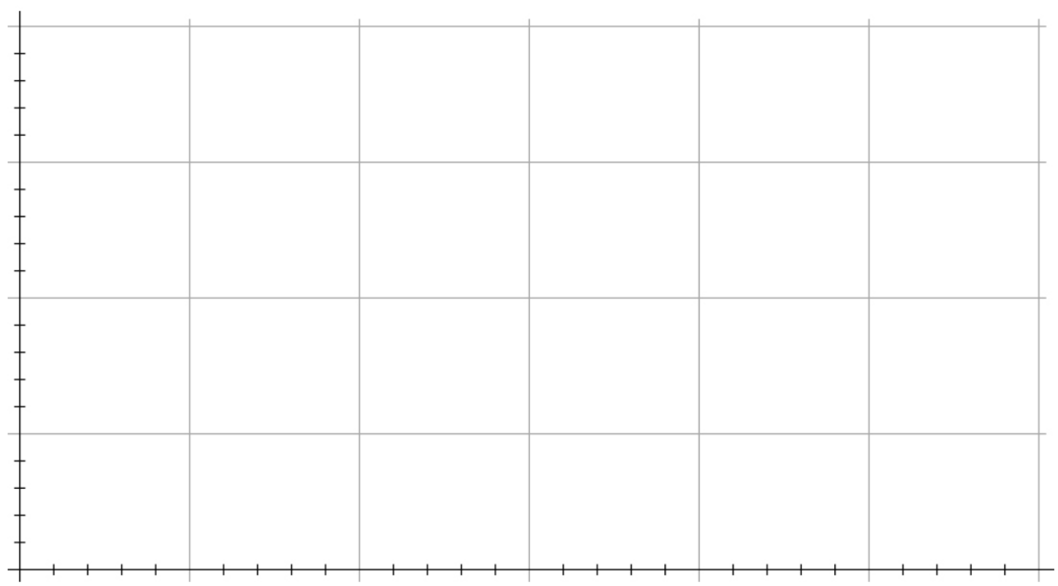
4. Calculate the magnitude of the net force $\left|\vec{F}_{\text{net}}\right|$ acting on the system in each trial:

$$\left|\vec{F}_{\text{net}}\right| = mg$$

where m is the amount of hanging mass in each trial and g is earth's gravitational constant (g = 9.8 m/s^2). Record your results in Table 2.

5. Plot a graph of *system acceleration* versus *net force* in the blank Graph 3 axes. Be sure to label both axes with the correct scale and units.

Graph 3: Acceleration versus net force for a system with constant mass



Analysis Questions

1. Qualitatively, what effect did your object's or system's mass have on its acceleration? Support your answer with data.

2. What is the relationship (inverse, proportional, equal, squared, et cetera) between the mass of your object or system and its acceleration? How do you know?

3. Qualitatively, what was the effect on your object's or system's acceleration as the net force acting on it increased? Support your answer with data.

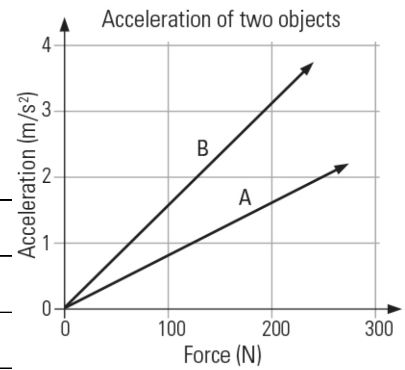
4. What is the relationship (inverse, proportional, equal, squared, et cetera) between your object's or system's acceleration and the net force acting on it? How do you know?

5. There are two common mathematical expressions for Newton's Second Law. One of these expressions is given below. How does your data support this mathematical relationship?

$$\vec{a} = \frac{\vec{F}_{net}}{m}$$

Synthesis Questions

1. Two different carts are accelerated by a net force. The graph shows their respective accelerations as a function of this net force. What can you conclude about the mass of cart A compared to the mass of cart B? How do you know?



2. We know from experience that the harder we throw a ball (apply more force), the faster it will be moving (greater initial velocity resulting from acceleration). If you throw a 1 kg softball as hard as you can, and it is traveling at 20 m/s when it leaves your hand, how fast do you think a 5 kg shot put would travel with the same throw?

3. If we launch a rocket that has been designed to produce a constant force, will the acceleration at initial launch be the same as the acceleration just before the fuel is completely expended? Explain your answer.

4. A 1,000.0 kg rocket is traveling straight up with its engine producing a force of 39,240 N. If the rocket experiences a retarding force from air resistance equal to –1,227 N, what is its acceleration?

❓ 5. A teacher challenges her students to find the mass of their physics book using the system shown at right and their understanding of Newton's Second Law. A motion sensor measures the cart's acceleration due to three different hanging masses: 0.020 kg, 0.040 kg, and 0.060 kg. The acceleration and force data are provided in the table. The mass of the cart is 0.300 kg. Use the provided information to find the mass of the physics book. Show all of your work and explain your reasoning and process for deriving the book's mass.

Table: Acceleration of a cart with varying net force and constant mass

Trial	Net Force Acting on the Cart (N)	Acceleration of the Cart (m/s^2)
1	0.196	0.131
2	0.392	0.261
3	0.588	0.392

3. ATWOOD'S MACHINE

Connections to the AP® Physics 1 Curriculum*

The lab activity correlates to the following pieces of the AP® Physics 1 framework:

Big Idea	3	Enduring Understanding	B	Essential Knowledge	1

Learning Objective 1: The student is able to predict the motion of an object subject to forces exerted by several objects using an application of Newton's Second Law in a variety of physical situations with acceleration in one dimension.
Science Practices: 6.4, 7.2

Learning Objective 2: The student is able to design a plan to collect and analyze data for motion (static, constant, or accelerating) from force measurements and carry out an analysis to determine the relationship between the net force and the vector sum of the individual forces.
Science Practices: 4.2, 5.1

Time Requirement

Preparation Time: 10 minutes

Lab Activity: 45 minutes

Prerequisites

Students should be familiar with the following concepts:

- Determining the acceleration of an object from the slope of its velocity versus time graph.
- Calculating the gravitational force on an object.
- From a free-body diagram, deriving a mathematical expression for an object's acceleration in one dimension from the net force on the object and the object's mass.

Driving Question | Objective

How is the acceleration of the two masses of an Atwood's machine affected by their difference in mass and by their total mass? Experimentally determine the mathematical relationship between the acceleration of an Atwood's machine, the difference between its two masses, and the sum of those two masses.

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Procedural Overview

The Structured version of this lab activity is divided into two parts:

Part 1 – Students transfer masses from the heavier side of the Atwood's machine to the lighter side to vary the mass difference $m_2 - m_1$ (and thus net force) while keeping the total mass $m_2 + m_1$ constant. In each trial they determine the acceleration from the slope of a plot of the speed versus time while the masses were moving freely. Plotting acceleration versus the mass difference for all Part 1 trials will result in a straight line, and from that students are expected to determine the proportional relationship

$$a_y \propto m_2 - m_1 \quad (\text{total mass held constant})$$

Part 2 – Students remove equal masses from both sides of the Atwood's machine to vary the total mass $m_2 + m_1$ while keeping the mass difference $m_2 - m_1$ (and thus net force) constant and determine the resulting acceleration as in Part 1. Plotting acceleration versus the inverse total mass (1/total mass) will result in a straight line, and from that students are expected to determine the inverse proportional relationship

$$a_y \propto \frac{1}{m_2 + m_1} \quad (\text{mass difference held constant})$$

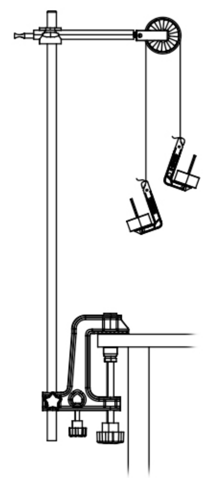
Students are then asked to combine the two discovered proportionalities into an equation relating the three variables,

$$a_y = k\frac{m_2 - m_1}{m_2 + m_1}$$

and to determine the value of the proportionality constant *k*, which should be found to be near the value of the free fall acceleration due to gravity *g*.

Finally, students are asked to derive the equation for the theoretical acceleration of an Atwood's machine and compare it to their experimentally-derived expression.

Pre-Lab Discussion and Activity

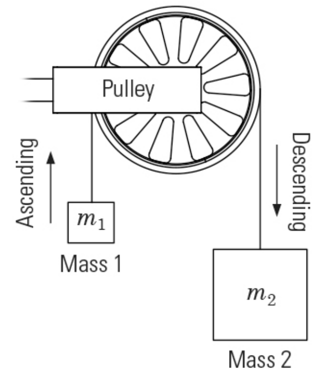


Demonstrate an Atwood's machine like that shown here, which is the design used in the Structured version of the lab but without a photogate for acceleration measurements. Start with the same mass on each side. Solicit students' observations on both the design and current (static) behavior.

Ask students to predict what will happen when you add a small mass to one hanger. Add the mass and discuss the results, arriving at the conclusion that the two masses accelerated, and their accelerations were of the same magnitude in opposite directions, since they were connected by a taut thread. Solicit ideas for measuring the acceleration.

Next, ask students to suggest variables that might affect the acceleration of the system. Continue at least until the two independent variables from the Driving Question—mass difference and total mass—are suggested. Students may also mention variables that make the real Atwood's machine deviate from the ideal, such as the string and pulley masses and the pulley friction. This provides a natural opportunity to describe an ideal Atwood's machine.

Sketch an idealized Atwood's machine, noting the assumptions that the pulley is frictionless and the string and pulley are massless. Especially for students who will do the Guided Inquiry or Student Designed versions of this lab, solicit experiment designs or approaches for reducing the deviations of the real Atwood's machine from the ideal. See Teacher Tips 1 and 2 for some possibilities.



NOTE: You do not need to direct the discussion toward a Newton's Second Law analysis in the pre-lab, as this is addressed in the Analysis Questions when students compare their experimentally-discovered and analytically-derived expressions for the acceleration of an Atwood's machine. The connection between Atwood's machine and Newton's Second Law would thus fit better in a post-lab discussion.

Materials and Equipment

- Data collection system
- PASCO Smart Gate photogate[1]
- PASCO Super Pulley with Mounting Rod[1]
- Mass and Hanger Set
- Table clamp or large base
- Support rod, 60-cm or taller
- Right angle clamp
- Thread, about 1 m
- Scissors

Probeware Resources

Below are web-link and QR codes that will direct you to instructional video resources for individual pieces of PASCO probeware, sensors, and other hardware used in the lab activity. These same links and codes are provided to students in their activity handouts.

[1]www.pasco.com/ap21

PASCO Smart Gate

Safety

Follow these important safety precautions in addition to your regular classroom procedures:

- To prevent the masses from flying off, gently stop the hangers before they strike the pulley or tabletop.

Teacher Tips

Tip 1 – Choosing Thread Type

- Use the lightest possible thread to more closely approximate an ideal Atwood's machine. The use of even light braided fishing line introduces significant errors compared to light thread.

Tip 2 – Choosing Mass Values

- The specific mass values m_1 and m_2 described in the Structured version of this lab activity were selected to both reduce errors and to make the changing of the masses easier when using the specific masses in PASCO's Mass and Hanger Set. When using the Guided Inquiry or Student Designed version, you may want to have a discussion before the lab about potential sources of error—including unaccounted for thread and pulley masses and pulley friction—to guide students toward the use of larger masses and larger mass differences. Errors are found to increase significantly with mass differences below the Structured procedure's minimum difference of 15 g. The use of relatively large total masses reduces the relative effect of the thread and pulley masses.

Tip 3 – Attaching the Hangers

- Although the PASCO mass hangers can be hung by simply wrapping the thread in the notch, hanging them from loops tied in the thread is recommended for this activity due to the potential for collisions between hangers or with the tabletop.

Tip 4 – Using Other PASCO Sensors

- The procedure outlined in the Structured version of this lab activity uses a PASCO Smart Gate with a pulley to determine the acceleration of the Atwood's machine. Older photogates will work fine in place of the Smart Gate. If your data collection system has digital ports, connect the photogate directly. Otherwise, use a digital adapter to connect your photogate to the data collection system's PASPORT port.

  Use of a rotary motion sensor in place of the Smart Gate is not recommended, as its three-step pulley has a greater rotational inertia and its sample rate can interfere with a proper determination of linear speed at some speeds. Use of a motion sensor in place of the Smart Gate is also not recommended, as the hanging masses do not provide a large enough target for the motion sensor, the sensor's minimum measuring distance requires additional space, and the sensor screen can be damaged by heavy falling masses.

Tip 5 – Determining Acceleration from Speed

- The procedure outlined in the Structured version of this lab activity uses a PASCO Smart Gate with a pulley to measure the linear speed of the Atwood's machine. The procedure directs students to find the acceleration from the slope of the speed versus time data, as the more prominent noise in a direct plot of acceleration versus time can make it more difficult to select the region in which the masses were moving freely.

Sample Data

Below are sample data, acquired using the experimental setup and procedure outlined in the Structured version of the lab activity, and answers to questions in the Data Analysis section.

Data Analysis

PART 1 – VARYING MASS DIFFERENCE

Student Part 1 graphs of speed versus time with varying mass difference will look similar to:

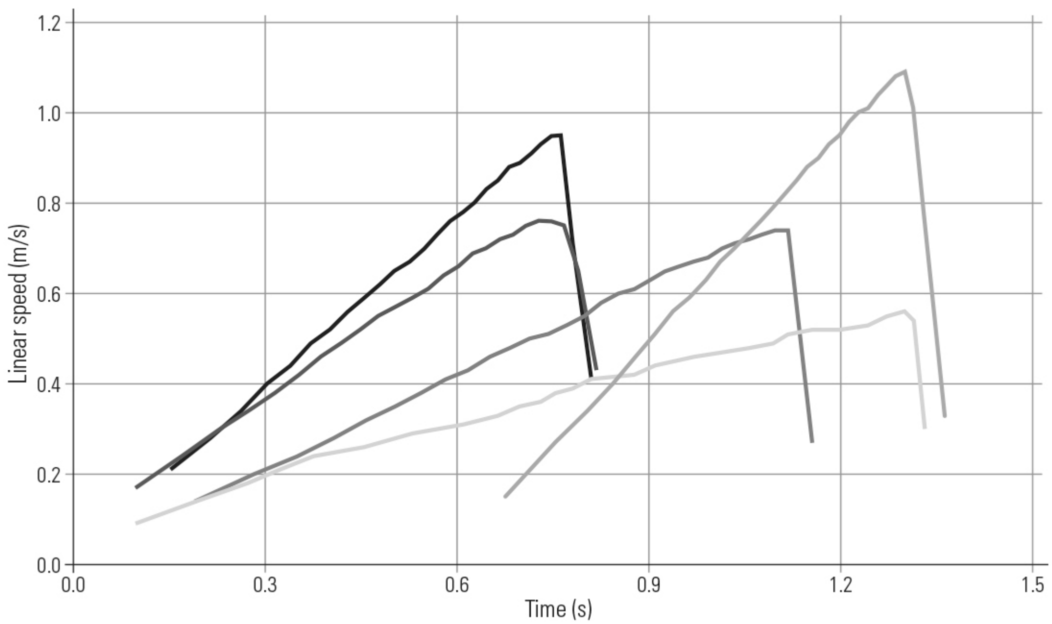


Table 1: Varying mass difference with constant total mass

Trial	Acceleration (m/s²)	Mass 1 (kg)	Mass 2 (kg)	Mass Difference (kg)	Total Mass (kg)
1	1.53	0.145	0.200	0.055	0.345
2	1.25	0.150	0.195	0.045	0.345
3	0.959	0.155	0.190	0.035	0.345
4	0.681	0.160	0.185	0.025	0.345
5	0.409	0.165	0.180	0.015	0.345

1. Calculate the difference between Mass 2 and Mass 1, $m_2 - m_1$, and total mass, $m_2 + m_1$, for each trial in Part 1. Record the results in Table 1 in units of kilograms (kg).

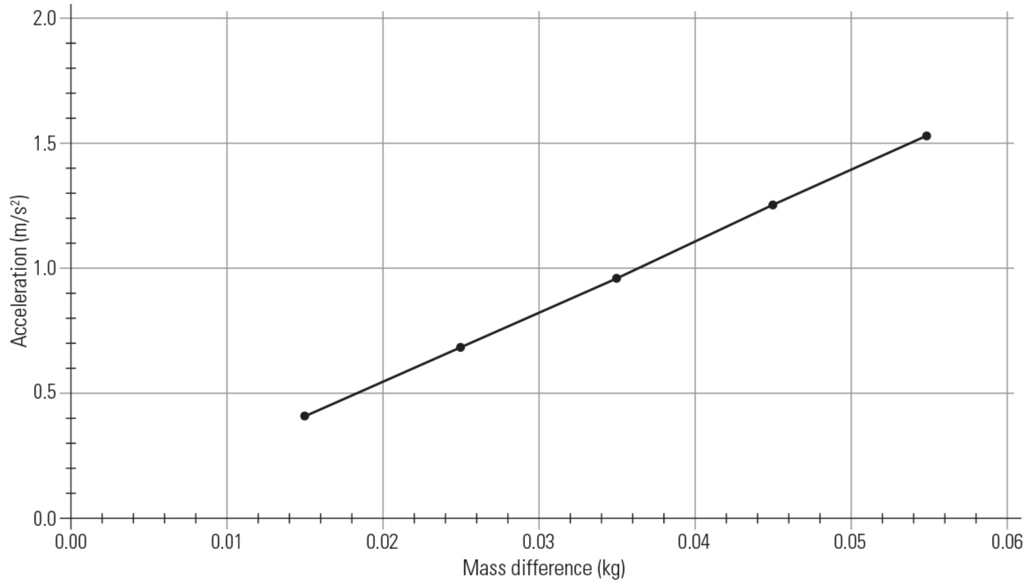
   Calculation using sample data for Trial 1:

   $m_2 - m_1$ = 0.200 kg – 0.145 kg = 0.055 kg

   $m_2 + m_1$ = 0.200 kg + 0.145 kg = 0.345 kg

2. Plot a graph of *acceleration* versus *mass difference* in Graph 1. Be sure to label both axes with the correct scale and units.

Graph 1: Acceleration versus mass difference with constant total mass



PART 2 – VARYING TOTAL MASS

Student Part 2 graphs of speed versus time with varying total mass will look similar to:

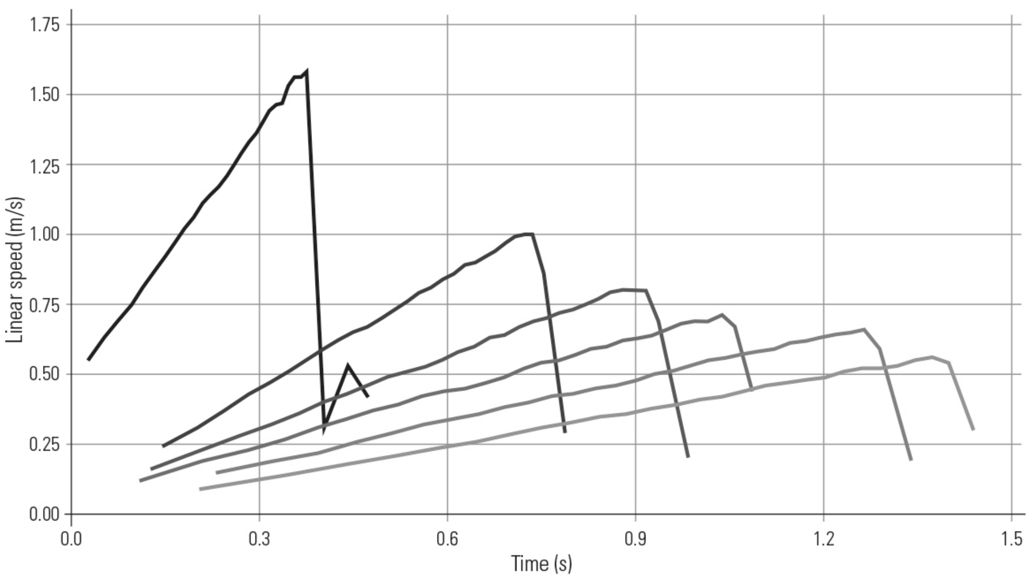


Table 2: Varying total mass with constant mass difference

Trial	Acceleration (m/s^2)	Mass 1 (kg)	Mass 2 (kg)	Mass Difference (kg)	Total Mass (kg)	1/Total Mass (kg^{-1})
5	0.409	0.165	0.180	0.015	0.345	2.90
6	0.503	0.135	0.150	0.015	0.285	3.51
7	0.638	0.105	0.120	0.015	0.225	4.44
8	0.859	0.075	0.090	0.015	0.165	6.06
9	1.32	0.045	0.060	0.015	0.105	9.52
10	3.05	0.015	0.030	0.015	0.045	22.2

3. Calculate the mass difference, $m_2 - m_1$, and total mass, $m_2 + m_1$, for each trial in Part 2. Record the results in Table 2 in units of kilograms (kg).

   Calculation using sample data for Trial 5:

   $m_2 - m_1$ = 0.180 kg – 0.165 kg = 0.015 kg

   $m_2 + m_1$ = 0.180 kg + 0.165 kg = 0.345 kg

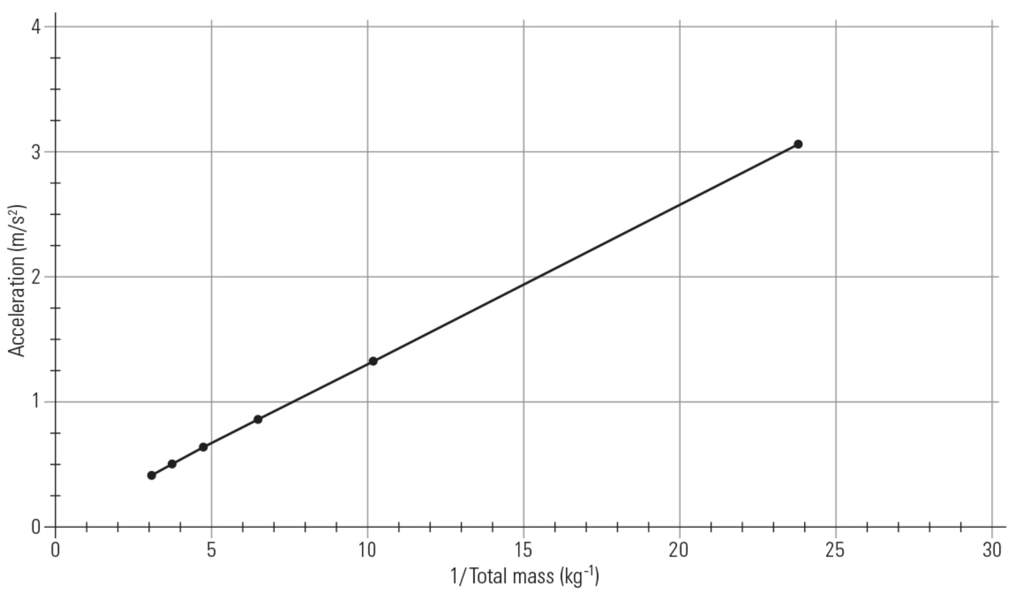
4. Calculate the inverse of the total mass (1/total mass) for each trial in Part 2. Record the results in Table 2 in units of inverse kilograms (kg^{-1}).

   Calculation using sample data for Trial 5:

   1/total mass = 1/0.345 kg = 2.90 kg^{-1}

5. Plot a graph of *acceleration* versus *1/total mass* in Graph 2. Be sure to label both axes with the correct scale and units.

Graph 2: Acceleration versus 1/total mass with constant mass difference



Guided Inquiry Questions

Below are sample responses to the Guiding Questions found in the Guided Inquiry version of this lab activity.

1. If the objective of this lab is to experimentally determine how the difference in mass and the total mass of the two Atwood's machine masses affect the system's acceleration, what should the dependent and independent variables be in your experiment?

   The objective statement indicates three variables: acceleration, difference in mass, and total mass. Students will explore how the difference in mass and the total mass affect the acceleration of an Atwood's machine, so acceleration should be their dependent variable and difference in mass and total mass should be their independent variables.

2. What equipment do you have available and how will you set it up to measure each variable?

   The system's motion can be measured with different tools, including a photogate and pulley, motion sensor, rotary motion sensor, or video-motion analysis software and a camera. See Teacher Tip 4, Using Other PASCO Sensors, for cautions about using a motion sensor or rotary motion sensor.

   Acceleration can be determined from the slope of a speed-versus-time graph or by taking an average of acceleration values. See Teacher Tip 5, Determining Acceleration from Speed, for suggestions on this.

   Mass values can be read directly from the labels on the masses themselves with an accuracy of ±2% or better for the PASCO Mass and Hanger Set, or students may choose to determine the masses more accurately with a balance.

3. How will you change each independent variable while collecting data? Should you change more than one variable at a time? Describe the steps you will take to change each variable.

   Independent variables should be changed one at a time while the others are held constant to isolate the effect of each variable. Measure acceleration and vary the mass difference while holding the total mass constant by transferring masses between the two hangers. Measure acceleration and vary the total mass while holding the mass difference constant by removing or adding the same amount of mass from or to each hanger.

4. An ideal Atwood's machine is frictionless and its string and pulley are massless. How will you design your experiment to approximate these ideals as closely as you reasonably can?

   Friction can be reduced by using a pulley with a high quality bearing. The relative effect of remaining friction can be minimized by applying forces much larger than those frictional forces by using larger mass differences. See Teacher Tip 2, Choosing Mass Values, for suggested minimum mass differences. The effect of the mass of the string can be reduced by using light thread and by using larger hanging masses. The effect of the pulley mass can be reduced by using a small, light pulley.

Assessment Questions: Sample Responses

Sample responses to the Analysis and Synthesis questions found in each version of the lab activity:

Analysis Questions

1. For each part of your experiment, list each variable and indicate whether it was held constant, increased, or decreased.

   Answers for the Structured version:

   Part 1 – independent variable: mass difference (decreased); dependent variable: acceleration (decreased); constant: total mass

   Part 2 – independent variable: total mass (decreased); dependent variable: acceleration (increased); constant: mass difference

   Answers for the Guided Inquiry and Student Designed versions of this activity should include the independent variables mentioned above.

2. How did changing the difference in mass between the two sides affect the acceleration of the Atwood's machine?

   As the mass difference decreased, with the total mass held constant, the acceleration also decreased.

❓ 3. Based on your data, express the relationship between the acceleration a_y and mass difference $m_2 - m_1$ by completing this proportionality statement:

$a_y \propto$ $m_2 + m_1$ (total mass held constant)

❓ 4. How did changing the sum of the two hanging masses affect the acceleration of the Atwood's machine?

As the total mass decreased, with the mass difference held constant, the acceleration increased.

❓ 5. Based on your data, express the relationship between the acceleration a_y and total mass $m_2 + m_1$ by completing this proportionality statement:

$a_y \propto \frac{1}{m_2 + m_1}$ (mass difference held constant)

❓ 6. Combine the two relationships above into a single proportionality expressing the relationship between the Atwood's machine's acceleration a_y the mass difference $m_2 - m_1$ and the total mass $m_2 + m_1$:

$a_y \propto \frac{m_2 - m_1}{m_2 + m_1}$

❓ 7. Convert the proportionality statement above into an equation by introducing a proportionality constant k:

$a_y = k\frac{m_2 - m_1}{m_2 + m_1}$

❓ 8. Use your data to determine the proportionality constant k. Be sure to specify its units. Briefly explain how you determined its value and units.

Rearranging the equation in the previous question to solve for k:

$$k = a_y \frac{m_2 + m_1}{m_2 - m_1}$$

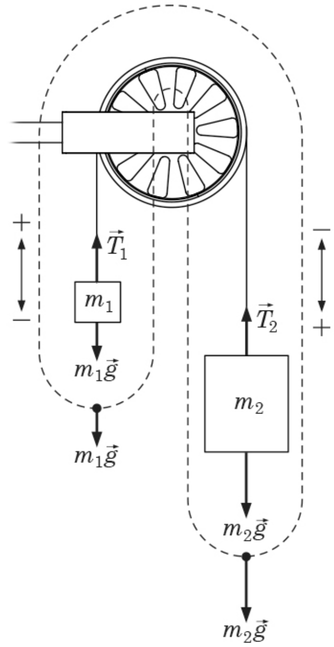
Calculation using sample data for Trial 1:

$$k_{\text{Trial 1}} = 1.53\ \text{m/s}^2 \times \frac{0.345\ \text{kg}}{0.055\ \text{kg}} = 9.60\ \text{m/s}^2$$

Using the accelerations and masses from Part 1 to calculate k in each of the first five trials, the results are the following: 9.60, 9.58, 9.45, 9.40, and 9.41. The average of these values is 9.49:

$k_{ave} = 9.49\ \text{m/s}^2$

In the expression for k, the units for mass cancel, so k must have the same units as the acceleration a, which are m/s^2.



9. Consider this free-body diagram of an Atwood's machine. Assume that the masses of the string and pulley are negligible. The analysis can be simplified if the system is defined to consist of the two masses linked together, as indicated by the dashed line. This allows you to disregard the string tension as an internal force and consider only the two forces m_1g and m_2g acting on the system. You can also consider the system to be moving in one dimension, with *positive* defined in the direction of m_1 ascending and m_2 descending, as indicated.

   Apply Newton's Second Law, $\vec{a} = \frac{\vec{F}_{\text{net}}}{m}$, to derive an expression for the acceleration a of the system in terms of the masses m_1 and m_2.

   In one dimension, $a_y = \frac{F_y}{m}$

   For an Atwood's machine, $a_y = \frac{m_2g - m_1g}{m_2 + m_1}$,

   or $a_y = g\frac{m_2 - m_1}{m_2 + m_1}$

10. How does the expression for acceleration that you determined from your data analysis compare to the equation derived above from Newton's Second Law? Justify your answer.

    The equation derived from the data matches that derived from Newton's Second Law, if the proportionality constant k is the free fall acceleration due to gravity g. The determined value of k, using sample data, was 9.49 m/s^2, which is only 3% less than the value of g, so it is reasonable to claim that the constant of proportionality k is the acceleration due to gravity.

Synthesis Questions

1. One way to check whether a derived relationship is reasonable is to consider whether it behaves as expected in extreme or *limiting* cases. Determine whether the relationship you derived between the acceleration a of an Atwood's machine and its two hanging masses m_2 and m_1 reduces to a reasonable form when the two masses are equal. Explain your reasoning.

   $a_y = g\frac{m_2 - m_1}{m_2 + m_1}$

   If $m_2 = m_1 \equiv m$, then

   $a_y = g\frac{m - m}{m + m} = g\frac{0}{2m} = 0$

   It makes sense that the acceleration is zero when the masses are equal, since there would be no net force on the system if the two masses balance each other, so for this case, the derived relationship is reasonable.

2. Similarly, determine whether the relationship you derived between the acceleration a of an Atwood's machine and its two hanging masses m_2 and m_1 reduces to a reasonable form when the mass m_2 is much greater than m_1. Explain your reasoning.

   $a_y = g\frac{m_2 - m_1}{m_2 + m_1}$

   If $m_2 \gg m_1$, then we can consider m_1 to go to zero:

   $a_y = g\frac{m_2 - 0}{m_2 + 0} = g\frac{m_2}{m_2} = g$

   This is reasonable, since m_2 would experience free-fall acceleration if there was no opposing mass m_1.

❷ 3. A planetary rover carries an Atwood's machine with 100 g one side and 110 g on the other. If the system's acceleration is measured to be 0.176 m/s² on a certain planet, what is the acceleration due to gravity on that planet? Which planet in the solar system is it?

After rearranging the Atwood's machine equation to solve for g:

$$g = a_y \frac{m_2 + m_1}{m_2 - m_1}$$

$$g = 1.76\ \text{m/s}^2 \left(\frac{0.110\ \text{kg} + 0.100\ \text{kg}}{0.110\ \text{kg} - 0.100\ \text{kg}} \right) = 3.70\ \text{m/s}^2$$

That is the acceleration due to gravity on the surface of Mars.

❷ 4. What ratio of masses m_2/m_1 would produce an Atwood's machine whose acceleration is half that of an object in free fall?

$$a_y = g \frac{m_2 - m_1}{m_2 + m_1}$$

For $a_y = \frac{1}{2} g$,

$$\frac{1}{2} g = g \frac{m_2 - m_1}{m_2 + m_1} \text{ or}$$

$$m_2 + m_1 = 2(m_2 - m_1)$$

$$m_2 = 3m_1$$

$$\frac{m_2}{m_1} = 3$$

❷ 5. Elevators cars have counterweights to reduce the amount of work motors need to do to lift the car. You might idealize an elevator system without its motor as an Atwood's machine. If a particular elevator's counterweight mass is 1,000 kg and its elevator car and passengers have a combined mass of 1,200 kg, what acceleration would the passengers experience if the motors and safety braking mechanisms failed? If the elevator car accelerated from rest at a height of 12 m above the ground floor, how long would it take for the car to reach the ground floor? What would be its speed on impact?

First determine the acceleration:

$$a_y = g \frac{m_2 - m_1}{m_2 + m_1} = 9.8\ \text{m/s}^2 \times \left(\frac{1200\ \text{kg} - 1000\ \text{kg}}{1200\ \text{kg} + 1000\ \text{kg}} \right) = 0.89\ \text{m/s}^2$$

Then, using $y = y_0 + v_{y0} t + \frac{1}{2} a_y t^2$, with $y - y_0 \equiv \Delta y$ and $v_{y0} = 0$, the time it would take is:

$$t = \sqrt{\frac{2\Delta y}{a_y}} = \sqrt{\frac{2 \times 12\ \text{m}}{0.89\ \text{m/s}^2}} = 5.2\ \text{s}$$

Using $v_y = v_{y0} + a_y t$, with $v_{y0} = 0$, the elevator's speed on impact is:

$$v_y = a_y t = 0.89\ \text{m/s}^2 \times 5.2\ \text{s} = 4.6\ \text{m/s}$$

Extended Inquiry Suggestions

Here are five ideas for extending and reinforcing the concepts in this activity:

- Discuss the approaches students took to determine the proportionality constant k in their experimentally-derived combined expression for the acceleration. Possible techniques include calculating k from the acceleration and mass data from a single trial, calculating k across multiple trials and averaging, and determining the value of k as the slope of a linear fit to a graph of acceleration versus the ratio mass difference/total mass for all trials.
- Discuss why the cart and mass-over-pulley configuration used in the Newton's Second Law lab is sometimes described as a "modified Atwood's machine".
- Reinforce the application of Newton's Second Law by repeating the derivation of the Atwood's machine acceleration, this time considering both the gravitational and (common) string tension forces on each of the two masses m_1 and m_2.

  Mass 1: $a_{1y} = \frac{\sum F_{1y}}{m_1} = \frac{(T_1 - m_1 g)}{m_1}$

  Mass 2: $a_{2y} = \frac{\sum F_{2y}}{m_2} = \frac{(m_2 g - T_2)}{m_2}$

  Since the thread is taut, $a_{1y} = a_{2y} \equiv a_y$

  Solving for the tensions, $T_1 = m_1 g + m_1 a_y$ and $T_2 = m_2 g - m_2 a_y$

  If the thread is massless and the pulley massless and frictionless, then $T_1 = T_2$

  So $m_1 g + m_1 a_y = m_2 g - m_2 a_y$

  or $a_y = g\frac{m_2 - m_1}{m_2 + m_1}$

- From Newton's Second Law, derive an expression for the tension in the thread connecting the two masses. How does this expression reduce when the masses are the same? How about when one mass is much larger than the other?

  Starting as above with Newton's Second Law applied to each mass and $T_1 = T_2 \equiv T$

  equating the two accelerations gives $a_{1y} = a_{2y}$

  so $\frac{T - m_1 g}{m_1} = \frac{m_2 g - T}{m_2}$

  Solving for the tension, $T = \frac{2m_2 m_1 g}{m_2 + m_1}$

  When the masses are the same, $m_1 = m_2 \equiv m$

  so $T = \frac{2mmg}{m + m} = mg$

  This makes sense, since the tension on each side of the thread is simply equal and opposite to the weight of each mass when they are stationary.

  When $m_2 \gg m_1$, we can consider m_1 to approach zero,

  so $T = \frac{2m_2(0)g}{m_2 + 0} = 0$

  This is reasonable, since no string tension would be opposing the free fall of Mass 2.

- After students have studied the work–energy theorem, use it to derive the same expression for the acceleration of an Atwood's machine as was obtained in this lab from Newton's Second Law.

  Consider the masses starting from rest and moving a distance y. The work done by gravity to displace the masses should be equal to the change in kinetic energy,

  so $W = K_f - K_i$

  or $m_2gy - m_1gy = \left(\frac{1}{2}m_2v_{2f}^2 + \frac{1}{2}m_1v_{1f}^2\right) - \left(\frac{1}{2}m_2v_{2i}^2 + \frac{1}{2}m_1v_{1i}^2\right)$

  With the masses starting from rest, $v_{2i} = v_{1i} = 0$

  and the thread taut, $v_{2f} = v_{1f} \equiv v_f$

  this becomes $m_2gy - m_1gy = \frac{1}{2}m_2v_f^2 + \frac{1}{2}m_1v_f^2$

  Solving for the speed squared, $v_f^2 = \frac{2g\left(m_2 - m_1\right)y}{m_2 + m_1}$

  With zero initial speed, the kinematic equation $v_f^2 = v_i^2 + 2a_yy = 2a_yy$

  can be substituted in to give $2a_yy = \frac{2g\left(m_2 - m_1\right)y}{m_2 + m_1}$

  Solving for the acceleration, $a_y = g\frac{m_2 - m_1}{m_2 + m_1}$

NAME PERIOD DATE

3. ATWOOD'S MACHINE

STRUCTURED

Driving Question | Objective

How is the acceleration of the two masses of an Atwood's machine affected by their difference in mass and by their total mass? Experimentally determine the mathematical relationship between the acceleration of an Atwood's machine, the difference between its two masses, and the sum of those two masses.

Materials and Equipment

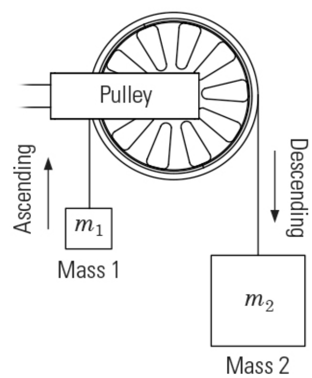
- Data collection system
- PASCO Smart Gate photogate[1]
- PASCO Super Pulley with Mounting Rod[1]
- Mass and Hanger Set
- Table clamp or large base
- Support rod, 60-cm or taller
- Right angle clamp
- Thread, about 1 m
- Scissors

[1] www.pasco.com/ap21

PASCO Smart Gate

Background

An Atwood's machine consists of two masses connected by a light thread over a pulley. If the mass m_2 in the diagram at right is greater than m_1, m_2 will accelerate downward and m_1 will accelerate upward. If the string connecting the masses is taut and does not stretch, the two masses will experience the same acceleration and they can be considered to be a single system. The acceleration of this system will depend on both the mass difference between the two masses and the total mass of the system, which is the sum of the two masses.



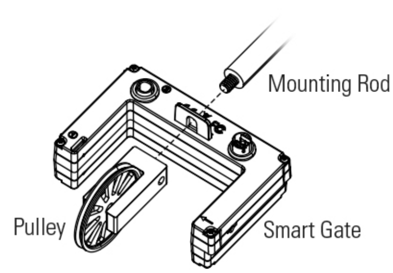
Procedure

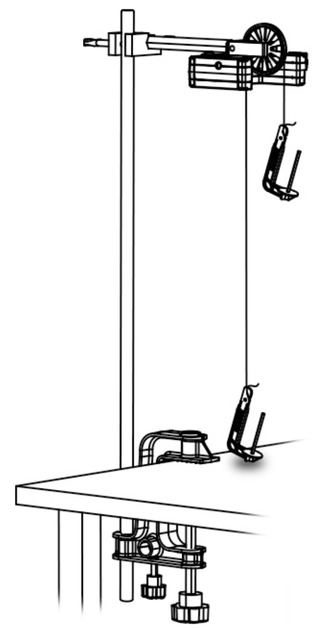
Part 1 – Varying Mass Difference

SET UP

1. Assemble and mount the table clamp and support rod on the edge of a table, or assemble the base and support rod and place it on the tabletop.

2. Attach the pulley to the tab of the photogate using the mounting rod so that the spinning pulley spokes will interrupt the beam of the photogate.





3. Attach the mounting rod horizontally on the support rod using the right angle clamp. Place it near the top of the rod with the pulley over the tabletop.

4. Cut a length of thread about 15 cm longer than the distance from the top of the pulley to the tabletop.

5. Place the thread on the pulley, threading one end through the gap between the pulley and its frame. Tie a loop on this end of the thread and place an empty mass hanger on it. Rest the attached hanger on the tabletop.

6. Tie a loop on the other end of the thread just below the pulley. Place the other empty mass hanger on this loop.

7. Connect the photogate to your data collection system.

8. Configure the data collection system for a Photogate with Pulley or Smart Pulley (Linear) with the default spoke arc length of 0.015 m to measure the linear speed.

9. Create a graph display of Linear Speed on the y-axis with Time on the x-axis.

10. Add 140 g of mass (suggested masses: 100-g + two 20-g) to the 5-g hanger resting on the tabletop, for a total of 145 g hanging from the string on the side of the pulley closer to the mounting rod.

NOTE: Throughout this activity, the mass hanging on the side of the pulley closer to the mounting rod will be the lesser of the Atwood's machine's two masses and will be referred to as Mass 1.

11. Support the suspended hanger with your hand to prevent it from dropping as you add the following masses to it. Add 195 g of mass (suggested masses: 100-g + 50-g + 20-g + 10-g + three 5-g) to the 5-g suspended hanger, for a total of 200 g hanging from the thread on this side. Continue to support it with your hand.

NOTE: Throughout this activity, the mass hanging on the side of the pulley farther from the mounting rod will be the greater mass and will be referred to as Mass 2.

12. Before you collect data, practice releasing and catching the masses: Slightly lower Mass 2 so Mass 1 lifts just off the tabletop. Once any swinging has settled, release Mass 2 and then gently catch the rising mass just before it strikes the pulley. Once you are done practicing, return Mass 1 to the tabletop and continue holding the greater mass suspended just below the pulley height.

COLLECT DATA

13. Begin data recording.

14. Release Mass 2 and then catch the rising Mass 1 just before it strikes the pulley.

15. Stop data recording.

    NOTE: If the two mass hangers collided, delete the run and record another.

16. Gently lower Mass 2 to rest on the tabletop.

17. Use the tools on your data collection system to determine the slope of a linear fit to your Linear Speed versus Time data during the time when the masses were moving freely. Record this as the *acceleration* of the system in Table 1.

18. Also in Table 1, record the mass of Mass 1 (including the hanger) and that of Mass 2 (including the hanger).

19. Repeat data collection four more times, transferring 5 g from Mass 2 (the greater mass) to Mass 1 between each trial.

Part 2 – Varying Total Mass

COLLECT DATA

20. Continue with the Part 1 setup for Part 2.

21. Copy the data from your final trial (Trial 5) in Part 1 into the first row of Table 2.

    NOTE: This eliminates the need to repeat data collection for this same mass combination in Part 2.

22. Remove 30 g from each of the mass hangers, leaving a total of 135 g (including hanger mass) on Mass 1 and 150 g (including hanger mass) on Mass 2.

23. Repeat the data collection steps from Part 1 to determine the system's acceleration. Record the acceleration in Table 2 as Trial 6. Also record the masses Mass 1 and Mass 2.

24. Repeat data collection four additional times, removing an additional 30 g from both sides between each trial.

Data Analysis

Part 1 – Varying Mass Difference

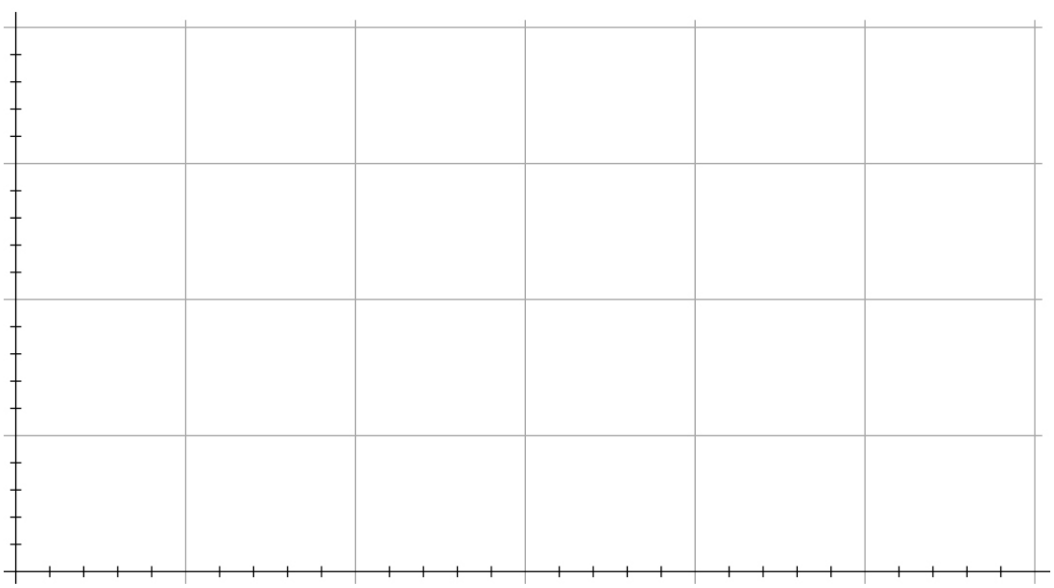
Table 1: Varying mass difference with constant total mass

Trial	Acceleration (m/s^2)	Mass 1 (kg)	Mass 2 (kg)	Mass Difference (kg)	Total Mass (kg)
1					
2					
3					
4					
5					

1. Calculate the difference between Mass 2 and Mass 1, $m_2 - m_1$, and total mass, $m_2 + m_1$, for each trial in Part 1. Record the results in Table 1 in units of kilograms (kg).

2. Plot a graph of *acceleration* versus *mass difference* in Graph 1. Be sure to label both axes with the correct scale and units.

Graph 1: Acceleration versus mass difference with constant total mass



Part 2 – Varying Total Mass

Table 2: Varying total mass with constant mass difference

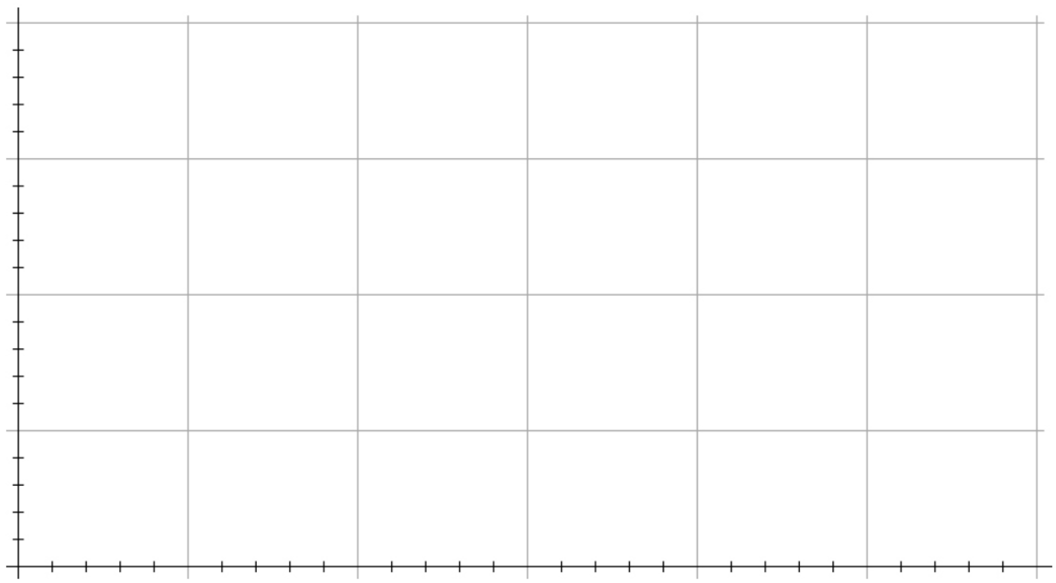
Trial	Acceleration (m/s^2)	Mass 1 (kg)	Mass 2 (kg)	Mass Difference (kg)	Total Mass (kg)	1/Total Mass (kg^{-1})
5						
6						
7						
8						
9						
10						

3. Calculate the mass difference, $m_2 - m_1$, and total mass, $m_2 + m_1$, for each trial in Part 2. Record the results in Table 2 in units of kilograms (kg).

4. Calculate the inverse of the total mass (1/total mass) for each trial in Part 2. Record the results in Table 2 in units of inverse kilograms (kg^{-1}).

5. Plot a graph of *acceleration* versus *1/total mass* in Graph 2. Be sure to label both axes with the correct scale and units.

Graph 2: Acceleration versus 1/total mass with constant mass difference



Analysis Questions

1. For each part of your experiment, list each variable and indicate whether it was held constant, increased, or decreased.

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2. How did changing the difference in mass between the two sides affect the acceleration of the Atwood's machine?

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3. Based on your data, express the relationship between the acceleration a_y and mass difference $m_2 - m_1$ by completing this proportionality statement:

   $a_y \propto$ ____________________ (total mass held constant)

4. How did changing the sum of the two hanging masses affect the acceleration of the Atwood's machine?

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5. Based on your data, express the relationship between the acceleration a_y and total mass $m_2 + m_1$ by completing this proportionality statement:

   $a_y \propto$ ____________________ (mass difference held constant)

6. Combine the two relationships above into a single proportionality expressing the relationship between the Atwood's machine's acceleration a_y the mass difference $m_2 - m_1$ and the total mass $m_2 + m_1$:

   $a_y \propto$ ____________________

7. Convert the proportionality statement above into an equation by introducing a proportionality constant k:

   $a_y =$ ____________________

8. Use your data to determine the proportionality constant k. Be sure to specify its units. Briefly explain how you determined its value and units.

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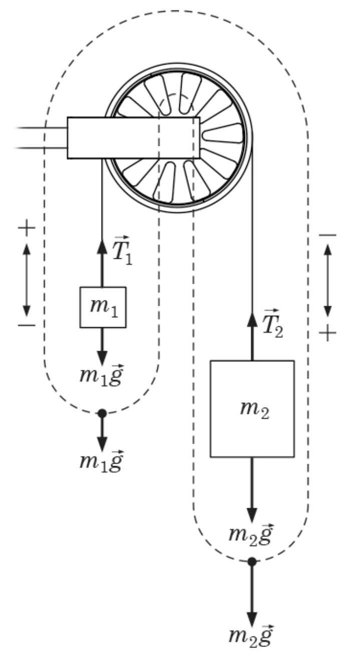
9. Consider this free-body diagram of an Atwood's machine. Assume that the masses of the string and pulley are negligible. The analysis can be simplified if the system is defined to consist of the two masses linked together, as indicated by the dashed line. This allows you to disregard the string tension as an internal force and consider only the two forces m_1g and m_2g acting on the system. You can also consider the system to be moving in one dimension, with *positive* defined in the direction of m_1 ascending and m_2 descending, as indicated.



Apply Newton's Second Law, $\vec{a} = \dfrac{\vec{F}_{\text{net}}}{m}$, to derive an expression for the acceleration a_y of the system in terms of the masses m_1 and m_2.

10. How does the expression for acceleration that you determined from your data analysis compare to the equation derived above from Newton's Second Law? Justify your answer.

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Synthesis Questions

1. One way to check whether a derived relationship is reasonable is to consider whether it behaves as expected in extreme or *limiting* cases. Determine whether the relationship you derived between the acceleration a of an Atwood's machine and its two hanging masses m_2 and m_1 reduces to a reasonable form when the two masses are equal. Explain your reasoning.

2. Similarly, determine whether the relationship you derived between the acceleration a of an Atwood's machine and its two hanging masses m_2 and m_1 reduces to a reasonable form when the mass m_2 is much greater than m_1. Explain your reasoning.

3. A planetary rover carries an Atwood's machine with 100 g one side and 110 g on the other. If the system's acceleration is measured to be 0.176 m/s^2 on a certain planet, what is the acceleration due to gravity on that planet? Which planet in the solar system is it?

4. What ratio of masses m_2/m_1 would produce an Atwood's machine whose acceleration is half that of an object in free fall?

❓ 5. Elevators cars have counterweights to reduce the amount of work motors need to do to lift the car. You might idealize an elevator system without its motor as an Atwood's machine. If a particular elevator's counterweight mass is 1,000 kg and its elevator car and passengers have a combined mass of 1,200 kg, what acceleration would the passengers experience if the motors and safety braking mechanisms failed? If the elevator car accelerated from rest at a height of 12 m above the ground floor, how long would it take for the car to reach the ground floor? What would be its speed on impact?

4. COEFFICIENTS OF FRICTION

Connections to the AP® Physics 1 Curriculum*

The lab activity correlates to the following pieces of the AP® Physics 1 framework:

Big Idea	3	Enduring Understanding	C	Essential Knowledge	4

Learning Objective 1: The student is able to make claims about various contact forces between objects based on the microscopic cause of those forces.
Science Practices: 6.1

Learning Objective 2: The student is able to explain contact forces (tension, friction, normal, buoyant, spring) as arising from interatomic electric forces and that they therefore have certain directions.
Science Practices: 6.2

Time Requirement

Preparation Time: 10 minutes

Lab Activity: 45 minutes

Prerequisites

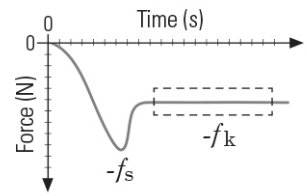
Students should be familiar with the following concepts:

- Friction is a contact force between two surfaces that opposes the motion of the surfaces. Frictional force has two types: *static*, which exists between two non-moving surfaces, and *kinetic*, which exists between two moving surfaces. Static frictional force is usually greater than kinetic frictional force.
- Newton's Second Law: The acceleration of an object is proportional to the net force acting on that object.
- Students should be able to analyze the motion of an object and draw free-body diagrams describing the forces acting on that object.

Driving Question | Objective

How can the coefficients of kinetic and static friction between two surfaces be determined? Experimentally determine the static and kinetic friction coefficients between two contacting surfaces.

Procedural Overview



Students use a force sensor to measure the force applied to start an object in motion and drag it across a surface at a constant speed (constant speed is assured by monitoring the velocity of the object using a motion sensor). Assuming the object's speed is constant (net force is zero while the object is stationary and when it is moving), students can assume that the force applied to the object to start it in motion and drag it across the surface is equal and opposite to the frictional force experienced by the object. Using their graphs of applied force versus time, students identify the magnitude of the force corresponding to the static f_s and kinetic f_k frictional forces and record values for each.

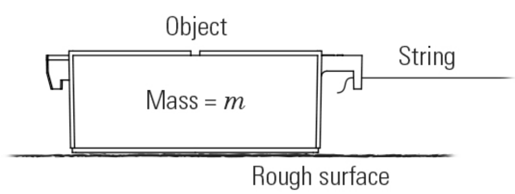
In each trial, mass is added to the object, increasing the normal force and thus increasing both frictional forces. Students plot frictional (applied) force versus normal force for both static and kinetic force values. Using the slope of each best fit line, students determine experimental values for the coefficient of static friction and kinetic friction.

Pre-Lab Discussion and Activity

This is a pre-lab suggestion and activity.

Pre-Lab Questions

Use the following figure when answering the pre-lab questions. Assume that the object is resting motionless on the rough surface and is free to move in any direction.



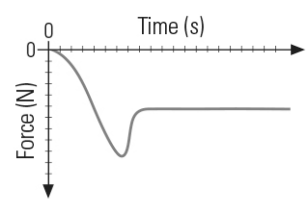
1. A person pulls on the string and the object does not move. What forces are acting on or opposing the motion of the object? How do the magnitudes of the forces compare to each other?

   The force due to gravity and the normal force are equal in magnitude but opposite in direction. The applied force and the static frictional forces are equal in magnitude but opposite in direction.

2. What must happen to the forces in order for the object to move?

   The applied force must be greater than the static frictional force.

3. The graph to the right shows the force applied to the string as the object is started in motion and dragged across the rough surface at a constant speed. What part of the graph corresponds to the point at which the object started in motion?

   This motion starts at the peak of the graph.

4. What is occurring in the flat section of the graph with respect to the forces? Describe the corresponding speed and acceleration.

   The applied force and the kinetic frictional forces are equal but opposite in direction. The speed is constant and the acceleration is zero.

Materials and Equipment

- Data collection system
- Masses (5), 250-g
- Thread, 20 cm
- Balance, 0.1-g resolution, 2,000-g capacity (1 per class)
- PASCO Motion Sensor[1]
- PASCO High Resolution Force Sensor with hook[2]
- PASCO Discover Friction Accessory tray[3]

[3]There are four different trays in this set, each with a different surface material on the bottom. Choose one tray.

Probeware Resources

Below are web-link and QR codes that will direct you to instructional video resources for individual pieces of PASCO probeware, sensors, and other hardware used in the lab activity. These same links and codes are provided to students in their activity handouts.

[1]www.pasco.com/ap18	[2]www.pasco.com/ap22	[3]www.pasco.com/ap09
PASCO Motion Sensor	PASCO High Resolution Force Sensor	PASCO Discover Friction Accessory

Teacher Tips

Tip 1 – Help Smooth Student Data

- In the structured version of this activity, students determine static frictional force by recording force versus time data using a force sensor as they pull a tray filled with mass. Often students' graphs will look messy with jagged spikes and erratic increases and decreases in force measurements as they start the tray in motion. To achieve best results, students should initially pull with a *very* slow, constantly increasing force until the applied force just overcomes the static frictional force.
- Another way to help student graphs look smoother is to increase the sample rate on their data collection system to 50 Hz.

Tip 2 – Apply Force in the Direction of Motion

- When dragging an object using a force sensor, students should always apply force in the direction of motion: parallel to the surface across which the object is being dragged. Applying force at downward or upward angles will cause the applied force to be greater or less than the actual frictional force.

Tip 3 – Maintain Constant Speed without the Motion Sensor

- In the structured version of this activity, students use a motion sensor to monitor the velocity of the tray so as to keep speed constant. However, the same effect can be achieved without a motion sensor by students who can carefully monitor their force versus time graphs, making certain that their measured force stays constant with respect to time once the tray is in motion.

Tip 4 – Distribute Mass Evenly

- As students increase the normal force of their objects by increasing its mass, they must be careful to distribute the mass evenly so as to minimize any error due to inconsistencies in the contacting surface of their object.

Tip 5 – Inclined Surfaces

- If students choose to perform this experiment using an inclined plane, they must be certain to pay close attention to the angle of inclination and include it in all of their net force calculations, most importantly calculations involving normal force.

Sample Data

Below are sample data acquired using the experimental setup and procedure outlined in the Structured version of the lab activity, and answers to questions in the Data Analysis section.

Data Analysis

Student graphs of force versus time with varying normal force will look similar to this graph:

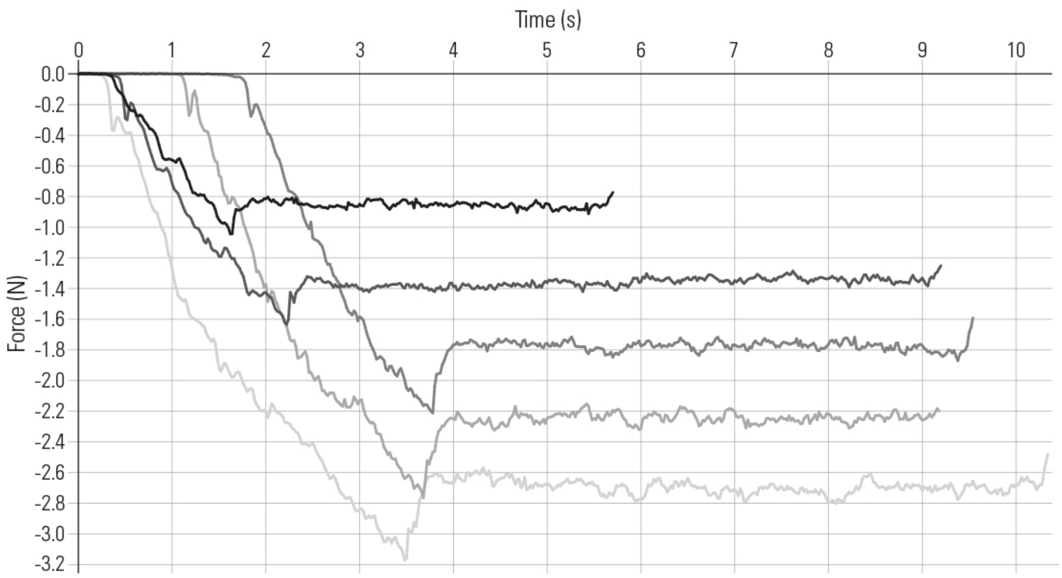


Table 1: Mass and frictional force data

Trial	Mass (kg)	Normal Force (N)	Static Frictional Force (N)	Kinetic Frictional Force (N)
1	0.334	3.28	1.040	0.849
2	0.582	5.71	1.630	1.355
3	0.832	8.16	2.208	1.770
4	1.082	10.6	2.760	2.241
5	1.332	13.1	3.167	2.690

1. Use your mass data from Table 1 to calculate the magnitude of the normal force acting on the tray in each trial. Record these values in Table 1. For your calculations, use $g = 9.81\ \text{m/s}^2$.

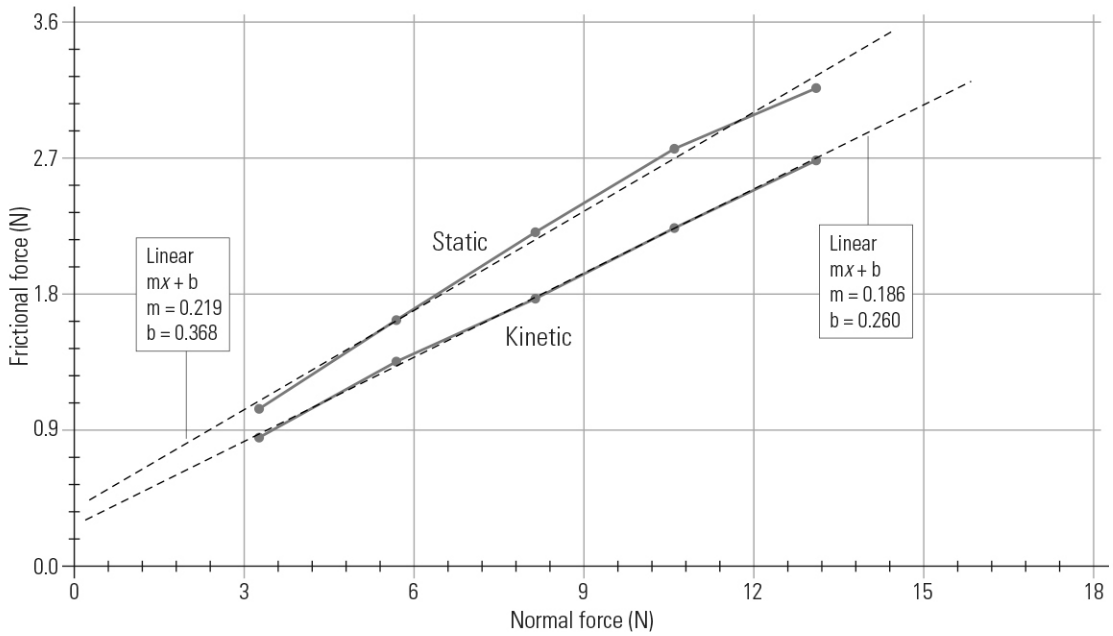
   Sample calculation for Trial 1:

   $$\left|\vec{F}_n\right| = 0.334\ \text{kg} \times 9.81\ \text{m/s}^2$$

   $$\left|\vec{F}_n\right| = 3.28\ \text{N}$$

2. Plot a graph of *frictional force* versus *normal force* in Graph 1. Be sure to label both axes with the correct scale and units. This plot should have two curves: one for static frictional force and a second for kinetic frictional force. Indicate in the graph which curve corresponds to which frictional force.

Graph 1: Static and kinetic frictional force versus normal force



3. Draw a line of best fit for each curve. Record the equation of each line here:

   Best fit line equation (static friction): $f_s = (0.219)F_n + 0.368$ N

   Best fit line equation (kinetic friction): $f_k = (0.186)F_n + 0.260$ N

4. Use the slope from each best fit line to determine the measured value for the coefficient of static friction and the coefficient of kinetic friction.

   Coefficient of static friction: 0.219

   Coefficient of kinetic friction: 0.186

Guided Inquiry Questions

Below are sample responses to the Guiding Questions found in the Guided Inquiry version of this lab activity.

STATIC FRICTION

1. What physical quantities can be measured or calculated to help determine the coefficient of static friction? (If you are unsure of the answers to questions 1 and 2, skip to question 3. Later, return to answer questions 1 and 2.)

   Students can choose to measure or calculate the magnitude of the normal force (measure the normal force by measuring the weight of the object; calculate the magnitude of the normal force using the equation F_n = mass × 9.81 m/s^2). If students are using an inclined plane, normal force measurements and calculations must include a factor of cos(θ), where θ is the angle of inclination.

2. Of these physical quantities, which will be easier to measure than calculate? How will you measure it?

   Normal force is a quantity that can be measured or calculated using F_n = mass × 9.81 m/s^2. Students will find it much easier to calculate normal force, while it will be easier to measure any applied forces than calculate them. Students can measure frictional force indirectly using a force sensor to measure the applied force. Given Newton's Third Law, the frictional force will be equal to the measured applied force as long as the object is not accelerating (constant speed).

❷ 3. It is not possible to directly measure the force of friction. Therefore, how can you set up the equipment to measure the force of friction indirectly?

Students can indirectly measure frictional force by measuring the applied force used to start their object in motion and drag it across a surface. This assumes that the surface they are dragging across is level and the speed at which the object moves, once in motion, is constant.

Some students may choose to use an inclined plane where they measure the angle at which the object just begins to slide down the plane (the point at which gravitational force is equal to static frictional force) and the angle at which the object slides with constant speed after having been given a slight push (the point at which gravitational force is equal to kinetic frictional force).

❷ 4. Imagine measuring the force needed to push an object across a rough surface. At what point are you measuring the maximum static frictional force?

Maximum static frictional force is measured at the point where the students' object just begins to move.

❷ 5. Up until and including the point of the maximum static frictional force, what is the relationship between the opposing forces that you measure?

The static frictional force and the applied tension force are equal and in opposite directions until the object begins to move.

KINETIC FRICTION

❷ 6. Is it possible to apply a force to an object and still have it move with constant speed (that is not zero)? If so, how is that possible? What forces are involved?

Yes. If the opposing forces on the friction tray are equal in magnitude, then the tray will move with constant speed.

❷ 7. What measurement(s) are similar in the determination of the coefficient of kinetic friction and the coefficient of static friction?

Students should make the same measurements when determining both static and kinetic coefficients of friction (normal force and frictional force).

Assessment Questions: Sample Responses

Sample responses to the Analysis and Synthesis questions found in each version of the lab activity:

Analysis Questions

❷ 1. Describe the two contacting surfaces for which you determined the coefficients of static and kinetic friction, and indicate your results for the experimental values for each.

Student may use any two surfaces; however, the PASCO Discover Friction Accessory Tray set uses the following four surfaces:

- Cork
- Felt fabric
- Plastic composite #1
- Plastic composite #2

Frictional coefficient values will vary depending on the second surface on which the tray is dragged. Students should expect static and kinetic coefficient values in the range 0.100–0.600 when using the friction trays.

2. Find another lab group that tested the same (or similar) two contacting surfaces and compare your coefficients of static and kinetic friction to theirs. Calculate the percent difference in each.

$$\text{Percent difference} = \frac{\left|\text{Group}_2 - \text{Group}_1\right|}{\left|\left(\text{Group}_1 + \text{Group}_2\right)/2\right|} \times 100$$

Sample calculation:

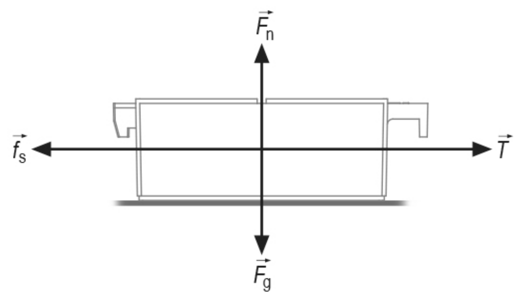
$$\text{Percent difference} = \frac{\left|0.219 - 0.202\right|}{\left|\left(0.202 + 0.219\right)/2\right|} \times 100 = 8.08\%$$

3. What are some factors that may have caused your values to be greater or less than the other group's? How could you have corrected these factors?

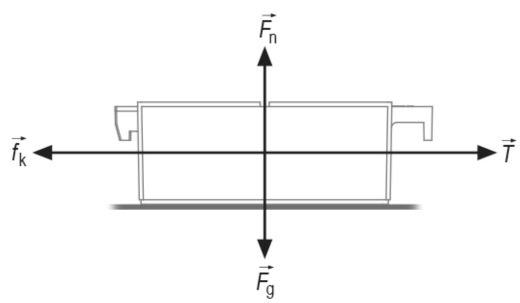
Some factors may include, but are not limited to:

- Dust gathered on one surface versus the other
- Inconsistent surface shape/material
- Uneven distribution of mass along or within the object being dragged
- Surface materials may not be identical
- Force wasn't applied in the direction of motion (parallel to the surface across which the object is being dragged)

4. Draw and label a force diagram (free-body diagram) that represents the forces on the object the moment that the object in this experiment experienced the maximum static frictional force.

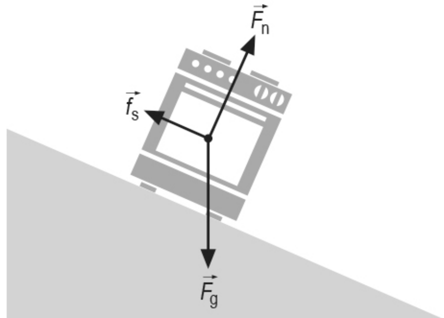


5. Draw and label a force diagram (free-body diagram) that corresponds to the forces on the object the moment the object in this experiment maintained constant speed.

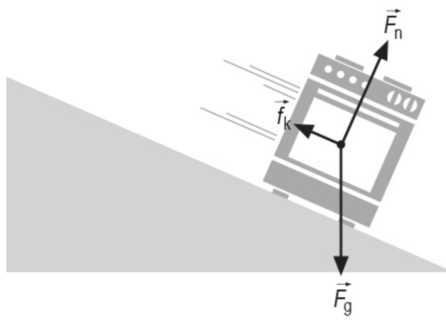


Synthesis Questions

1. Draw a diagram showing a real world situation in which an object is motionless on an incline. Overlay a labeled force diagram on the object.



2. Draw a diagram showing the same object from the previous question skidding down the same incline (accelerating slightly down the incline). Assume the incline now has a much more slippery surface, but not frictionless. Overlay a labeled force diagram on the object.



3. Compare the two coefficients of friction: static and kinetic. Which is greater? Do you expect this coefficient of friction to be greater in most situations? Explain.

   Experimentation will show that the coefficient of static friction is greater, and is expected to be so in most situations. The explanation is complex, as it involves the inertia of moving an object that is not moving versus continuing to move an object that is already moving, which requires less force. In addition, the irregularities and impurities between surfaces are difficult to generalize. Experimentation shows that the differences between static and kinetic friction dissipate with smooth objects.

4. It takes a force of 22 N to push a 10.0 kg box horizontally with a constant speed over concrete. What is the coefficient of kinetic friction between the box and the concrete?

   Since the object is moving with constant speed, the applied force equals the kinetic frictional force:

$$f_k = \mu_k F_n$$

$$\mu_k = \frac{f_k}{mg}$$

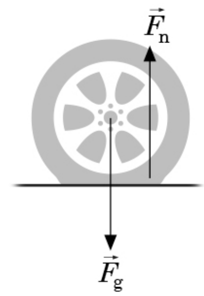
$$\mu_k = \frac{22\text{ N}}{(10.0\text{ kg})(9.8\text{ m/s}^2)} = 0.22$$

Extended Inquiry Suggestions

The microscopic view of friction is often glossed over in many high school physics classes. An extended inquiry activity may include the study of the irregularities between contact surfaces.

Since students often think about the friction involved in wheels, another extended inquiry activity may include *rolling friction*. Rolling friction is due to the deformation of the wheel material as it rolls. For example, round objects made of steel are less likely to deform than round objects made of rubber, therefore rigid rolling object experience much less rolling friction.

Observe the exaggerated picture below of a car tire moving to the right. The tire becomes deformed as it rolls, shifting the normal force forward. Because this normal force is no longer pointed at or through the rotational axis of the wheel, a torque is generated that opposes the rotational motion of the wheel as it rolls, thereby slowing it down.



NAME PERIOD DATE

4. COEFFICIENTS OF FRICTION

STRUCTURED

Driving Question | Objective

How can the coefficients of kinetic and static friction between two surfaces be determined? Experimentally determine the static and kinetic friction coefficients between two contacting surfaces.

Materials and Equipment

- Data collection system
- Masses (5), 250-g
- Thread, 20 cm
- Balance, 0.1-g resolution, 2,000-g capacity (1 per class)
- PASCO Motion Sensor[1]
- PASCO High Resolution Force Sensor with hook[2]
- PASCO Discover Friction Accessory tray[3]

[3]There are four different trays in this set, each with a different surface material on the bottom. Choose one tray.

[1]www.pasco.com/ap18 — PASCO Motion Sensor

[2]www.pasco.com/ap22 — PASCO High Resolution Force Sensor

[3]www.pasco.com/ap09 — PASCO Discover Friction Accessory

Background

You may have noticed that pushing a heavy cardboard box over a smooth surface (like carpet) requires less effort than pushing it over a rough surface (like cement). This effect is due to the different frictional forces between the box and the smooth surface, and the box and the rough surface.

To quantify or compare the amount of friction between any two surfaces, scientists employ the *coefficient of friction*. There are two types of friction: *static* and *kinetic*. Hence, there are two coefficients of friction: the coefficient of static friction and the coefficient of kinetic friction.

When you apply a relatively small force to the heavy cardboard box, you may notice that it does not move. The force that opposes your applied force is called *static friction*. As you apply an increasing force, the static frictional force matches your applied force up to some maximum amount. The following is the equation for the maximum static frictional force:

$$f_s = \mu_s F_n \qquad (1)$$

Where f_s is the magnitude of maximum static frictional force, μ_s is the coefficient of static friction, and F_n is the magnitude of the normal force.

Once you get the heavy cardboard box moving at constant speed, *kinetic friction* then opposes your applied force. The following is the equation for the kinetic frictional force:

$$f_k = \mu_k F_n \qquad (2)$$

Where f_k is the magnitude of kinetic frictional force, μ_k is the coefficient of kinetic friction, and F_n is the magnitude of the normal force.

Please note that the AP Physics 1 equation table combines the above equations into one equation:

$$\left|\vec{F}_{\text{f}}\right| \leq \mu\left|\vec{F}_{\text{n}}\right| \tag{3}$$

Where the frictional force $\vec{F}_{\text{f}}$ and normal force $\vec{F}_{\text{n}}$ are vector quantities and μ is the given *coefficient of friction* between the two surfaces in question.

RELEVANT EQUATIONS

$$f_{\text{s}} = \mu_{\text{s}} F_{\text{n}} \tag{1}$$

$$f_{\text{k}} = \mu_{\text{k}} F_{\text{n}} \tag{2}$$

$$\left|\vec{F}_{\text{f}}\right| \leq \mu\left|\vec{F}_{\text{n}}\right| \tag{3}$$

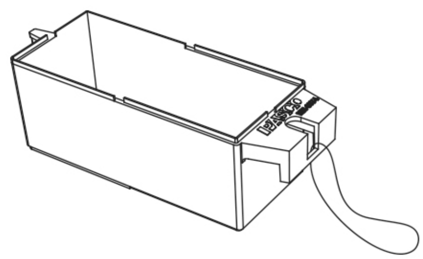
Procedure

SET UP

1. Set the motion and force sensors on the lab table with the front of the motion sensor aimed parallel to the table's surface. Make certain the switch on the top of the motion sensor is set to the cart icon.

2. Connect both sensors to the data collection system and then create two graph displays: one graph display of velocity versus time, and the second graph display of force versus time.

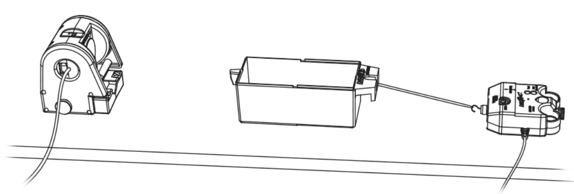
3. Thread a 20-cm long piece of thread through the slot on the front of the friction tray. Tie the ends to make a loop.



4. Make certain the hook accessory is attached to the front of the force sensor and the sensor is sitting flat on the lab table. Press the "ZERO" button on the top of the force sensor.

COLLECT DATA

5. Place 250 g of mass in the friction tray, and then measure the combined mass of the friction tray plus the mass using a balance. Record the mass in Table 1 next to Trial 1.



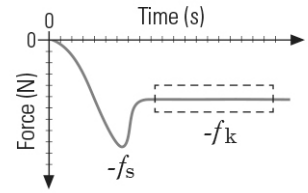
6. Set the tray with the masses on the lab table in front of the motion sensor about 15 cm away from the front of the sensor with the thread end of the tray facing away.

7. Attach the force sensor to the thread loop using the hook on the front of the sensor.

8. Start recording data, and then, using the force sensor to pull, very slowly begin pulling the tray away from the motion sensor. The tray will not move at first as the force increases to some maximum, at which point the tray will begin to slide.

9. Once it begins to slide, continue slowly dragging the tray for approximately 3 seconds while watching your velocity versus time graph, making certain to keep velocity as constant as possible as you drag. Stop recording data after the 3 seconds.

10. Observe the Force versus Time graph. If the procedure was carried out properly, the force increases (negatively) to a peak, decreases, and then levels off, similar to the graph at right.



11. Use the tools on your data collection system to determine the maximum static frictional force f_s between the tray and the lab table (the peak of the graph). Record this value in Table 1.

    NOTE: Your graph of force versus time shows the force applied by the force sensor, not the frictional force. The applied force measured by the sensor can be assumed to be equal and opposite to the frictional force: $F_f = -F_{applied}$

12. Use the tools on your data collection system to determine the average of the data in your graph corresponding to the kinetic frictional force f_k between the tray and the lab table (the leveled-off section of the graph). Record this value in Table 1. Remember: $F_f = -F_{applied}$

13. Repeat the previous data collection steps 4 more times, each time adding an additional 250-g mass to the friction tray. Record the resulting mass, maximum static frictional force, and average kinetic frictional force for each trial in Table 1.

    NOTE: Distribute the mass evenly in the tray.

Data Analysis

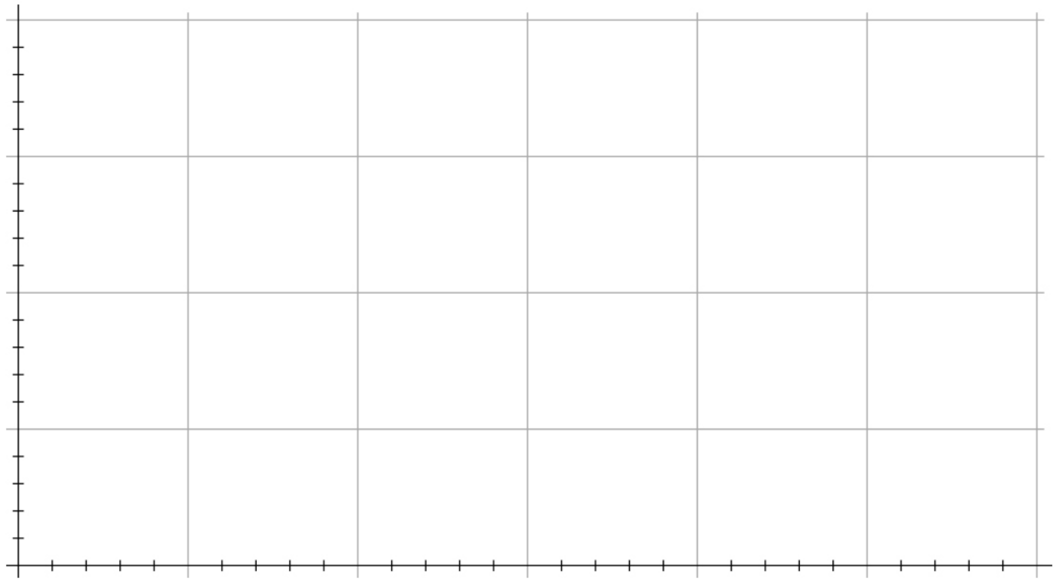
Table 1: Mass and frictional force data

Trial	Mass (kg)	Normal Force (N)	Static Frictional Force (N)	Kinetic Frictional Force (N)
1				
2				
3				
4				
5				

1. Use your mass data from Table 1 to calculate the magnitude of the normal force acting on the tray in each trial. Record these values in Table 1. For your calculations, use $g = 9.81\ m/s^2$.

2. Plot a graph of *frictional force* versus *normal force* in Graph 1. Be sure to label both axes with the correct scale and units. This plot should have two curves: one for static frictional force and a second for kinetic frictional force. Indicate in the graph which curve corresponds to which frictional force.

Graph 1: Static and kinetic frictional force versus normal force



3. Draw a line of best fit for each curve. Record the equation of each line here:

    Best fit line equation (static friction): ________________________________

    Best fit line equation (kinetic friction): _______________________________

4. Use the slope from each best fit line to determine the measured value for the coefficient of static friction and the coefficient of kinetic friction.

    Coefficient of static friction: ________________________

    Coefficient of kinetic friction: ________________________

Analysis Questions

1. Describe the two contacting surfaces for which you determined the coefficients of static and kinetic friction, and indicate your results for the experimental values for each.

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____

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2. Find another lab group that tested the same (or similar) two contacting surfaces and compare your coefficients of static and kinetic friction to theirs. Calculate the percent difference in each.

$$\text{Percent difference} = \frac{|\text{Group}_2 - \text{Group}_1|}{|(\text{Group}_1 + \text{Group}_2)/2|} \times 100$$

❓ 3. What are some factors that may have caused your values to be greater or less than the other group's? How could you have corrected these factors?

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____

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❓ 4. Draw and label a force diagram (free-body diagram) that represents the forces on the object the moment that the object in this experiment experienced the maximum static frictional force.

❓ 5. Draw and label a force diagram (free-body diagram) that corresponds to the forces on the object the moment the object in this experiment maintained constant speed.

Synthesis Questions

❓ 1. Draw a diagram showing a real world situation in which an object is motionless on an incline. Overlay a labeled force diagram on the object.

❷ 2. Draw a diagram showing the same object from the previous question skidding down the same incline (accelerating slightly down the incline). Assume the incline now has a much more slippery surface, but not frictionless. Overlay a labeled force diagram on the object.

❷ 3. Compare the two coefficients of friction: static and kinetic. Which is greater? Do you expect this coefficient of friction to be greater in most situations? Explain.

❷ 4. It takes a force of 22 N to push a 10.0 kg box horizontally with a constant speed over concrete. What is the coefficient of kinetic friction between the box and the concrete?

5. TWO-DIMENSIONAL MOTION: PROJECTILES

Connections to the AP® Physics 1 Curriculum*

The lab activity correlates to the following pieces of the AP® Physics 1 framework:

Big Idea	3	Enduring Understanding	E	Essential Knowledge	1

Learning Objective 3: The student is able to use force and velocity vectors to determine qualitatively or quantitatively the net force exerted on an object and qualitatively whether kinetic energy of that object would increase, decrease, or stay the same.
Science Practices: 1.4, 2.2

Learning Objective 4: The student is able to apply mathematical routines to determine the change in kinetic energy of an object given the force on the object and the displacement of the object.
Science Practices: 2.2

Time Requirement

Preparation Time: 10 minutes

Lab Activity: 50 minutes

Prerequisites

Students should be familiar with the following concepts:

- The 1-dimensional motion of an object undergoing constant acceleration can be described using kinematic equations.
- The 2-dimensional velocity of a projectile is described using component vectors in the vertical and horizontal directions, and each component vector can be analyzed independent of the other except that both must stay constrained to the same time frame.
- Students should be familiar with and able to draw free-body force and component vector diagrams of objects in motion.

Driving Question | Objective

What is the range of a projectile launched horizontally? Develop a plan to measure the variables that affect the two-dimensional motion of a projectile launched horizontally, and then use those variables to accurately predict and test the projectile's horizontal range.

Procedural Overview

Students assemble and mount a projectile launcher to their lab table and then make measurements of the variables (initial height and initial velocity) affecting the range of the projectile as it is launched horizontally. Students use their measurements to calculate a predicted value for the range of their projectile, and then they test their prediction. Students qualitatively analyze the accuracy and precision of their launcher based on the predicted range and the distribution of their actual range values.

Pre-Lab Discussion and Activity

The first of the following two pre-lab discussions can be delivered to all student groups, that is, those performing the Structured, Guided Inquiry, and Student Designed versions of the activity. The second discussion is intended for only those students using the Guided Inquiry or Student Designed versions of this lab activity.

1. Demonstrate to students the division between vertical and horizontal motion by dropping a ball and throwing an identical ball simultaneously from the same height. If the thrown ball is thrown horizontally (all of its initial velocity is horizontal) both balls will hit the ground at exactly the same time. Ask students, "Why do the balls strike the ground at the same time?"

   The solution involves the independence of horizontal and vertical vectors. Although the object that is thrown has an initial velocity, its initial velocity in the vertical direction is zero, as is the initial vertical velocity of the dropped ball. Because the horizontal velocity does not affect the vertical velocity, both balls strike the ground at the same time.

   As an extension, show students a slow motion video or a computer animation of an object launched at an angle from ground level and allowed to land at ground level. Draw horizontal and vertical velocity vectors at evenly spaced positions to emphasize the independence of the vectors.

2. One of the objectives of this lab activity is for students to measure the variables that affect the range of a horizontally-launched projectile. Students not using the Structured version of this lab may not know which variables affect the projectile's range (in the Structured version, these variables are outlined for the student). You may choose to have these students research and derive the equation for the range of a projectile launched horizontally, or to give them this mathematical derivation as a classroom discussion or lecture. Having the derivation will help students identify the variables that affect the projectile's range, clarifying the variables that must be measured in this activity. This derivation is outlined in the Background section of the Structured version handout.

Materials and Equipment

- Data collection system
- PASCO Smart Gate photogate[1]
- PASCO Mini Launcher[2]
- PASCO Photogate Mounting Bracket[2]
- Launcher loading rod[2]
- Mini launcher table clamp[2]
- Steel ball, 1.6-cm diameter[2]
- White paper, 1 sheet
- Carbon paper, 1 sheet
- Cardboard, square piece, 10 × 10 inch
- Meter stick

Probeware Resources

Below are web-link and QR codes that will direct you to instructional video resources for individual pieces of PASCO probeware, sensors, and other hardware used in the lab activity. These same links and codes are provided to students in their activity handouts.

[1]www.pasco.com/ap21	[2]www.pasco.com/ap05
PASCO Smart Gate	PASCO Mini Launcher

Safety

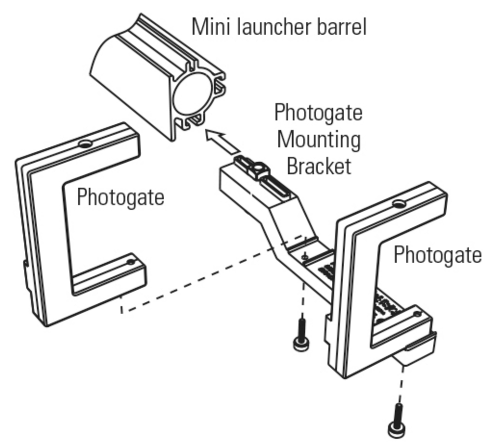
Follow these important safety precautions in addition to your regular classroom procedures:

- Wear safety goggles at all times.
- Do not look into the launcher.
- Do not aim the launcher at others.
- Use only what the teacher provides as the plunger to load the projectile launcher.

Teacher Tips

Tip 1 – Using Two Older PASCO Photogates With or Without a PASCO Digital Adapter

- The setup and procedure outlined in the Structured version of this lab activity utilizes a PASCO Smart Gate, but two older PASCO photogates will work seamlessly in its place. If your data collection system has digital inputs, the PASCO Digital Adapter is not needed; the photogates can be connected directly to the digital inputs on the data collection system. If there are no digital inputs on your data collection system, you will need to connect the older PASCO photogates using a PASCO Digital Adapter.
- When using two older PASCO photogates instead of the PASCO Smart Gate, you will need to mount both photogates to the projectile launcher as in the diagram to the right, and use the "Photogate Timing" pre-configured timer within the data collection system (see Teacher Tip 2).



Tip 2 – Use the Older "Photogate Timing" Setting

- Students using any version of this lab activity will need to measure the initial horizontal velocity of their projectile. When using two older PASCO photogates with a PASCO data collection system, students should use the pre-configured "Photogate Timing" timer/setting. This setting automatically includes measurements of time between gates and velocity/speed between gates. Students will need to enter in the data collection system the spacing between their photogates, which, if using a PASCO Photogate Bracket, will be 10 cm.

Tip 3 – Zero Angle of Inclination

- When testing their range predictions, students must first ensure that their projectile launchers have zero inclination angle (perfectly horizontal). Having even a slight inclination angle can cause the range of their test shots to vary greatly from their prediction.
- Every PASCO mini projectile launcher comes with a small angle indicator attached to the barrel of the launcher. Students can use the angle indicator to set the inclination angle to zero. Or, if they prefer, students can use a bubble level to make sure the barrel of the launcher is perfectly horizontal.

Tip 4 – Measure Height to the Projectile

- Students will need to measure the change in height of their projectile to accurately calculate its range. Change in height measurements should be made relative to the actual projectile. For example: if the target is the floor, height should be measured as the vertical distance from the floor to the bottom of the projectile as it leaves the barrel of the launcher (the PASCO mini launcher has a small diagram on the side of its barrel indicating the position of the projectile just as it leaves the launcher).

Sample Data

Below are sample data, acquired using the experimental setup and procedure outlined in the Structured version of the lab activity, and answers to questions in the Data Analysis section.

Data Analysis

Height of launcher at projectile exit point (m): 1.250

Table 1: Initial velocity of projectile launched horizontally

Trial	Initial Velocity (m/s)
1	5.03
2	5.05
3	5.05
4	5.03
5	5.05
Ave Velocity (m/s)	5.04

1. Calculate the projectile's average initial velocity and record your result into Table 1.

   Calculation using sample data:

   (5.03 m/s + 5.05 m/s + 5.05 m/s + 5.03 m/s + 5.05 m/s) / 5 = 5.04 m/s

Calculate the Range

2. Using your average launch velocity and the measured height of the launcher, calculate the range of the projectile using the equation:

$$\Delta x = v_{x0}\sqrt{\frac{2\Delta y}{g}} \tag{4}$$

For your calculation assume that $g = 9.81\ \text{m/s}^2$. Show your work here:

Calculation using sample data:

$$\Delta x = (5.04\ \text{m/s})\sqrt{\frac{2(1.250\ \text{m})}{9.81\ \text{m/s}^2}}$$

$$\Delta x = 2.54\ \text{m}$$

Test the Range

3. Draw a circle with a radius of 8 cm in the center of a piece of white paper, and then tape the paper to the floor in front of the projectile launcher with the center of the circle at a distance equal to your predicted range.

4. Place carbon paper over the white paper, and then align the projectile launcher with the center of the paper.

5. Place the steel ball into the launcher and then use the push rod or plunger to load the ball as far into the launcher as possible (three clicks).

6. Launch the ball toward the paper. Place the steel ball back into the launcher and repeat the test four more times.

7. Remove the carbon paper. Observe the locations where the ball struck the paper.

Guided Inquiry Questions

Below are sample responses to the Guiding Questions found in the Guided Inquiry version of this lab activity.

❓ 1. For a projectile launched horizontally, neglecting air drag, what horizontal and vertical forces are acting on it while it is in motion?

Neglecting air resistance, a free falling object (which includes projectiles) experiences only a downward force from earth's gravity. This force is constant and points downward toward the earth's surface.

❓ 2. Assuming air resistance is negligible, is a projectile that is launched horizontally accelerating in both the horizontal and vertical directions after it is launched? How do you know?

Because free falling objects only experience a downward force from gravity, they will only accelerate downward in the direction of the force. There is no horizontal acceleration once the projectile has been launched because there are no horizontal forces acting on it.

❓ 3. Kinematic equations are used to describe the motion of an object undergoing constant acceleration. Can kinematic equations be used to describe the motion of a projectile? Justify your answer.

Because free falling objects experience a downward force from gravity, and gravity is constant, the motion of a projectile can be described using kinematic equations.

4. What kinematic equation(s) would you use to help calculate the range of a projectile launched horizontally, and what variables from these equations must you know to accurately calculate range?

The kinematic equations: $\Delta x = v_{x0}t$ and $\Delta y = v_{y0}t - \frac{1}{2}gt^2$ can be simplified and combined to produce an equation for the range Δx of a projectile that is dependent only on the change in height Δy of the projectile, the initial horizontal velocity of the projectile v_{x0} and earth's gravitational acceleration constant g :

$$\Delta x = v_{x0}\sqrt{\frac{2\Delta y}{g}} \quad (4)$$

All values must be known to accurately calculate the range of a projectile.

5. What equipment and techniques could you use to determine the variables listed in the response to the previous question?

Initial horizontal velocity and change in height of the projectile must be measured by students, while earth's gravitational constant is known: g = 9.8 m/s^2. Students may choose to use any method for measuring these variables; however, the suggested method for assembling equipment measuring each variable is outlined in the Structured version of this lab activity.

Assessment Questions: Sample Responses

Sample responses to the Analysis and Synthesis questions found in each version of the lab activity:

Analysis Questions

1. Assuming air resistance is negligible, what other variables affect the range of a projectile?

Variables affecting the range of a projectile include: launch height, launch velocity (both horizontal and vertical if there is a non-zero launch angle), and launch angle (if non-zero).

2. Qualitatively describe how close your predicted range is to your actual range in terms of accuracy (the relative distance between each test shot and the actual target) and precision (the grouping of the test shots). Use the tables below to help.

Accuracy	Accuracy Definition
Very high	All shots in 8 cm target
High	Four shots in 8 cm target
Moderate	Three shots in 8 cm target
Low	Two shots in 8 cm target
Very low	One or no shots in 8 cm target

Precision	Precision Definition
Very high	All shots within 3 cm radius
High	Four shots within 5 cm radius
Moderate	Two shots within 8 cm radius
Low	Two shots within 20 cm radius
Very low	No shots within 30 cm radius

Students should indicate the accuracy and precision in their test shots. Accuracy should be quantified using the relative distance between each test shot and the actual target: the farther from the actual target, the lower the accuracy. Precision is based on the grouping of the test shots: the farther apart the test shots are from each other, the worse the precision.

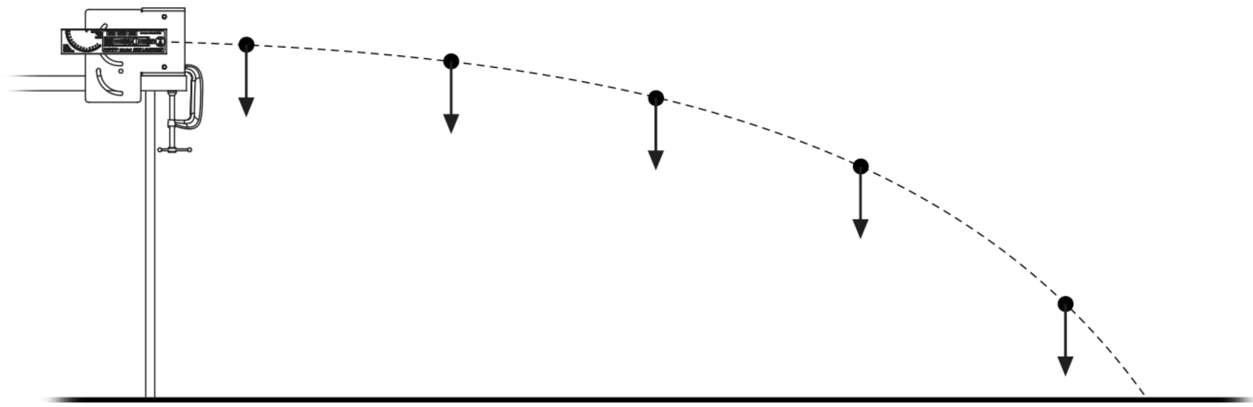
3. What are factors that may have caused your range prediction to be incorrect, and what could you have done differently to avoid them?

Sources of error may include, but are not limited to:

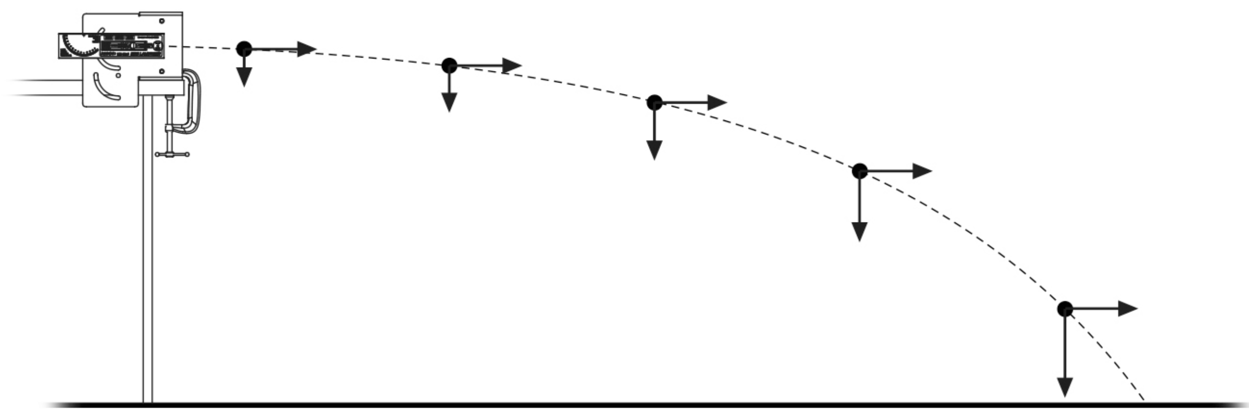
- A non-zero angle of inclination—positive angles will cause range test shots to be longer than the target; negative angles will cause range test shots to be shorter than the target. Students can use a bubble level or the built-in angle indicator on the barrel of the PASCO projectile launcher to ensure a zero-degree launch angle.
- Inaccurate measure of initial velocity—inaccuracies can arise from the measurement tools used to measure velocity, as well as the inability to have repeatable launch velocities from the launcher. Students should measure the initial velocity of their projectile using several trials to be averaged. This will help minimize the effect of variance in launch velocity used in their calculations. Students

should also pay close attention to the resolution and accuracy of their measuring devices, making sure that the measured values used in any calculations do not include any digits outside the accuracy or resolution of the measurement tool.

❓ 4. Sketch the complete trajectory of your projectile. Draw your projectile at five locations on its trajectory (evenly spaced). At each of these locations, draw the net force vector acting on the projectile at that location. Make sure the lengths of the vectors represent the relative magnitudes. Neglect any force from air drag.



❓ 5. Sketch the complete trajectory of your projectile. Redraw the projectile at the same five locations used in the previous question. At each of these locations, draw the projectile's horizontal and vertical component velocity vectors. Make sure the lengths of the vectors represent the relative magnitudes of the velocities. Neglect any force from air drag.



❓ 6. How did the kinetic energy of the projectile change in its trajectory? Use your sketches from the previous questions to explain how the kinetic energy of the projectile changed in its trajectory.

Students can correctly argue that the kinetic energy of the projectile increased as it was in free fall because its vertical velocity component increased as it fell; the length of the vertical velocity vector increased even though the horizontal vector did not. An increase in velocity indicates an increase in kinetic energy.

Students can correctly argue that the net force vector acting on the projectile was non-zero as it traversed the height Δy , which indicates that work was done on the projectile. Using the work-energy theorem, students can infer that work done on the projectile causes its kinetic energy to increase.

Students can correctly argue that the total energy of the system must be conserved. Because the system is relatively closed, and the potential energy of the projectile decreases as it falls, the projectile's kinetic energy must, in turn, increase.

Synthesis Questions

❓ 1. A ball player throws a ball horizontally. Neglecting air resistance, what variables affect the horizontal range of the ball?

Neglecting air resistance, the range of a ball thrown horizontally (having initial velocity in only the horizontal direction) will be affected only by the height from which it is thrown and the initial velocity of the ball.

2. For the same ball player, how would doubling the initial velocity affect the range, if at all?

Based on the equation for the range of a ball thrown horizontally:

$$\Delta x = v_{x0}\sqrt{\frac{2\Delta y}{g}} \tag{4}$$

Based on their proportional relationship, the range Δx of the ball will double if the initial velocity v_{x0} of the ball is doubled..

3. For the same ball player, how would quadrupling the height affect the range, if at all?

Based on their mathematical relationship: $\Delta x \propto \sqrt{\Delta y}$, the range Δx of the ball doubles if the height Δy of the ball quadruples.

4. For the same ball player, how would doubling the mass of the ball affect the range, if at all?

The mass of an object does not affect its acceleration if that object is in free fall.

Extended Inquiry Suggestions

Have students translate the equations and skills used in this activity to a two-dimensional application in which the projectile is not launched horizontally, but with a non-zero angle of inclination. Challenge students to land a projectile into an empty paint can at a range they calculate based on a fixed launch height, non-zero launch angle, and initial velocity assigned to them.

Set Up

Prior to class:

1. Mount one projectile launcher in the front of the class to a table with plenty of room in front of it to launch the projectile.

2. Lay down a measuring tape or meter sticks in a straight line away from the launcher the entire length of the projectile range. You may want to test fire the projectile and make sure that the measuring tape or meter sticks are aligned with the path of the projectile. The test-fired projectile should land on or very close to the measuring tape or meter sticks every shot.

3. Determine the number of student groups that will be performing the challenge and assign each group a reasonable initial launch angle between 45° and 90°, depending on the height of your classroom ceiling.

4. Use a data collection system (or other method) to measure the initial velocity of the projectile at each of the assigned launch angles (the initial velocity may change, depending on the launch angle). Record the initial velocity and launch angle on the whiteboard for students to use in their challenge.

CALCULATE THE RANGE

5. Assign each student group a launch angle and the corresponding initial velocity and have them calculate the range of the projectile:

   The horizontal range, Δx, for a projectile can be found using the following equation:

   $$\Delta x = v_0 \cos\theta t \qquad (1a)$$

   where $v_0 \cos\theta$ is the initial horizontal component velocity of the projectile and t is its time of flight. To find the time of flight t the following kinematic equation is needed:

   $$y_f = y_i + v_0 \sin\theta t - \frac{1}{2}gt^2 \qquad (2a)$$

   where y_f and y_i are the final and initial heights of the projectile, g is the acceleration due to gravity, and $v_0 \sin\theta$ is the vertical component of the projectile's initial velocity. Solving Equation 2c for t requires the use of the quadratic equation:

   $$0 = \left(y_i - y_f\right) + v_0 \sin\theta t - \frac{1}{2}gt^2$$

   $$t = \frac{-v_0 \sin\theta - \sqrt{\left(v_0 \sin\theta\right)^2 - 4\left(-g/2\right)\left(y_i - y_f\right)}}{2\left(-g/2\right)} \qquad (3a)$$

   Simplifying Equation 3a and substituting it into Equation 1a yields the formula for the range Δx of a projectile launched with an initial angle θ, initial velocity v_0 ,initial height y_i and landing at a final height of y_f :

   $$\Delta x = v_0 \cos\theta \left(\frac{v_0 \sin\theta + \sqrt{\left(v_0 \sin\theta\right)^2 + 2g\left(y_i - y_f\right)}}{g} \right) \qquad (4a)$$

6. Have student groups come up one at a time after they have calculated the range of the projectile, place the paint can at their calculated range, and then fire the projectile to test their calculations.

   If the projectile lands in the paint can, then their calculations were correct. Student groups who do not land the projectile in the paint can may not have accounted for the height of the can in their calculations: the projectile must clear the vertical walls of the paint can to land in it.

5. TWO-DIMENSIONAL MOTION: PROJECTILES

STRUCTURED

Driving Question | Objective

What is the range of a projectile launched horizontally? Develop a plan to measure the variables that affect the two-dimensional motion of a projectile launched horizontally, and then use those variables to accurately predict and test the projectile's horizontal range.

Materials and Equipment

- Data collection system
- PASCO Smart Gate photogate[1]
- PASCO Mini Launcher[2]
- PASCO Photogate Mounting Bracket[2]
- Launcher loading rod[2]
- Mini launcher table clamp[2]
- Steel ball, 1.6-cm diameter[2]
- White paper, 1 sheet
- Carbon paper, 1 sheet
- Cardboard, square piece, 10 × 10 inch
- Meter stick

[1]www.pasco.com/ap21

PASCO Smart Gate

[2]www.pasco.com/ap05

PASCO Mini Launcher

Background

The motion of a projectile can be described using kinematics applied in both the vertical and horizontal directions. Assuming air resistance is negligible, a projectile launched horizontally only experiences acceleration in the vertical *y* direction, while its velocity in the horizontal *x* direction remains constant until the projectile strikes a target. Given this, the horizontal range Δx for a projectile can be found using the following equation:

$$\Delta x = v_{x0}t \qquad (1)$$

where v_{x0} is the initial horizontal velocity of the projectile and t is its time of flight. To find the time of flight t, the vertical motion of the projectile must be analyzed. Given that the initial height of the projectile is Δy and the acceleration it experiences in the vertical direction is from gravity, the following kinematic equation can be used to isolate t:

$$\Delta y = v_{y0}t + \frac{1}{2}a_y t^2 \qquad (2)$$

where a_y is the acceleration due to gravity and v_{y0} is the vertical component of the projectile's initial velocity. Assuming that the projectile's initial vertical velocity is zero, the first term drops out and Equation 2 can be rearranged to solve for t (substituting g for the acceleration in the vertical direction):

$$t = \sqrt{\frac{2\Delta y}{g}} \qquad (3)$$

Combining equations 1 and 3 produces the mathematical relationship between launch height Δy and initial velocity v_{x0} for a free falling projectile launched horizontally:

$$\Delta x = v_{x0}\sqrt{\frac{2\Delta y}{g}} \qquad (4)$$

Safety

Follow these important safety precautions in addition to your regular classroom procedures:

- Wear safety goggles at all times.
- Do not look into the launcher.
- Do not aim the launcher at others.
- Use only what the teacher provides as the plunger to load the projectile launcher.

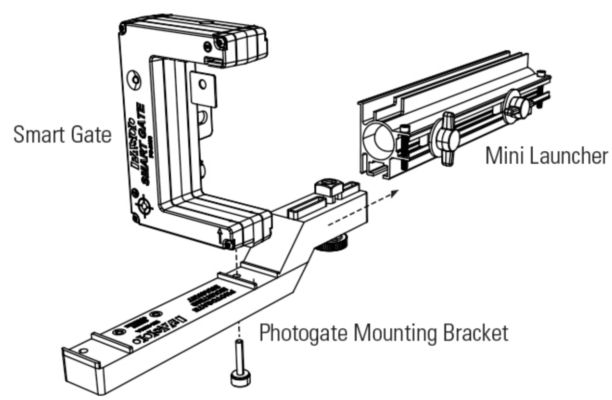
Procedure

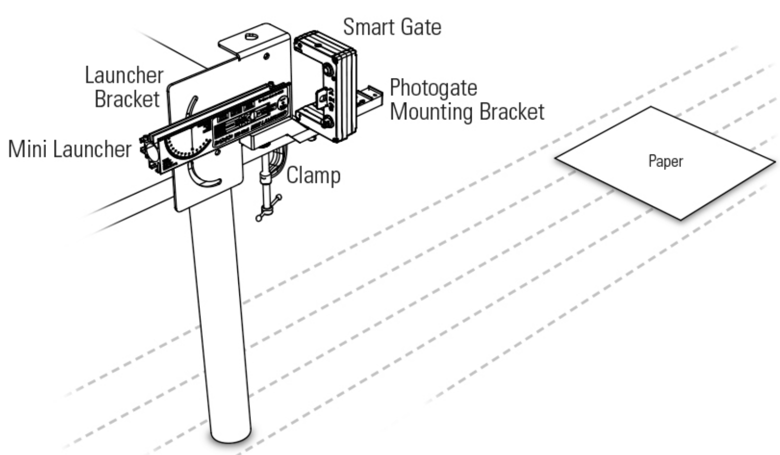
SET UP

1. Connect the photogate mounting bracket and photogate to the mini launcher as in the figure to the right.

2. Choose one corner of a table to mount the projectile launcher. Make sure a distance of about 3 m is clear on the floor around the table in the direction you plan to launch the projectile.

3. Clamp the launcher to the corner of the table using the table clamp, and then adjust the angle of the mini launcher to zero degrees (horizontal launch).





4. Connect the photogate to the data collection system.

5. Configure the data collection system to use the photogate timing to measure the initial speed of the projectile passing through the photogate, and then display this measurement in a digits display.

COLLECT DATA

6. Measure the height of the point the projectile will exit the launcher relative to the floor. Record this value in the Data Analysis section below.

7. Place the steel ball into the launcher, and use the push rod or plunger to load the ball as far into the launcher as possible (three clicks).

8. Hold a piece of cardboard a few centimeters past the photogate to block the ball.

9. Start recording data, and then pull up on the yellow cord attached to the trigger to launch the ball. Make sure it strikes the cardboard before it lands on the ground.

10. Stop data recording.

11. Record the initial speed of the projectile in Table 1 in the Data Analysis section.

12. Repeat the data collection steps 4 additional times. Record the initial speed of the projectile for each trial into Table 1.

Data Analysis

Height of launcher at projectile exit point (m): ____________________

Table 1: Initial velocity of projectile launched horizontally

Trial	Initial Velocity (m/s)
1	
2	
3	
4	
5	
Ave Velocity (m/s)	

1. Calculate the projectile's average initial velocity and record your result into Table 1.

CALCULATE THE RANGE

2. Using your average launch velocity and the measured height of the launcher, calculate the range of the projectile using the equation:

$$\Delta x = v_{x0}\sqrt{\frac{2\Delta y}{g}} \tag{4}$$

For your calculation assume that $g = 9.81\ \text{m/s}^2$. Show your work here:

TEST THE RANGE

3. Draw a circle with a radius of 8 cm in the center of a piece of white paper, and then tape the paper to the floor in front of the projectile launcher with the center of the circle at a distance equal to your predicted range.

4. Place carbon paper over the white paper, and then align the projectile launcher with the center of the paper.

5. Place the steel ball into the launcher and then use the push rod or plunger to load the ball as far into the launcher as possible (three clicks).

6. Launch the ball toward the paper. Place the steel ball back into the launcher and repeat the test four more times.

7. Remove the carbon paper. Observe the locations where the ball struck the paper.

Analysis Questions

❓ 1. Assuming air resistance is negligible, what other variables affect the range of a projectile?

____

____

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❓ 2 Qualitatively describe how close your predicted range is to your actual range in terms of accuracy (the relative distance between each test shot and the actual target) and precision (the grouping of the test shots). Use the tables below to help.

Accuracy	Accuracy Definition
Very high	All shots in 8 cm target
High	Four shots in 8 cm target
Moderate	Three shots in 8 cm target
Low	Two shots in 8 cm target
Very low	One or no shots in 8 cm target

Precision	Precision Definition
Very high	All shots within 3 cm radius
High	Four shots within 5 cm radius
Moderate	Two shots within 8 cm radius
Low	Two shots within 20 cm radius
Very low	No shots within 30 cm radius

____

____

____

❓ 3. What are factors that may have caused your range prediction to be incorrect, and what could you have done differently to avoid them?

____

____

____

❓ 4. Sketch the complete trajectory of your projectile. Draw your projectile at five locations on its trajectory (evenly spaced). At each of these locations, draw the net force vector acting on the projectile at that location. Make sure the lengths of the vectors represent the relative magnitudes.

❓ 5. Sketch the complete trajectory of your projectile. Redraw the projectile at the same five locations used in the previous question. At each of these locations, draw the projectile's horizontal and vertical component velocity vectors. Make sure the lengths of the vectors represent the relative magnitudes of the velocities.

❓ 6. How did the kinetic energy of the projectile change in its trajectory? Use your sketches from the previous questions to explain how the kinetic energy of the projectile changed in its trajectory.

Synthesis Questions

❓ 1. A ball player throws a ball horizontally. What variables affect the horizontal range of the ball?

❷ 2. For the same ball player, how would doubling the initial velocity affect the range, if at all?

❷ 3. For the same ball player, how would quadrupling the height affect the range, if at all?

❷ 4. For the same ball player, how would doubling the mass of the ball affect the range, if at all?

6. CONSERVATION OF MECHANICAL ENERGY

Connections to the AP® Physics 1 Curriculum*

The lab activity correlates to the following pieces of the AP® Physics 1 framework:

Big Idea	5	Enduring Understanding	B	Essential Knowledge	4

Learning Objective 1: The student is able to describe and make predictions about the internal energy of systems.
Science Practices: 6.4, 7.2

Learning Objective 2: The student is able to calculate changes in kinetic energy and potential energy of a system using information from representations of that system.
Science Practices: 1.4, 2.1, 2.2

Time Requirement

Preparation Time: 10 minutes

Lab Activity: 50 minutes

Prerequisites

Students should be familiar with the following concepts:

- Conservative forces are those which store energy in a useful form when objects within a system interact. Gravity is an example of a conservative force: when work is done to lift an object from one height to another, energy is stored as gravitational potential energy (GPE) that can be converted to kinetic energy. Friction is an example of a non-conservative force: when work is done on a system with friction, it is converted to thermal energy which is not available for later use.
- Newton's Second Law of Motion for linear motion states that a net force causes an object to accelerate.
- Objects near earth's surface accelerate due to the gravitational field, with a magnitude of 9.8 m/s^2.

Driving Question | Objective

How do the potential and kinetic energies of an object in a closed system change as its motion changes due to a conservative force? Design an experiment to explore how a cart's kinetic energy, gravitational potential energy, and total mechanical energy change as it rolls down an inclined track.

Procedural Overview

In the Structured version of this lab activity, students release a cart from several different heights on an inclined track and measure the different velocities of the cart, using a photogate, as it passes through a fixed reference point near the bottom of the track.

Using their measurements of height, students calculate the cart's initial gravitational potential energy at the point from which the cart was released in each trial. Using both the speed and height of the cart at the reference point, students calculate the kinetic and gravitational potential energy of the cart at the reference point for each trial. Assuming that the total mechanical energy of the cart when it was released is equal to the cart's initial gravitational potential energy, students compare that value to the sum of the cart's kinetic and gravitational potential energy at the reference point.

Student data should show that the total mechanical energy of the cart when it was released is equal to its total mechanical energy at the reference point. Students then use this equality to establish that the total mechanical energy of the cart system is conserved.

Pre-Lab Discussion and Activity

This is a pre-lab suggestion and activity.

Explain to students that energy is a quantity that can never be created or destroyed although it may change form. Begin a discussion with your students by inquiring about the energy present in a pen or pencil held out in front of you at some height above the ground. Students will likely agree that the pen or pencil has no energy because it is not moving. Then inquire how the pen or pencil will react if you release it into free fall. Students may assume that the object's speed governs its total energy. Rebut this by explaining that energy is neither created nor destroyed, "So how is it that the pen or pencil's energy is increasing, but nothing is adding energy to the system?"

Explain to students that they are correct in assuming there is an increasing energetic quantity related to the speed of the object; however, there must be some other form of energy present that is changing form to compensate for this increase.

Use this opportunity to introduce the concept of *potential energy* and how the pen or pencil actually has "potential" energy present due to the conservative force from gravity, and that this energy is not dependent on the speed of the object, but rather, on the height of the object from the floor. The higher the object is above the floor, the more potential it has to convert its energy into kinetic energy. If the object is at rest on the ground, it has no energy.

Materials and Equipment

- Data collection system
- PASCO Smart Gate photogate[1]
- PASCO Photogate Bracket[1]
- PASCO Dynamics Cart Picket Fence[2]
- Table clamp or large base
- Support rod, 45-cm
- Meter stick
- PASCO Dynamics Track[3]
- PASCO Dynamics Track Rod Clamp[4]
- PASCO Dynamics Track End Stop[5]
- PASCO Dynamics Cart[6]
- PASCO Angle Indicator[7]
- Balance, 0.1-g resolution, 2,000-g capacity (1 per class)

Probeware Resources

Below are web-link and QR codes that will direct you to instructional video resources for individual pieces of PASCO probeware, sensors, and other hardware used in the lab activity. These same links and codes are provided to students in their activity handouts.

[1]www.pasco.com/ap21	[2]www.pasco.com/ap16	[3]www.pasco.com/ap08	[4]www.pasco.com/ap17
PASCO Smart Gate	PASCO Dynamics Cart Picket Fence	PASCO PAStrack	PASCO Dynamics Track Rod Clamp
[5]www.pasco.com/ap11	[6]www.pasco.com/ap07	[7]www.pasco.com/ap14	
PASCO Dynamics Track End Stop	PASCO PAScar	PASCO Angle Indicator	

Safety

Follow this important safety precaution in addition to your regular classroom procedures:

- The cart can roll off the end of the track and possibly knock objects off the lab bench. Make certain a member of your group catches the cart as it rolls to the end of the track.

Teacher Tips

Tip 1 – Catch the Cart at the Bottom of the Track

- Students following the procedure outlined in the Structured version of this activity will measure the speed of a cart rolling down an inclined track using a photogate and the PASCO picket fence system. Using this system, each time the cart and picket fence passes through the photogate beam a speed measurement is recorded by the data collection system. If the cart bounces off the end stop at the end of the track and passes back through the photogate, multiple speed measurements will be recorded. To avoid recording multiple speed measurements, students should use their hands to catch the cart just before or just after it hits the end stop at the end of the track.

Tip 2 – Avoid Inclination Angles > 10°

- If students choose to design their setup using an inclined cart and track system similar to the setup outlined in the Structured version of this activity, they should avoid inclining the track greater than 10°. Angles of inclination greater than 10° can potentially produce final cart velocities so great that the cart or end stop may be damaged if they collide.

Tip 3 – Measuring Height for Determining GPE

- When determining the GPE of an object, the measured height of the object is not always relative to the earth's surface. Students must choose a *zero*, or point of reference that is consistent throughout all of their calculations. All height measurements must be made relative to this zero point.
- Height measurements needed to calculate the GPE of a cart should be made to the center of mass (CM) of the cart in the vertical direction, as this is a location that mathematically represents the entire mass of the cart; however, students using a setup and procedure similar to that outlined in the Structured version of this lab activity can measure the height relative to any point on the cart as long as they are consistent in measuring the height to the same point in each trial and the cart travels on a straight track with no curvature.

  Measuring height from a point not near the CM while using a curved track may cause calculated values of GPE to deviate from the theoretical when the speed of the cart is low. This is because the vertical position of the cart's CM changes differently compared to positions away from it as the cart rotates on the way up or down the curved track.

Tip 4 – Measuring Speed with Other PASCO Sensors

- The procedure outlined in the Structured version of this lab activity makes use of a PASCO Smart Gate to measure the instantaneous speed of the cart at a discrete point in its path; however, students can use a variety of PASCO sensors that measure motion. These include:

  - PASCO Motion Sensor—This sensor requires that students measure the velocity of the cart in a continuous fashion. Students who choose to do this must place the motion sensor at the top of the incline in-line with, but away from the direction of, the cart's path. Make sure the switch on the top of the motion sensor is set to the cart icon.

  - PASCO Photogate with or without a PASCO Digital Adapter—The setup and procedure outlined in the Structured version of this lab activity uses a PASCO Smart Gate, but a single older PASCO photogate will work seamlessly in its place. If your data collection system has digital inputs, the PASCO Digital Adapter is not needed; the photogate can be connected directly to the digital inputs on the data collection system. If there are no digital inputs on your data collection system, you will need to connect the older PASCO photogate using a PASCO Digital Adapter.

  - Photogate setting—When using an older PASCO photogate instead of the PASCO Smart Gate, students should use the "Photogate Timing" pre-configured timer within the data collection system. This setting automatically includes measurements of "time in gate" and "speed in gate" (or "velocity in gate"). Students will need to enter in the data collection system the length of the flag in the picket fence being used, which should appear correctly defaulted to 2.5 cm.

Sample Data

Below are sample data, acquired using the experimental setup and procedure outlined in the Structured version of the lab activity, and answers to questions in the Data Analysis section.

Data Analysis

Mass of the cart and picket fence (kg): 0.2704

Height of the cart at the reference point (m): 0.072

GPE of the cart at the reference point (J): 0.19

1. Use the mass of the cart m, earth's gravitational acceleration $g = 9.8\ \text{m/s}^2$, the height of the cart at the reference point, and Equation 2 to calculate the gravitational potential energy $U_{\text{g ref}}$ of the cart at the reference point. This is the GPE of the cart at the reference point for each trial. Record the result above.

   Calculation using sample data for Trial 1:

   $U_{\text{g ref}} = mgy_{\text{ref}}$

   $U_{\text{g ref}} = 0.2704\ \text{kg} \times 9.8\ \text{m/s}^2 \times 0.072\ \text{m} = 0.19\ \text{J}$

Table 1: Mechanical energy of a cart on an inclined track

Trial	Initial Height (m)	Speed at Reference Point (m/s)	Initial Gravitational Potential Energy (J)	Kinetic Energy at Reference Point (J)	Total Mechanical Energy at Reference Point (J)
1	0.112	0.82	0.30	0.091	0.28
2	0.128	1.02	0.34	0.14	0.33
3	0.145	1.18	0.38	0.19	0.38
4	0.161	1.32	0.43	0.24	0.43
5	0.179	1.44	0.47	0.28	0.47

2. Using the cart's initial height from Table 1, calculate the initial gravitational potential energy of the cart for each trial. Record your results in Table 1.

   Calculation using sample data for Trial 1:

   $U_{\text{gi}} = mgy_{\text{i}}$

   $U_{\text{gi}} = 0.2704\ \text{kg} \times 9.8\ \text{m/s}^2 \times 0.112\ \text{m} = 0.30\ \text{J}$

❷ 3. Assume that the cart's total mechanical energy when it was released was equal to its initial gravitational potential energy in each trial. Why can you make this assumption?

   At the moment the cart is released, its speed is considered to be zero. If the speed is zero, according to Equation 3 in the Structured version Background section, the kinetic energy of the cart will also be zero. If the cart's kinetic energy is zero, according to Equation 1, the cart's total mechanical energy is then equal to its gravitational potential energy.

4. Use the mass m of the cart, the cart's speed v_{ref} at the reference point, and Equation 3 to calculate the cart's kinetic energy K_{ref} at the reference point. Record your results in Table 1.

   Calculation using sample data for Trial 1:

   $$K_{ref} = \frac{1}{2}mv_{ref}^2$$

   $$K_{ref} = \frac{1}{2}\left[0.2704\text{ kg} \times (0.82\text{ m/s})^2\right]$$

   $$K_{ref} = 0.091\text{ J}$$

5. Calculate the cart's total mechanical energy E_{ref} at the reference point by summing its kinetic energy and gravitational potential energy at the reference point. Record your results into Table 1.

   Calculation using sample data for Trial 1:

   $$E_{ref} = U_{g\,ref} + K_{ref}$$

   $$E_{ref} = mgy_{ref} + \frac{1}{2}mv_{ref}^2$$

   $$E_{ref} = 0.19\text{ J} + 0.091\text{ J} = 0.28\text{ J at the reference point}$$

❓ 6. How does the cart's mechanical energy when it was released compare to its mechanical energy at the reference point?

   Student data should show that the total mechanical energy pf the cart when it was released (the initial gravitational potential energy) is equal to or nearly equal to (within less than 10%), the cart's total mechanical energy at the reference point.

Guided Inquiry Questions

Below are sample responses to the Guiding Questions found in the Guided Inquiry version of this lab activity.

❓ 1. How will you assemble your cart and track? Should you use a large, small, or somewhere in between incline angle? Justify your response.

   The incline of the track is irrelevant in terms of the theory—Mechanical energy will be conserved regardless of the incline angle; however, students should consider three important points and find a satisfactory compromise between them to achieve best results:

   - If the incline angle of the track is too steep, the cart will have a very high speed if it travels the entire length of the track. If the speed is too high, the cart could become dangerous if it flies off the end of the track, or it could be damaged, or both.
   - The method students choose to measure the speed of the cart (speed is a measured quantity used to calculate kinetic energy) may impose resolution and accuracy limitations at high inclines and high cart speeds. For example, when using the 2.5 cm band on the cart picket fence (as outlined in the procedure in the Structured version of this lab activity) the PASCO Photogate has a timing resolution of 0.1 ms, which implies that the accuracy of the photogate increases the slower the cart passes through the photogate. Using a small track incline angle will reduce the speed of the cart and increase the accuracy.
   - Although most setups will involve a low friction cart and track system, some may have a greater frictional effect than others. This introduces a non-conservative force into the system that opposes gravity, resulting in kinetic energy values lower than expected. This effect is greatly reduced by using larger track incline angles.

❓ 2. How will you measure the kinetic energy of the cart on the inclined track? What tools will you use and how will you use them?

   Kinetic energy is a quantity that cannot be directly measured using conventional measurement techniques. Students should use a photogate or other timing device to measure the speed of the cart at one or various points, or continuously. The procedure outlined in the Structured version of this lab activity makes use of a PASCO Smart Gate to measure the instantaneous speed of the cart at a discrete point in its path, which student then use to calculate the kinetic energy of the car.

❓ 3. Should you measure the cart's kinetic energy in a continuous fashion throughout the cart's motion, or can you simplify your experiment by measuring it in one or two specific locations? Explain your answer.

Students using a continuous data acquisition method may choose to measure the cart’s speed or kinetic energy in a continuous fashion. This is completely acceptable as long as students can compare those kinetic energy values to the cart’s corresponding GPE and total mechanical energy values occurring at the same time.

Students may find it simpler to choose one, two, or three discrete locations in the cart’s path to indirectly measure the cart’s GPE, kinetic energy, and total mechanical energy, and then compare those values.

❓ 4. How will you measure the gravitational potential energy of the cart on the inclined track? What tools will you use and how will you use them?

Gravitational potential energy is a quantity that cannot be directly measured using conventional measurement techniques. Students should use measurements of the cart’s height to determine its gravitational potential energy at one or various points, or continuously.

Height measurements used to calculate GPE can be made relative to any surface (earth’s surface, top of lab table, floor, et cetera) as long as all height measurements are made relative to that surface.

❓ 5. If measuring the height of the cart is part of your strategy for determining the cart's gravitational potential energy, to what point on the cart should you measure the height: to the front of the cart, the back of the cart, et cetera, or does it not matter? Justify your answer.

Students should measure height to the center of mass (CM) of the cart in the vertical direction, as this is a location that mathematically represents the entire mass of the cart; however, students using a setup and procedure similar to that outlined in the Structured version of this lab activity can measure the height relative to any point on the cart as long as they are consistent in measuring the height to the same point in each trial and the cart travels on a straight track. Measuring height from a point not near the CM, while using a curved track, may cause the calculated GPE to deviate from the theoretical when the speed of the cart is low. This is because the vertical position of the cart’s CM changes differently compared to positions farther from the CM as the cart rotates on the way up or down the curved track.

❓ 6. Should you measure the cart's gravitational potential energy in a continuous fashion throughout the cart's motion or can you simplify your experiment by measuring it in one or two specific locations? Explain your answer.

Students using a continuous data acquisition method may choose to measure the cart’s height (to calculate GPE) in a continuous fashion. This is completely acceptable as long as students can compare those GPE values to the cart’s corresponding kinetic energy and total mechanical energy values at the same time.

Students may find it simpler to choose one, two, or three discrete locations in the cart’s path to indirectly measure the cart’s GPE, kinetic energy, and total mechanical energy, and then compare those values.

❓ 7. How will you measure the total mechanical energy of the cart on the inclined track?

Total mechanical energy is a quantity that cannot be directly measured using conventional measurement techniques. Students should calculate the cart’s mechanical energy by summing their calculated values of GPE and kinetic energy.

Assessment Questions: Sample Responses

Sample responses to the Analysis and Synthesis questions found in each version of the lab activity:

Analysis Questions

❓ 1. In your experiment, how did your cart’s gravitational potential energy and kinetic energy change as it rolled down the track in each trial?

Student data should show that the GPE of a cart rolling down an inclined track decreases as the cart descends (height decreases), while the cart’s kinetic energy increases (speed increases).

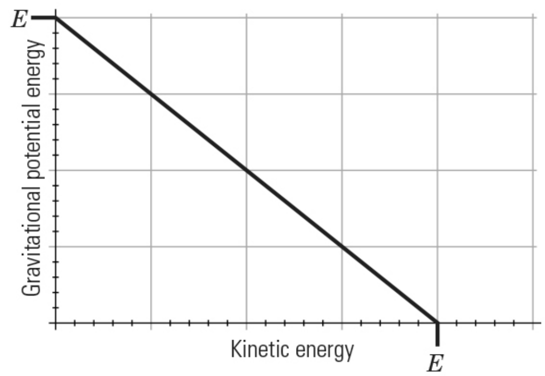
❓ 2. How did your cart’s total mechanical energy change as it rolled down the track?

Student data should show that the total mechanical energy of a cart rolling down an inclined track is equal or nearly equal at any point in its path. This is a result of the law of conservation of energy.

❷ 3. Does your data show that the cart's total mechanical energy was conserved? Justify your answer.

Students should indicate that the total mechanical energy of their cart system was conserved, and that this is supported by their data because the total mechanical energy of the cart was equal or nearly equal at any point in its path.

❷ 4. If you were to plot a graph of your cart's *gravitational potential energy* versus its *kinetic energy* as it rolled down the track, what would be the shape of the curve? Sketch the curve in the blank graph below.



The graph should be a straight line.

Synthesis Questions

❷ 1. Describe how your sketch from the previous question would be different if the cart started with twice as much potential energy.

The line would have identical slope but the y-intercept would be shifted up and the x-intercept would be shifted to the right by a factor of two.

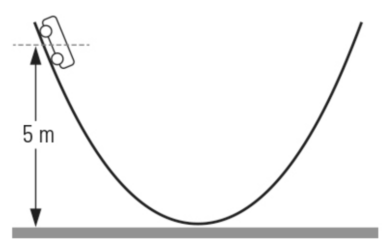
❷ 2. What would be the speed of a cart at the bottom of a very long frictionless inclined plane if it was released from rest at a height of 10.0 m above the bottom of the track? Assume the bottom of the track has height $y_{\text{bottom}} = 0$. Show your work.

$$mgy_1 + \frac{1}{2}mv_1^2 = mgy_{\text{bottom}} + \frac{1}{2}mv_{\text{bottom}}^2$$

$$mgy_1 = \frac{1}{2}mv_{\text{bottom}}^2$$

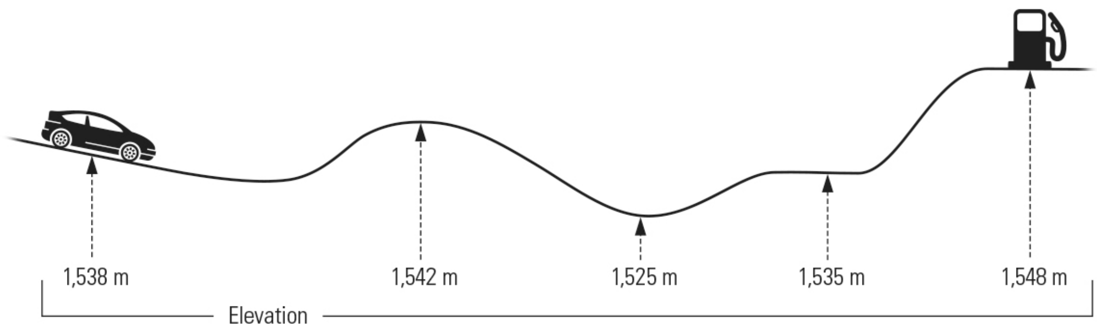
$$v_{\text{bottom}} = \sqrt{2gy_1} = \sqrt{2(9.8\text{ m/s})(10.0\text{ m})} = 14\text{ m/s}$$

❷ 3. A cart is released from a height of 5 m on the left side of a U-shaped track with identical inclines on the left and right. If the cart is released from rest, how far up (height) the right side of the track will the cart travel? Assume that friction is negligible. Justify your answer.



The initial total mechanical energy of the cart is equal to mgy_1 where y_1 is the starting height of the cart on the left side of the track. Because the total mechanical energy of the cart is conserved, and the speed of the cart at the peak of its height y_2 on the right side of the track is zero, all of the cart's mechanical energy must be equal to GPE at that point. Making $mgy_1 = mgy_2$, which simplifies to $y_1 = y_2$, indicating that the cart will travel 5 m up the right side.

❷ 4. The car shown in the picture below (mass = 998 kg) has just run out of gas while moving at a speed of 15 m/s. Assuming that friction is negligible, will the car make it to the gas station if it coasts the whole way? Justify your answer.



$$E_1 = E_2 = E_3 = ...E_n$$

$$mgy_1 + \frac{1}{2}mv_1^2 = mgy_2 + \frac{1}{2}mv_2^2$$

$$v_2 = \sqrt{2\left(g\left(y_1 - y_2\right) + \frac{1}{2}v_1^2\right)} = \sqrt{2\left(9.8\text{ m/s}\left(1{,}538\text{ m} - 1{,}542\text{ m}\right) + \frac{1}{2}\left(15\text{ m/s}\right)^2\right)} = 12\text{ m/s}$$

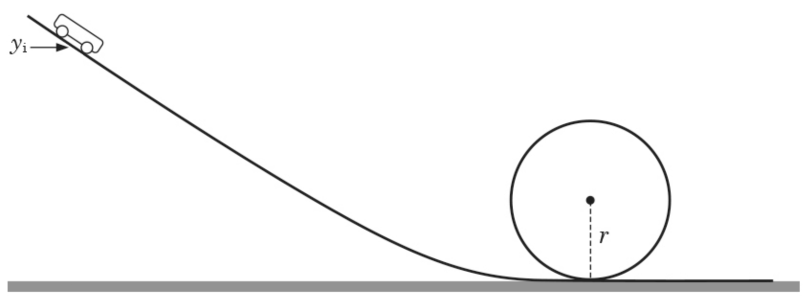
The car makes it over the first hill with a speed of 12 m/s, and because the second hill is lower than the car's original starting height, it makes it over that one as well. For the third hill:

$$mgy_1 + \frac{1}{2}mv_1^2 = mgy_3 + \frac{1}{2}mv_3^2$$

$$v_3 = \sqrt{2\left(g\left(y_1 - y_3\right) + \frac{1}{2}v_1^2\right)} = \sqrt{2\left(9.8\text{ m/s}\left(1{,}538\text{ m} - 1{,}548\text{ m}\right) + \frac{1}{2}\left(15\text{ m/s}\right)^2\right)} = 5.4\text{ m/s}$$

The car makes it to the gas station with a speed of 5.4 m/s.

❷ 5. In the figure below, a cart is released from rest and rolls down an inclined track and around a loop with radius r = 25.0 cm. From what minimum height y_i must the cart be released so that it does not fall off the track when it is upside down at the top of the loop? Show your work.



For the cart to stay on the track at the top of the loop, the centripetal force must be equal to the gravitational force:

$mg = \frac{mv_{\text{top of loop}}^2}{r}$, which implies that the cart must have a speed $v_{\text{top of loop}} = \sqrt{gr}$. This means that the cart must have a kinetic energy at the top of the loop equal to $\frac{1}{2}m(\sqrt{gr})^2$. Because energy is conserved:

$E_i = E_{\text{top of loop}}$ or

$$mgy_i = \frac{1}{2}mv_{\text{top of loop}}^2 + mgy_{\text{top of loop}}$$

$$mgy_i = \frac{1}{2}m\left(\sqrt{gr}\right)^2 + mg(2r)$$

$$y_i = \frac{1}{2}r + 2r = \frac{5}{2}r = 62.5\text{ cm}$$

Extended Inquiry Suggestions

Conservation of energy in spring systems is a natural progression from this lab.

Explore with students the potential and kinetic energies involved with spring systems and how they differ from those involving gravity.

A suggested activity to reinforce this concept:

A known amount of mass hangs from a spring suspended above the floor or lab bench. Using the law of conservation of energy, have students determine the height to which the mass should be raised so that after the mass is released it just touches the floor or lab bench at the bottom of its descent before the spring pulls the mass back toward equilibrium; then have students test their value.

NAME PERIOD DATE

6. CONSERVATION OF MECHANICAL ENERGY

STRUCTURED

Driving Question | Objective

How do the potential and kinetic energies of an object in a closed system change as its motion changes due to a conservative force? Design an experiment to explore how a cart's kinetic energy, gravitational potential energy, and total mechanical energy change as it rolls down an inclined track.

Materials and Equipment

- Data collection system
- PASCO Smart Gate photogate[1]
- PASCO Photogate Bracket[1]
- PASCO Dynamics Cart Picket Fence[2]
- Table clamp or large base
- Support rod, 45-cm
- Meter stick
- PASCO Dynamics Track[3]
- PASCO Dynamics Track Rod Clamp[4]
- PASCO Dynamics Track End Stop[5]
- PASCO Dynamics Cart[6]
- PASCO Angle Indicator[7]
- Balance, 0.1-g resolution, 2,000-g capacity (1 per class)

[1]www.pasco.com/ap21
PASCO Smart Gate

[2]www.pasco.com/ap16
PASCO Dynamics Cart Picket Fence

[3]www.pasco.com/ap08
PASCO PAStrack

[4]www.pasco.com/ap17
PASCO Dynamics Track Rod Clamp

[5]www.pasco.com/ap11
PASCO Dynamics Track End Stop

[6]www.pasco.com/ap07
PASCO PAScar

[7]www.pasco.com/ap14
PASCO Angle Indicator

Background

Mechanical energy is described as the energy associated with an object's motion and position. A free falling object on earth, isolated from any outside influences, experiences two forms of energy that both contribute to the object's total mechanical energy: gravitational potential energy U_g, and kinetic energy K. The object's total mechanical energy E is equal to the sum of its gravitational potential energy and kinetic energies:

$$E = U_g + K \quad (1)$$

Gravitational potential energy (GPE) is described as the energy stored in an object due to its position in a gravitational field. In the case of a free falling object with mass m, gravitational potential energy is due only to the attraction from earth's gravitational field g, and is based on the height y the object is from earth's surface:

$$U_g = mgy \quad (2)$$

Because the falling object is in the presence of a net conservative force (gravity), it experiences acceleration related to Newton's Second Law as it falls. As the object's speed increases due to this acceleration, its kinetic energy K increases in the form:

$$K = \frac{1}{2}mv^2 \tag{3}$$

Imagine that this object was initially at rest, then lifted to some height y above the ground and released. At the top of the object's fall, the kinetic energy would initially be zero because its speed is zero, but the object would have a non-zero mechanical energy because its potential energy is non-zero. Once the object was released, the kinetic energy would increase as it falls, and keep increasing until it hits the ground, while its gravitational potential energy decreases as it falls, and is equal to zero when it hits the ground.

RELEVANT EQUATIONS

$$U_g = mgy \tag{2}$$

$$K = \frac{1}{2}mv^2 \tag{3}$$

$$E = U_g + K = mgy + \frac{1}{2}mv^2$$

Safety

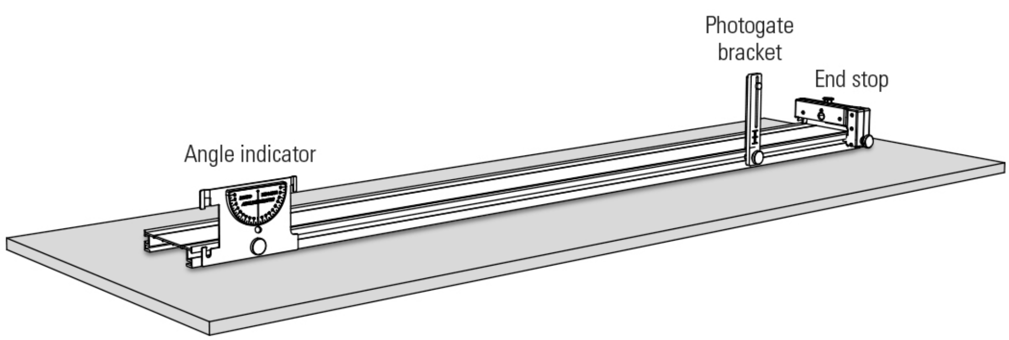
Follow this important safety precaution in addition to your regular classroom procedures:

- The cart can roll off the end of the track and possibly knock objects off the lab bench. Make certain a member of your group catches the cart as it rolls to the end of the track.

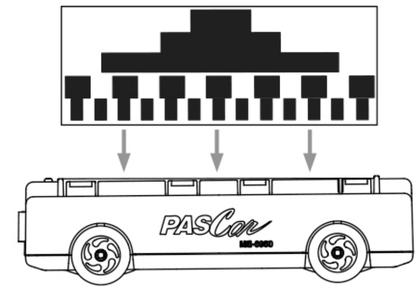
Procedure

SET UP

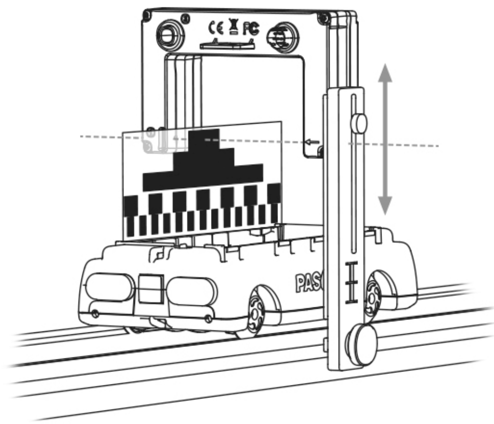
1. Attach the angle indicator, end stop, and photogate bracket to the dynamics track as in the picture below. The end stop should be at the very end of the track and the photogate bracket should be 20 cm from the end stop.



2. Insert the dynamics cart picket fence into the recessed slots on the top of the dynamics cart, oriented with the 2.5-cm solid band as the top-most pattern on the picket fence, and then place the cart on the track.



Insert picket fence into slot in top of dynamics cart

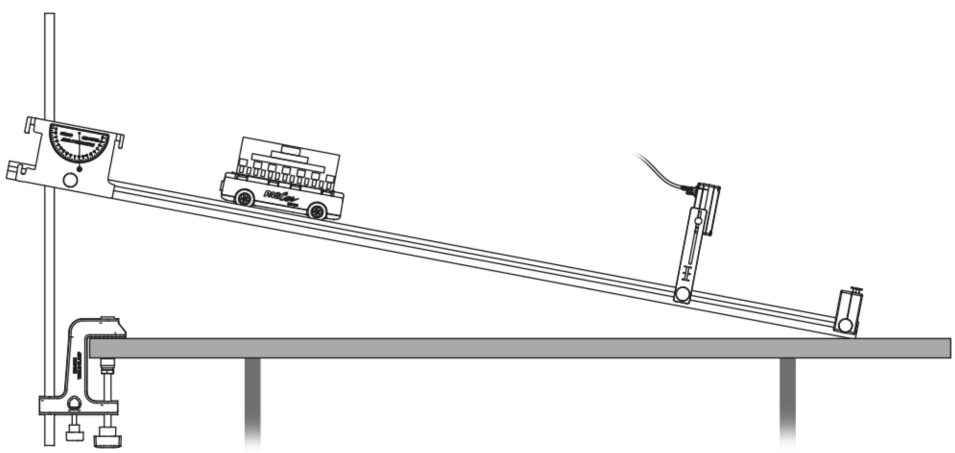


3. Attach the photogate to the photogate bracket. Adjust the height of the photogate on the bracket so the 2.5-cm band on the picket fence passes through the photogate beam as the cart rolls through the photogate.

4. Remove the cart and attach the side of the dynamics track without the end stop to the support rod using the dynamics track rod clamp. Incline the track 10°.

5. Connect the photogate to the data collection system.

6. Configure the data collection system to use photogate timing to measure the speed (or velocity) of the cart as the picket fence passes through the photogate, and then display this measurement in a digits display.

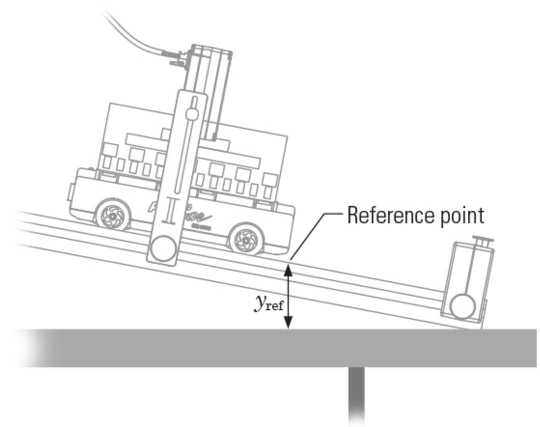


COLLECT DATA

7. Use the balance to measure the mass of the cart and picket fence. Record this mass at the top of the Data Analysis section below.

8. Place the cart on the track and hold it in place with the center of the picket fence aligned with the photogate as in the picture. Make note of the position of the cart's front edge; this will be the reference point from which you will move the cart up the track.

   NOTE: It may be helpful to place a small piece of tape or make a small mark on the track noting the position of the cart's front edge (the reference point).



9. Use the meter stick to measure the height y_{ref} of the cart at the reference point. This height should be measured from the top of the lab table to the top of the track at the front edge of the cart, as shown. Record this value at the top of the Data Analysis section below.

10. Slide the cart up the track 20 cm from the reference point and hold it in place.

11. Use the meter stick to measure the new height of the cart (from the top of the lab table to the top of the track at the front edge of the cart). Record this height in Table 1 next to Trial 1.

12. Begin recording data on the data collection system, and then release the cart, letting it roll freely down the track and through the photogate.

13. Catch the cart just before it hits the end stop, and then stop recording data.

14. Record the speed of the cart at the reference point into Table 1 next to Trial 1 in the Data Analysis section.

15. Repeat the data collection steps four more times, releasing the cart 10 cm farther up the track in each trial. Record the initial height and the speed for each trial into Table 1 in the Data Analysis section.

Data Analysis

Mass of the cart and picket fence (kg): ________________________

Height of the cart at the reference point (m): ________________________

GPE of the cart at the reference point (J): ________________________

1. Use the mass of the cart m, earth's gravitational acceleration $g = 9.8\ \text{m/s}^2$, the height of the cart at the reference point, and Equation 2 to calculate the gravitational potential energy $U_{g\,ref}$ of the cart at the reference point. This is the GPE of the cart at the reference point for each trial. Record the result above.

Table 1: Mechanical energy of a cart on an inclined track

Trial	Initial Height (m)	Speed at Reference Point (m/s)	Initial Gravitational Potential Energy (J)	Kinetic Energy at Reference Point (J)	Total Mechanical Energy at Reference Point (J)
1					
2					
3					
4					
5					

2. Using the cart's initial height from Table 1, calculate the initial gravitational potential energy of the cart for each trial. Record your results in Table 1.

3. Assume that the cart's total mechanical energy when it was released was equal to its initial gravitational potential energy in each trial. Why can you make this assumption?

4. Use the mass m of the cart, the cart's speed v_{ref} at the reference point, and Equation 3 to calculate the cart's kinetic energy K_{ref} at the reference point. Record your results in Table 1.

5. Calculate the cart's total mechanical energy E_{ref} at the reference point by summing its kinetic energy and gravitational potential energy at the reference point. Record your results into Table 1.

6. How does the cart's mechanical energy when it was released compare to its mechanical energy at the reference point?

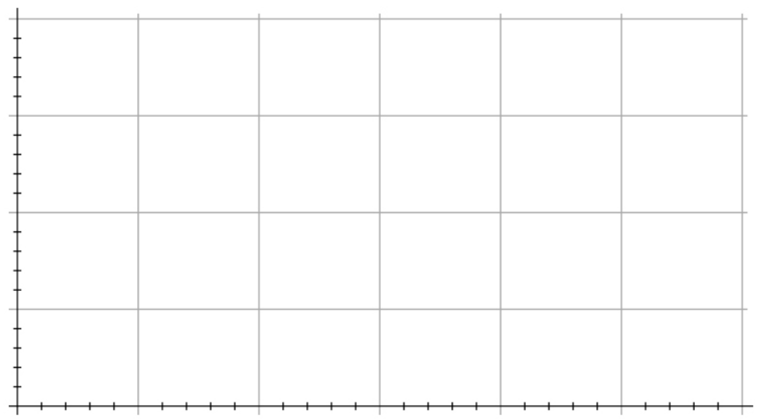
Analysis Questions

1. In your experiment, how did your cart's gravitational potential energy and kinetic energy change as it rolled down the track in each trial?

2. How did your cart's total mechanical energy change as it rolled down the track?

❓ 3. Does your data show that the cart's total mechanical energy was conserved? Justify your answer.

❓ 4. If you were to plot a graph of your cart's *gravitational potential energy* versus its *kinetic energy* as it rolled down the track, what would be the shape of the curve? Sketch the curve in the blank graph below.

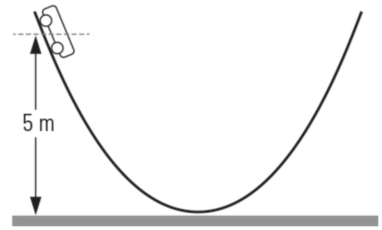


Synthesis Questions

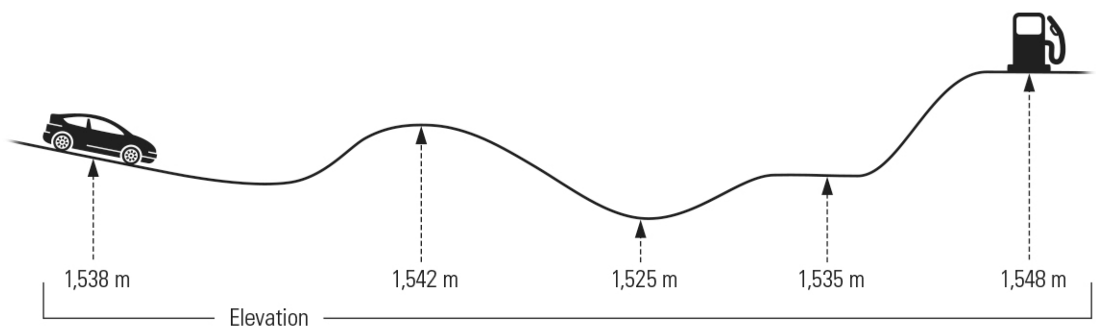
❓ 1. Describe how your sketch from the previous question would be different if the cart started with twice as much potential energy.

❓ 2. What would be the speed of a cart at the bottom of a very long frictionless inclined plane if it was released from rest at a height of 10.0 m above the bottom of the track? Assume the bottom of the track has height $y_{bottom} = 0$. Show your work.

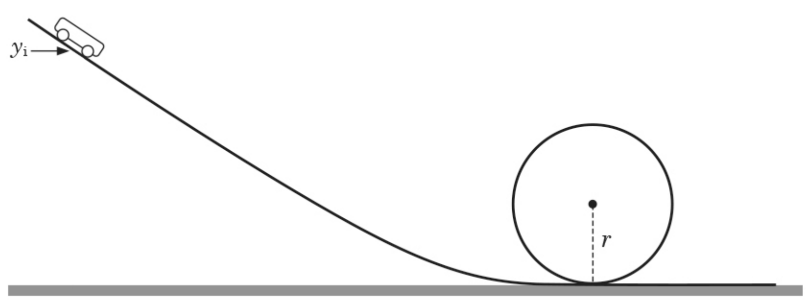
❷ 3. A cart is released from a height of 5 m on the left side of a U-shaped track with identical inclines on the left and right. If the cart is released from rest, how far up (height) the right side of the track will the cart travel? Assume that friction is negligible. Justify your answer.



❷ 4. The car shown in the picture below (mass = 998 kg) has just run out of gas while moving at a speed of 15 m/s. Assuming that friction is negligible, will the car make it to the gas station if it coasts the whole way? Justify your answer.



❷ 5. In the figure below, a cart is released from rest and rolls down an inclined track and around a loop with radius r = 25.0 cm. From what minimum height y_i must the cart be released so that it does not fall off the track when it is upside down at the top of the loop? Show your work.



7. WORK AND KINETIC ENERGY

Connections to the AP® Physics 1 Curriculum*

The lab activity correlates to the following pieces of the AP® Physics 1 framework:

Big Idea	4	Enduring Understanding	C	Essential Knowledge	2

Learning Objective 1: The student is able to make predictions about the changes in the mechanical energy of a system when a component of an external force acts parallel or antiparallel to the direction of the displacement of the center of mass.
Science Practices: 6.4

Learning Objective 2: The student is able to apply the concepts of conservation of energy and the work-energy theorem to determine qualitatively and/or quantitatively that work done on a two-object system in linear motion will change the kinetic energy of the center of mass of the system, the potential energy of the systems, and/or the internal energy of the system.
Science Practices: 1.4, 2.2, 7.2

Time Requirement

Preparation Time: 10 minutes

Lab Activity: 50 minutes

Prerequisites

Students should be familiar with the following concepts:

- Kinetic energy is a component on an object's total mechanical energy and is equal to $1/2mv^2$ where m is the object's mass and v is the object's speed.
- Conservative forces are forces that do work on an object regardless of the path the object is moved through, and that the net work done on an object whose total displacement is zero, is also zero. Gravity is a conservative force.
- Work is only done on an object by the component of a non-zero net force acting in the direction of the object's displacement.

Driving Question | Objective

How is the work done on an object by a non-zero net conservative force related to the change in that object's kinetic energy? Investigate the relationship between the change in kinetic energy of an object experiencing a non-zero net conservative force, and the work done by that net force on the object. Establish a measurement-based relationship between work and kinetic energy.

Procedural Overview

Students use a cart and track system to show that the change in kinetic energy of a cart as it rolls down an inclined track is equal to the work done on the cart by gravity to displace it down the track.

To show this, students release a cart on an inclined track from various positions and measure its speed and displacement at the bottom of the track. Assuming that the initial speed of the cart just as it was released was zero, students use their cart mass m and speed v measurements to calculate the change in kinetic energy ($\Delta K = 1/2m\Delta v^2$) of the cart as it traveled from the point where it was released to the bottom of the track in each trial. Students then use the displacement d, the angle θ between the track and the downward force from gravity, and the force F due to gravity ($F = mg$) acting on the cart to calculate the amount of work W done by gravity to displace it ($W = Fd\cos\theta$).

Students' calculated values for the work done by gravity on the cart and the cart's change in kinetic energy should be nearly equal in each trial. From this, students are expected to ascertain that the work done on the cart by gravity to displace it down the track is equal to the change in the cart's kinetic energy after being displaced.

Pre-Lab Discussion and Activity

All students performing this investigation will benefit from a brief discussion regarding work, but it will be most beneficial to students performing the Guided Inquiry or Student Designed version of this investigation. Part of their investigation requires that they use a non-zero net force to do work on an object. Understanding the fundamentals of work and its dependence on the directionality of the applied force will help students determine how, and in which direction, to apply the force.

To simplify their experiment as much as possible, explain that the work done on an object is determined by the product of the displacement of the object and the component of the applied force that is in the *same* direction as the displacement. Forces acting on an object that do not point in the same direction as the object's displacement do no (or negative) work. Using constant forces that act in the same direction as the displacement of their objects will simplify their experiments.

PRE-LAB QUESTIONS

These questions can be assigned as student homework prior to the lab.

❓ 1. If a weight lifter raises a weight (mass m) with constant velocity straight up to a height y, how much work has he or she done on the weight?

The weight lifter has done work equal to mgy. When the weight lifter raises the weight, he or she applies a force to displace it a distance y. Because the weight's velocity is constant, and the direction of the force applied by the lifter is vertical, the force applied to lift the weight is equal to the force of gravity on that weight. Because the applied force and displacement are in the same direction ($\theta = 0°$), the total work done on the weight by the lifter is $W = Fd\cos\theta = mgy$.

❓ 2. If the same weight lifter raises a weight with mass m straight up to a height y, walks forward some distance d without raising or lowering the weight, and then sets the weight back down on the ground, how much work has gravity done on the weight?

Gravity has done zero work. When the weight lifter raises the weight, gravity does work equal to $W = Fd\cos\theta$. Because the displacement y of the weight and the force applied by gravity mg are in opposite directions, θ is equal to 180°, making the work done by gravity to raise the weight equal to $-mgy$. When the weight lifter moves forward without changing the height of the weight, gravity does zero work because the force applied by gravity is acting perpendicular to its displacement ($\theta = 90°$). When the weight lifter lowers the weight, gravity does work equal and opposite to the work done while lifting it, making the sum of the work done on the weight by gravity equal to zero.

❓ 3. A box slides horizontally along a frictionless surface with constant positive velocity, and then onto a surface that applies a constant horizontal frictional force of –4.5 N. If the box comes to a stop on the frictional surface after 2 m, how much work was done on the box to stop it?

The work done on the box to stop it is equal to the product of the applied force and the displacement. Work = (−4.5 N) (2 m) = –9 J

Materials and Equipment

- Data collection system
- PASCO Smart Gate photogate[1]
- PASCO Photogate Bracket[1]
- PASCO Dynamics Cart Picket Fence[2]
- Table clamp or large base
- Support rod, 45-cm
- Meter stick
- PASCO Dynamics Track[3]
- PASCO Dynamics Track Rod Clamp[4]
- PASCO Dynamics Track End Stop[5]
- PASCO Dynamics Cart[6]
- PASCO Angle Indicator[7]
- Balance, 0.1-g resolution, 2,000-g capacity (1 per class)

Probeware Resources

Below are web-link and QR codes that will direct you to instructional video resources for individual pieces of PASCO probeware, sensors, and other hardware used in the lab activity. These same links and codes are provided to students in their activity handouts.

[1]www.pasco.com/ap21
PASCO Smart Gate

[2]www.pasco.com/ap16
PASCO Dynamics Cart Picket Fence

[3]www.pasco.com/ap08
PASCO PAStrack

[4]www.pasco.com/ap17
PASCO Dynamics Track Rod Clamp

[5]www.pasco.com/ap11
PASCO Dynamics Track End Stop

[6]www.pasco.com/ap07
PASCO PAScar

[7]www.pasco.com/ap14
PASCO Angle Indicator

Teacher Tips

Tip 1 – Catch the Cart at the Bottom of the Track

- Students following the procedure outlined in the Structured version of this activity will measure the speed of a cart rolling down an inclined track using a photogate and the PASCO picket fence system. Using this system, each time the cart and picket fence passes through the photogate beam a speed measurement is recorded by the data collection system. If the cart bounces off the end stop at the end of the track and passes back through the photogate, multiple speed measurements will be recorded. To avoid recording multiple speed measurements, students should use their hands to catch the cart just before or just after it hits the end stop at the end of the track.

Tip 2 – Avoid Inclination Angles > 10°

- If students choose to design their setup using an inclined cart and track system similar to the setup outlined in the Structured version of this activity, they should avoid inclining the track greater than 10°. Angles of inclination greater than 10° can potentially produce final cart velocities so great that the cart or end stop may be damaged if they collide.

Tip 3 – Measuring Speed with Other PASCO Sensors

- The procedure outlined in the Structured version of this lab activity makes use of a PASCO Smart Gate to measure the instantaneous speed of the cart at a discrete point in its path; however, students can use a variety of PASCO sensors that measure motion. These include:

- PASCO Motion Sensor—This sensor requires that students measure the velocity of the cart in a continuous fashion. Students who choose to do this must place the motion sensor at the top of the incline in-line with, but away from the direction of, the cart's path. Make sure the switch on the top of the motion sensor is set to the cart icon.

- PASCO Photogate with or without a PASCO Digital Adapter—The setup and procedure outlined in the Structured version of this lab activity uses a PASCO Smart Gate, but a single older PASCO photogate will work seamlessly in its place. If your data collection system has digital inputs, the PASCO Digital Adapter is not needed; the photogate can be connected directly to the digital inputs on the data collection system. If there are no digital inputs on your data collection system, you will need to connect the older PASCO photogate using a PASCO Digital Adapter.

- Photogate setting—When using an older PASCO photogate instead of the PASCO Smart Gate, students should use the "Photogate Timing" pre-configured timer within the data collection system. This setting automatically includes measurements of "time in gate" and "speed in gate" (or "velocity in gate"). Students will need to enter in the data collection system the length of the flag in the picket fence being used, which should appear correctly defaulted to 2.5 cm.

Tip 4 – Use the Correct θ

- If students choose to design their setup using an inclined cart and track system similar to the setup outlined in the Structured version of this activity, they should pay close attention to which angle is used in their calculations of work done by gravity. Students will likely assume that the angle θ used in the equation for work,

$$W = Fd\cos\theta \tag{2}$$

is the angle of inclination of the track; however, this is not correct. The angle θ used in Equation 2 is the angle between the applied force from gravity and the direction of the cart's displacement. In the case of the cart on an inclined track, this is equal to:

$\theta = 90° -$ Track inclination angle.

Sample Data

Below are sample data, acquired using the experimental setup and procedure outlined in the Structured version of the lab activity, and answers to questions in the Data Analysis section.

Data Analysis

Mass of cart and picket fence (kg) = 0.270

F_g (N) = 2.65

Table 1: Work and kinetic energy of a cart on an inclined track experiencing force from gravity

Trial	Distance Travelled by Cart (m)	Speed at Reference Point (m/s)	Work Done by Gravity (kg·m/s²)	Change in Kinetic Energy (J)
1	0.20	0.83	0.092	0.093
2	0.30	1.00	0.14	0.14
3	0.40	1.16	0.18	0.18
4	0.50	1.30	0.23	0.23
5	0.60	1.43	0.28	0.28

1. Calculate the magnitude of the gravitational force acting on the cart: $\left|\vec{F}_g\right| = F_g = mg$ where m is the mass of the cart and g is earth's gravitational constant 9.8 m/s². Record your result above.

   Calculation using sample data for Trial 1:

   $F_g = mg$

   $F_g = 0.270 \text{ kg} \times 9.8 \text{ m/s}^2 = 2.65 \text{ kg} \cdot \text{m/s}^2 = 2.65 \text{ N}$

2. Use Equation 2, F_g, and the distance travelled by the cart (in Table 1) to calculate the work done on the cart by gravity in each trial. Record your results for each trial into Table 1.

   NOTE: The angle θ in Equation 2 is not equal to the angle of inclination of the track; θ is equal to the angle between $\vec{F}_g$ and the direction of displacement of the cart. For example, if the angle of inclination is 10°, θ = 90° – 10° = 80°.

   Calculation using sample data for Trial 1:

   $W = F_g d \cos\theta$

   $W = 2.65 \text{ N} \times 0.20 \text{ m} \times \cos 80°$

   $W = 0.092 \text{ N} \cdot \text{m} = 0.092 \text{ kg} \cdot \text{m/s}^2$

3. Use Equation 3, the mass of the cart, and the speed data in Table 1 to calculate the change in kinetic energy of the cart from when it was released to when it reached the reference point in each trial. Assume the kinetic energy of the cart when it was released was zero in each trial. Record your results into Table 1.

   Calculation using sample data for Trial 1:

   $\Delta K = \frac{1}{2}m\Delta v^2 = \frac{1}{2}m(v_f - v_i)^2;\ v_i = 0 \text{ m/s}$

   $\Delta K = \frac{1}{2} \times 0.270 \text{ kg} \times \left(0.83 \text{ m/s}\right)^2$

   $\Delta K = 0.093 \text{ J}$

Guided Inquiry Questions

Below are sample responses to the Guiding Questions found in the Guided Inquiry version of this lab activity.

❓ 1. Based on the objective statement, what object will be the subject of your investigation and why?

   Students can choose any object to which a conservative force can be imparted; however, it will simplify their experiment if students choose an object whose speed can be easily measured in preparation for making calculations of its kinetic energy. Also, when students choose an object, they should consider how they will apply a conservative non-zero net force to it: objects that can be dropped or otherwise moved under the force of gravity will be good choices. Examples include a cart rolling down a near-frictionless track, a free-falling ball, or a pendulum swinging under the force of gravity.

❓ 2. How will you apply a non-zero net conservative force to your object that does work on the object?

   There are several ways to impart a conservative force to an object including: magnetic field attraction, electric field attraction, spring force (Hooke's Law), or gravitational field attraction. With any technique, students must be consistent and able to determine the amount of force being applied to their object so as to calculate the amount of work being done on their object. Students will greatly simplify their experiment if they choose to use gravitational force as a means of imparting a conservative force to their object.

3. How will you measure the change in kinetic energy of your object as the applied force does work? Can this be measured directly?

   Kinetic energy is a quantity that cannot be directly measured using conventional measurement techniques. Students should use a photogate or other timing device to measure the speed of their object at one or various points, or continuously. The procedure outlined in the Structured version of this lab activity makes use of a PASCO Smart Gate to measure the instantaneous speed of a cart at a discrete point in its path, which student then use to calculate the kinetic energy of the car.

4. What sources of error in your measurements do you expect, and how do you plan to prevent or minimize these?

   Students will encounter error when measuring the speed of their object depending on the measurement device(s) and technique(s) chosen. These error values should be clearly outlined in each student response and directly related to the measuring device's published accuracy and resolution. Errors propagating from the measurement tools are not easily avoided or prevented, but students may be able to improve their results by using tools with higher resolution and accuracy (for example, using a motion sensor that can measure instantaneous speed with high resolution versus a stopwatch that can only be used to measure average speed).

   Students will also encounter error intrinsic to their setup and procedure. Be certain that student responses are clear and each point of potential error is called out individually with a suitable solution to prevent or minimize it when collecting data.

Assessment Questions: Sample Responses

Sample responses to the Analysis and Synthesis questions found in each version of the lab activity:

Analysis Questions

1. How does the data for the work done on an object compare to the object's change in kinetic energy?

   Student values for work done on an object by a force should be equal to (or nearly equal to) their values for the change in kinetic energy of the object after the work was done. Depending on student setups, the acceleration experienced by their object (either positive or negative) will produce a change in kinetic energy that should equal the amount of work done to accelerate the object.

2. What were some unexpected factors that may have caused error in your measured values, and how could these have been avoided?

   For simplicity, students should attempt to isolate a single constant force that does work on their object in the direction of the object's displacement. Using variable forces like applied force from a spring or a rubber band will make calculating the work done on their object very difficult, if not impossible. If students have vastly different values between work done on their object and the change in the object's kinetic energy, it could be the result of assuming the applied force is constant when it is not, or the measured angle between the force and the displacement of their object is incorrect.

   Students may also encounter unexpected effects from friction. Friction can produce a force opposing the displacements of their objects (depending on the setup) which will also do work. If not accounted for, student values for the work done on their objects will be much greater than their values for change in kinetic energy.

3. How do the units associated with work compare to the units associated with change in kinetic energy?

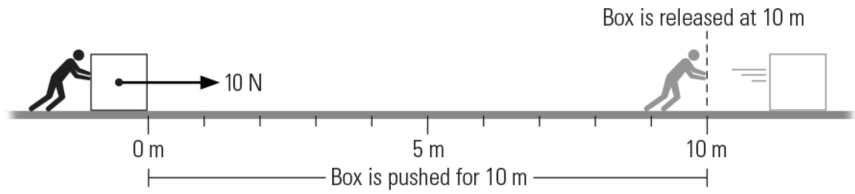
   The units for work are $N \cdot m$. Expanding these units yields $(kg \cdot m/s^2) \cdot m$, which equals $kg \cdot m^2/s^2$. The units for change in kinetic energy are $kg \cdot (m/s)^2$ which equals $kg \cdot m^2/s^2$. The units for work and change in kinetic energy are the same.

4. In one sentence, describe what you believe the mathematical relationship is between the work done by a non-zero net force on an object, and the change in that object's kinetic energy. Be specific, and use terms like "proportional to," "equal to," "inversely proportional to," and so on.

   Using their measurement-based relationship, students should ascertain that the work done on an object by a non-zero net force is equal to the change in that object's kinetic energy.

Synthesis Questions

❷ 1. A student pushes a 21-kg box horizontally along a frictionless surface for 10.0 m, and then releases the box to continue sliding. If the box is initially at rest, and the student pushes with a constant 15 N force, what is the box's speed when it is released?



$$W = K_f - K_i = \frac{1}{2}mv_f^2$$

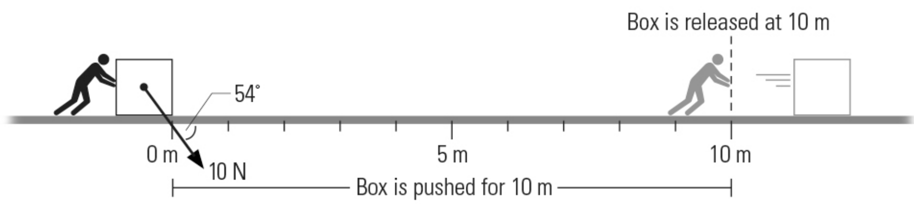
$$W = Fd\cos\theta = Fd$$

$$Fd = \frac{1}{2}mv_f^2$$

$$v_f = \sqrt{\frac{2Fd}{m}} = \sqrt{\frac{2(15\text{ N})(10.0\text{ m})}{(21\text{ kg})}}$$

$$v_f = 3.8\text{ m/s}$$

❷ 2. If the student from the previous question hadn't pushed the box horizontally, but rather, at an angle of 54° relative to the frictionless surface, what would have been the box's speed when it was released?



$$Fd\cos\theta = \frac{1}{2}mv_f^2$$

$$v_f = \sqrt{\frac{2Fd\cos\theta}{m}} = \sqrt{\frac{2(15\text{ N})(10.0\text{ m})\cos(54°)}{(21\text{ kg})}}$$

$$v_f = 2.9\text{ m/s}$$

3. Suppose an 18-wheel truck and trailer has a mass of 30,000 kg and is traveling with a speed of 24.5 m/s. If the driver slams on his brakes and begins to skid, what would the stopping distance be if the coefficient of kinetic friction between the truck's tires and the pavement is 0.50? Show you work.

$$W = K_f - K_i = -\frac{1}{2}mv_i^2;\ K_f = 0$$

$$W = Fd\cos\theta = -F_f d = -\mu_k mgd$$

$$-\mu_k mgd = -\frac{1}{2}mv_i^2$$

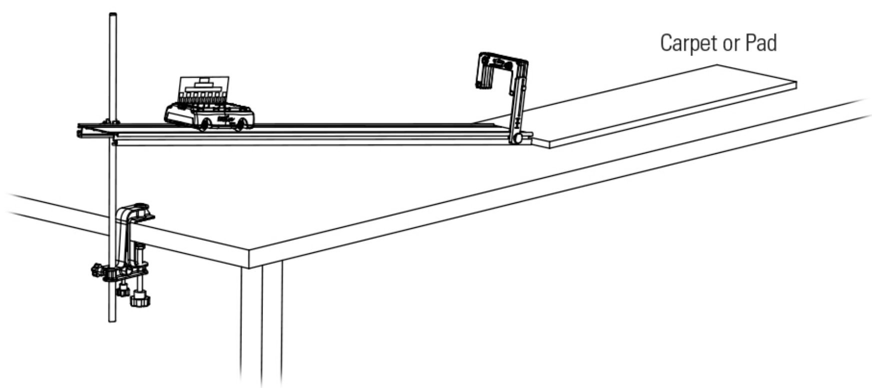
$$d = \frac{v_i^2}{2\mu_k g} = \frac{(24.5\ \text{m/s})^2}{2(0.50)(9.8\ \text{m/s}^2)}$$

$$d = 61\ \text{m}$$

Students may mistakenly think that the stopping distance of the truck (or cart) is dependent on its mass, but they should discover that the work being done by the frictional force is equal to the kinetic energy of the truck. Both quantities are proportional to mass, which results in a relationship between stopping distance and speed of the truck, but not mass.

Extended Inquiry Suggestions

Challenge students to use the same setup from this activity, with the addition of a narrow strip of carpet at the end of the track, and the skills they have built in this activity to calculate the distance needed to stop a cart as it rolls down an inclined track, off the end of the track and onto a flat "runaway" cart pad. Have students then test their calculations.



You will need to prepare the runaway cart pad(s) before class, which can be made of a semi-firm, but relatively soft material that will provide rolling or kinetic friction to the cart, but won't stop the cart as soon as it touches the pad. Some suggested pad materials are:

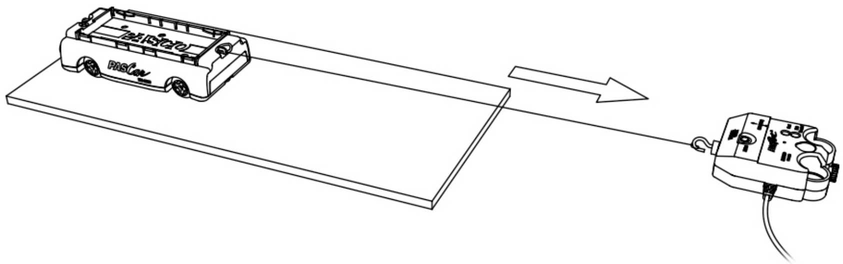
- Flat, dense foam padding
- Carpet padding
- Medium–low density (zero nap) carpet

Because the cart will have to transition as smoothly as possible from the track to the pad, try to use a material that will elevate the pad so its surface is flush with the end of the track.

You (or student groups) will need to determine the coefficient for rolling (kinetic) friction μ_k between the cart and the pad. This can be done by measuring the applied force needed to pull the cart across the runaway cart pad at a constant speed, and then dividing that value by the mass m of the cart and earth's gravitational acceleration g:

$$F_{\text{applied}} = -\mu_k F_n = -\mu_k mg$$

$$\mu_k = -\frac{F_{\text{applied}}}{mg}$$



Assign each group a different starting distance l up the inclined track for their cart. Tell students that they cannot use photogates or other sensors to measure the speed of their cart as it leaves the track, but rather, they must calculate their cart's speed using a conservation of energy argument involving the starting distance l and the track angle θ:

$$v = \sqrt{2gy} = \sqrt{2gl\sin\theta}$$

To calculate their stopping distance, students will use the following relationship:

$$W = K_f - K_i = -\frac{1}{2}mv^2$$

$$W = Fd\cos\theta = -F_f d = -\mu_k mgd$$

$$-\mu_k mgd = -\frac{1}{2}mv^2$$

$$d = \frac{v^2}{2\mu_k g} = \frac{l\sin\theta}{\mu_k}$$

NAME PERIOD DATE

7. WORK AND KINETIC ENERGY

STRUCTURED

Driving Question | Objective

How is the work done on an object by a non-zero net conservative force related to the change in that object's kinetic energy? Investigate the relationship between the change in kinetic energy of an object experiencing a non-zero net conservative force, and the work done by that net force on the object. Establish a measurement-based relationship between work and kinetic energy.

Materials and Equipment

- Data collection system
- PASCO Smart Gate photogate[1]
- PASCO Photogate Bracket[1]
- PASCO Dynamics Cart Picket Fence[2]
- Table clamp or large base
- Support rod, 45-cm
- Meter stick
- PASCO Dynamics Track[3]
- PASCO Dynamics Track Rod Clamp[4]
- PASCO Dynamics Track End Stop[5]
- PASCO Dynamics Cart[6]
- PASCO Angle Indicator[7]
- Balance, 0.1-g resolution, 2,000-g capacity (1 per class)

[1]www.pasco.com/ap21
PASCO Smart Gate

[2]www.pasco.com/ap16
PASCO Dynamics Cart Picket Fence

[3]www.pasco.com/ap08
PASCO PAStrack

[4]www.pasco.com/ap17
PASCO Dynamics Track Rod Clamp

[5]www.pasco.com/ap11
PASCO Dynamics Track End Stop

[6]www.pasco.com/ap07
PASCO PAScar

[7]www.pasco.com/ap14
PASCO Angle Indicator

Background

WORK

Work done on an object by a force is expressed as the product of the force and the magnitude of displacement of the object in the direction of that force:

$$\text{Work} = \text{Force} \times \text{Distance} \tag{1}$$

If a force is applied to an object whose resultant movement is not in the direction of the force, the work done on that object only includes that component of the force in the direction of the object's movement:

$$W = Fd\cos\theta \tag{2}$$

where d is the distance that the force F acts on the object, and θ is the angle between the applied force direction and the direction of displacement.

KINETIC ENERGY

An object's kinetic energy K is described by the equation:

$$K = \frac{1}{2}mv^2 \qquad (3)$$

where m is the mass of the object and v is the object's speed.

Although work and kinetic energy are different quantities, they are closely related to each other in a mechanical system. In this lab you will investigate the relationship between the change in kinetic energy of a cart experiencing a non-zero net force from gravity and the work done by gravity on the cart.

RELEVANT EQUATIONS

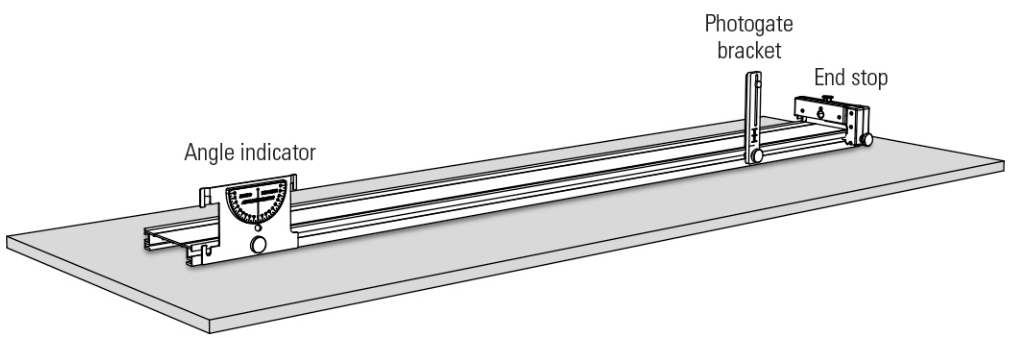
$$W = Fd\cos\theta \qquad (2)$$

$$K = \frac{1}{2}mv^2 \qquad (3)$$

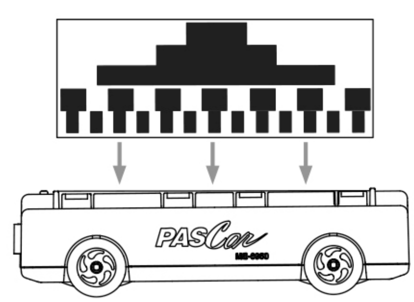
Procedure

SET UP

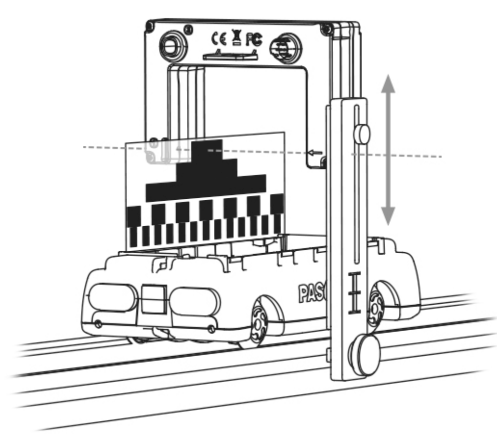
1. Attach the angle indicator, end stop, and photogate bracket to the dynamics track as in the picture below. The end stop should be at the very end of the track and the photogate bracket should be 20 cm from the end stop.



2. Insert the dynamics cart picket fence into the recessed slots on the top of the dynamics cart, oriented with the 2.5-cm solid band as the top-most pattern on the picket fence, and then place the cart on the track. The cart and picket fence will be the "object" on which work will be done in this investigation.



Insert picket fence into slot in top of dynamics cart

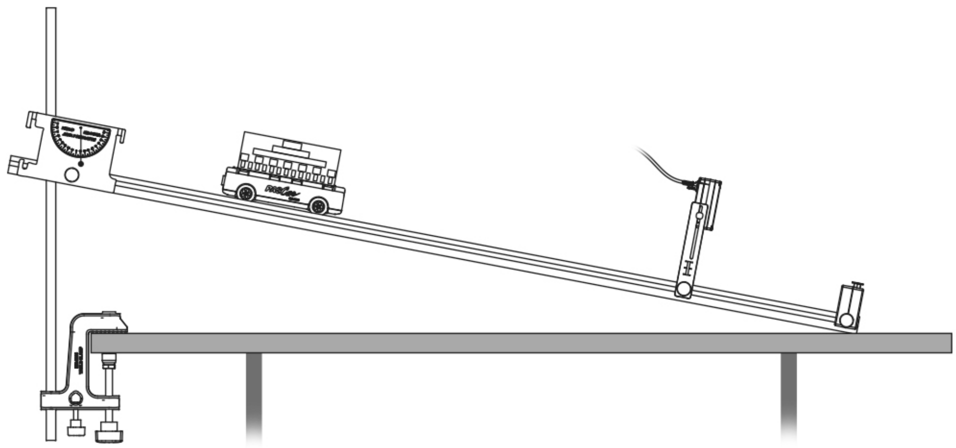


3. Attach the photogate to the photogate bracket. Adjust the height of the photogate on the bracket so the 2.5-cm band on the picket fence passes through the photogate beam as the cart rolls through the photogate.

4. Remove the cart and attach the side of the dynamics track without the end stop to the support rod using the dynamics track rod clamp. Incline the track 10°.

5. Connect the photogate to the data collection system.

6. Configure the data collection system to use photogate timing to measure the speed (or velocity) of the cart as the picket fence passes through the photogate, and then display this measurement in a digits display.

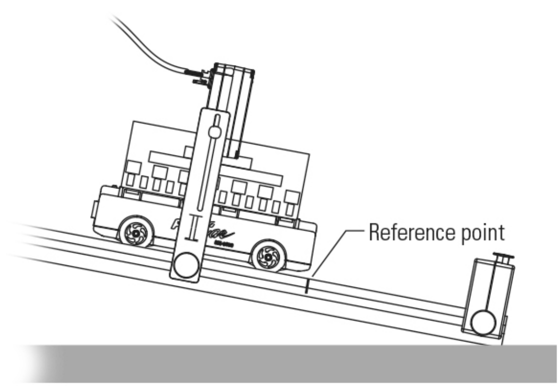


COLLECT DATA

7. Use the balance to measure the mass of the cart and picket fence. Record this mass in the Data Analysis section below.

8. Place the cart on the track and hold it in place with the center of the picket fence aligned with the photogate as in the picture. Make note of the position of the cart's front edge; this will be the reference point from which you will make distance measurements and speed measurements in each trial.

   NOTE: It may be helpful to place a small piece of tape or make a small mark on the track noting the position of the cart's front edge (the reference point).



9. Slide the cart up the track 20 cm from the reference point and hold it in place.

10. Begin recording data on the data collection system, and then release the cart, letting it roll freely down the track
and through the photogate.

11. Catch the cart just before it hits the end stop, and then stop recording data.

12. Record the speed of the cart measured at the reference point, and the distance the cart travelled from where it was released to the reference point, into Table 1 in the Data Analysis section.

13. Repeat the same data collection steps four more times, releasing the cart from 10 cm farther up the track in each trial. Record all data into Table 1 in the Data Analysis section.

Data Analysis

Mass of cart and picket fence (kg) = ________________________

F_g (N) = ________________________

Table 1: Work and kinetic energy of a cart on an inclined track experiencing force from gravity

Trial	Distance Travelled by Cart (m)	Speed at Reference Point (m/s)	Work Done by Gravity (kg·m/s²)	Change in Kinetic Energy (J)
1				
2				
3				
4				
5				

1. Calculate the magnitude of the gravitational force acting on the cart: $\left|\vec{F}_g\right| = F_g = mg$ where m is the mass of the cart and g is earth's gravitational constant 9.8 m/s². Record your result above.

2. Use Equation 2, F_g, and the distance travelled by the cart (in Table 1) to calculate the work done on the cart by gravity in each trial. Record your results for each trial into Table 1.

   NOTE: The angle θ in Equation 2 is not equal to the angle of inclination of the track; θ is equal to the angle between $\vec{F}_g$ and the direction of displacement of the cart. For example, if the angle of inclination is 10°, θ = 90° – 10° = 80°.

3. Use Equation 3, the mass of the cart, and the speed data in Table 1 to calculate the change in kinetic energy of the cart from when it was released to when it reached the reference point in each trial. Assume the kinetic energy of the cart when it was released was zero in each trial. Record your results into Table 1.

Analysis Questions

❷ 1. How does the data for the work done on an object compare to the object's change in kinetic energy?

____

____

____

❷ 2. What were some unexpected factors that may have caused error in your measured values, and how could these have been avoided?

____

____

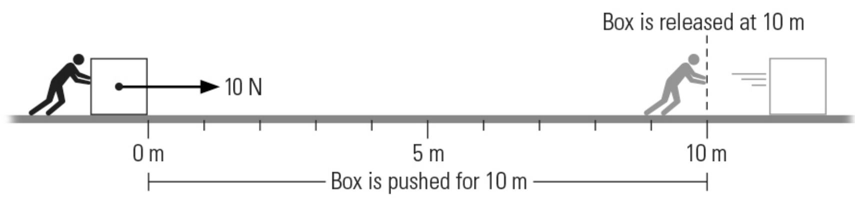
____

3. How do the units associated with work compare to the units associated with change in kinetic energy?

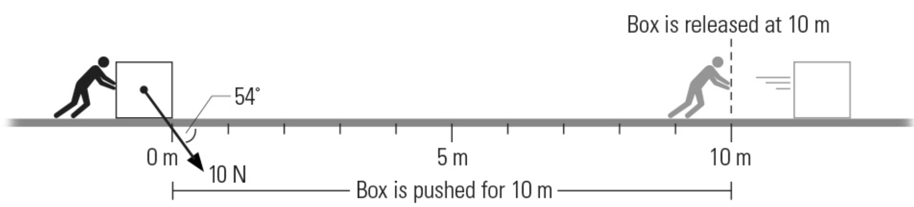
4. In one sentence, describe what you believe the mathematical relationship is between the work done by a non-zero net force on an object, and the change in that object's kinetic energy. Be specific, and use terms like "proportional to," "equal to," "inversely proportional to," and so on.

Synthesis Questions

1. A student pushes a box initially at rest, horizontally along a frictionless surface for 10.0 m and then releases the box to continue sliding. If the student pushes with a constant 15 N force, what is the box's speed when it is released?



2. If the student from the previous question hadn't pushed the box horizontally, but rather, at an angle of 54° relative to the frictionless surface, what would have been the box's speed when it was released?



❓ 3. Suppose an 18-wheel truck and trailer has a mass of 30,000 kg and is traveling with a speed of 24.5 m/s. If the driver slams on his brakes and begins to skid, what would the stopping distance be if the coefficient of kinetic friction between the truck's tires and the pavement is 0.50? Show you work.

8. CONSERVATION OF MOMENTUM

Connections to the AP® Physics 1 Curriculum*

The lab activity correlates to the following pieces of the AP® Physics 1 framework:

Big Idea	5	Enduring Understanding	D	Essential Knowledge	1

Learning Objective 3: The student is able to apply mathematical routines appropriately to problems involving elastic collisions in one dimension and justify the selection of those mathematical routines based on the conservation of momentum and restoration of kinetic energy.
Science Practices: 2.1, 2.2

Big Idea	5	Enduring Understanding	D	Essential Knowledge	2

Learning Objective 2: The student is able to plan data collection strategies to test the law of conservation of momentum in a two-object collision that is elastic or inelastic and analyze the resulting data graphically.
Science Practices: 4.1, 4.2, 5.1

Learning Objective 4: The student is able to analyze data that verify conservation of momentum in collisions with and without external friction force.
Science Practices: 4.1, 4.2, 4.4, 5.1, 5.3

Time Requirement

Preparation Time: 10 minutes

Lab Activity: 50 minutes

Prerequisites

Students should be familiar with the following concepts:

- The momentum $\vec{p}$ of an object is equal to the product of its mass m and velocity $\vec{v}$:

  $$\vec{p} = m\vec{v} \quad (1)$$

  For a linear system that is not influenced by outside forces, the total momentum of the system is conserved. This extends to objects experiencing two types of collisions: elastic and inelastic.
- An elastic collision is a type of collision in which the objects involved in the collision bounce off each other without the loss of kinetic energy from the total object system. An inelastic collision is a collision type in which the system consisting of the objects involved in the collision experiences a loss of kinetic energy due to other changes in the system, such as the objects sticking fully or partially to each other, or the deformation of the objects in the collision.
- Kinetic energy is a component of an object's total mechanical energy and is equal to $1/2mv^2$ where m is the object's mass and v is the object's speed.

Driving Question | Objective

How is the total linear momentum and kinetic energy of a two-object system affected by a collision? Experimentally demonstrate that linear momentum and kinetic energy are conserved in an elastic collision, and that linear momentum is conserved but kinetic energy is not conserved in an inelastic collision.

Procedural Overview

The Structured version of this lab activity is divided into two parts:

Part 1 – Elastic Collisions: Students gently push one cart into a stationary second cart, producing a perfectly elastic collision using the magnetic bumpers on the carts. The magnetic bumpers repel each other without the carts ever making contact. Students use a balance and two motion sensors to measure the mass of the carts and the velocity of each cart just before and just after the collision, and then use those values to calculate the total momentum and kinetic energy of the system before and after the collision. Students repeat this process two additional times, increasing the mass of the stationary cart each time. Student data should show that the momentum and kinetic energy of the two car system are the same before and after the elastic collision in each trial.

Part 2 – Inelastic Collisions: Students follow the same procedure as in Part 1, but use the Velcro® bumpers on the carts instead of the magnetic bumpers, producing an inelastic collision in which the two carts stick together. The mass of each cart is measured in each trial, along with each cart's velocity just before and just after the collision. Using their measured values for mass and velocity, students calculate the total momentum and kinetic energy of the system before and after the collision in each trial. Student data should show that the total momentum of the system is the same before and after the inelastic collision in each trial, but the kinetic energy is not.

Pre-Lab Discussion and Activity

This is a pre-lab suggestion and activity.

Begin this lab discussion by reminding students what momentum is, namely, mass × velocity, and that the total momentum of the system at any time is equal to the sum of the individual momentum of the objects in the system. A great starting point would be a discussion about billiard balls: what happens when a billiards player hits the cue ball (the white one) directly at another ball of identical mass so that they hit head on? Usually, the cue ball will stop, and the other ball will move away at the same speed as the cue ball's initial speed, indicating that the total momentum of the system before the collision is the same after the collision. This momentum conservation extends to all types of objects, masses, and velocities, and will hold true for both elastic and inelastic collision types; however, the total kinetic energy of the system may change in a collision depending on the collision type. Kinetic energy is conserved in perfectly elastic collisions, and is not conserved in inelastic collisions.

So, the total system momentum after a collision is the same as the total system momentum before the collision. Now consider the billiard ball that's moving after the collision. Does it keep moving forever? Of course not; it eventually slows down and comes to rest because of rolling friction with the surface of the billiards table. But if friction is minimized and accurate measurements can be taken both before and after the collision, students should be able to verify (within reasonable error ranges) that momentum is conserved.

The following questions can be sent home as pre-lab homework, or can be incorporated into the above discussion.

PRE-LAB QUESTIONS

Consider a billiards player hitting a cue ball with mass m and velocity $\vec{v}$ directly at another stationary ball of identical mass so that they hit head on. After the collision, the cue ball is stationary and the other ball has velocity $\vec{v}$.

1. What is the momentum of the cue ball initially?

   Mass (cue ball) × Velocity (cue ball)

2. What is the momentum of the stationary ball initially?

   Zero

3. What is the total momentum of the system (both balls together) before the collision?

   Mass (cue ball) × Velocity (cue ball)

4. What is the momentum of the cue ball after the collision?

   Zero

5. What is the momentum of the other ball after collision?

   Mass (other ball) × Velocity (other ball)

6. What is the total momentum of the system (both balls together) after the collision?

   Mass (other ball) × Velocity (other ball)

7. How does the momentum of the system before the collision compare to the momentum of the system after the collision?

   They are equal.

Materials and Equipment

- Data collection system
- PASCO Motion Sensor (2)[1]
- PASCO Dynamics Cart with magnetic and Velcro® bumpers (2)[2]
- PASCO Dynamics Track with feet[3]
- 250-g cart mass (2)[2]
- Balance, 0.1-g resolution, 2,000-g capacity (1 per class)

Probeware Resources

Below are web-link and QR codes that will direct you to instructional video resources for individual pieces of PASCO probeware, sensors, and other hardware used in the lab activity. These same links and codes are provided to students in their activity handouts.

[1]www.pasco.com/ap18 — PASCO Motion Sensor

[2]www.pasco.com/ap07 — PASCO PAScar

[3]www.pasco.com/ap08 — PASCO PAStrack

Teacher Tips

Tip 1 – Cart Speeds

- When using carts and tracks, students should avoid colliding their carts at velocities so great that the carts become de-railed from the track. Students following the procedure outlined in the Structured version of this lab activity will need to use higher, yet controlled, initial velocities for their moving cart in trials 2 and 3 within Parts 1 and 2 so that the final velocities of both carts are great enough to be measureable.

Tip 2 – Using Objects Other Than Carts

- The magnetic bumpers on the PASCO dynamics carts are used to produce perfectly elastic collisions between the carts, which will help deliver favorable data with minimal error. However, if your students do not have access to PASCO dynamics carts, it is possible to use any cart or object that that can produce a perfectly elastic collision. These objects include, but are not limited to:

    - Gliders on an air track system with rubber band bumpers

    - Frictionless carts with spring bumpers (the springs must have sufficient size and rigidity so as to not completely compress in the collision)

    - Billiard balls traveling in one-dimension

Tip 3 – Aiming the Motion Sensors

- Students following the setup and procedure outlined in the Structured version of this lab activity, as well as any students using motion sensors to make measurements, should be careful to aim each motion sensor directly at its target. In the case of the PASCO dynamics cart and track, a very slight downward angle between the motion sensor and cart can help to return better data and avoid crosstalk between two motion sensors aimed at each other.

Tip 4 – Use Only the Velocities Immediately Before and After the Collision

- In this activity, students will use a collision between two objects to demonstrate momentum and energy conservation. When making measurements of velocity to use in their calculations of both momentum and kinetic energy, students should only use values acquired immediately before and immediately after the collision. Using the velocity at these moments better represents the velocities associated with the collision. Velocities measured before or after these times may not be the actual velocity of the objects at the point the collision occurs.

Sample Data

Below are sample data, acquired using the experimental setup and procedure outlined in the Structured version of the lab activity, and answers to questions in the Data Analysis section.

Data Analysis

PART 1 – ELASTIC COLLISION

Student Part 1 graphs of velocity versus time for both carts undergoing an elastic collision will look similar to:

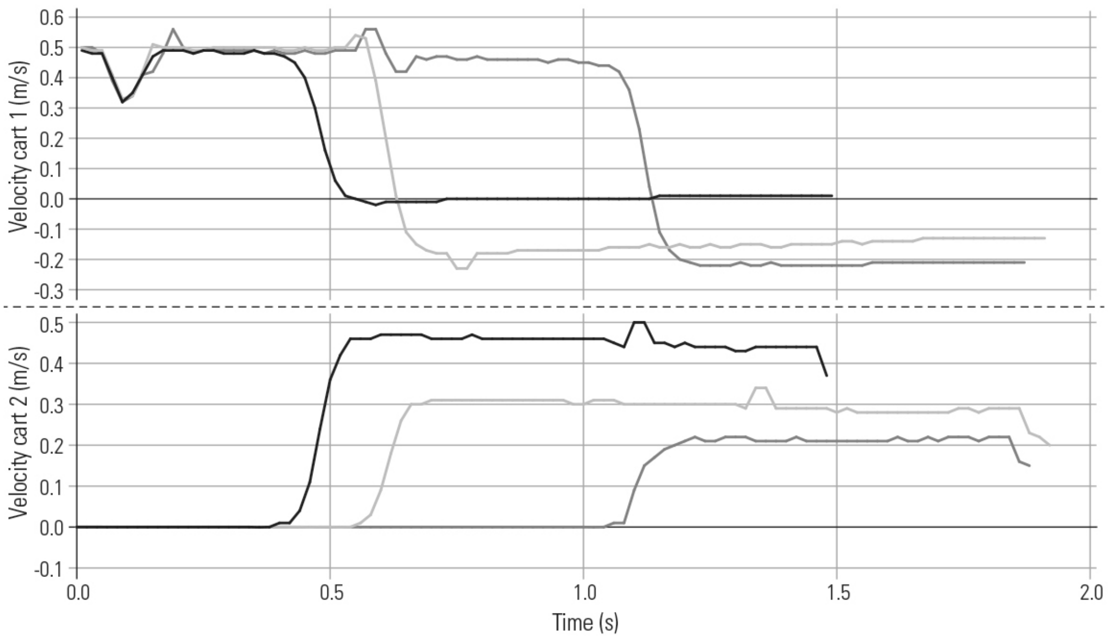


Table 1: Cart 1 Elastic collision data

Trial	Mass (kg)	Initial Velocity (m/s)	Final Velocity (m/s)	Initial Momentum (kg·m/s)	Final Momentum (kg·m/s)	Initial Kinetic Energy (J)	Final Kinetic Energy (J)
1	0.249	0.36	0.00	0.090	0.00	0.016	0.00
2	0.249	0.50	−0.16	0.12	−0.040	0.031	0.0032
3	0.249	0.45	−0.23	0.11	−0.057	0.025	0.0066

Table 2: Cart 2 Elastic collision data

Trial	Mass (kg)	Initial Velocity (m/s)	Final Velocity (m/s)	Initial Momentum (kg·m/s)	Final Momentum (kg·m/s)	Initial Kinetic Energy (J)	Final Kinetic Energy (J)
1	0.257	0.00	0.33	0.00	0.085	0.00	0.014
2	0.516	0.00	0.32	0.00	0.17	0.00	0.026
3	0.775	0.00	0.21	0.00	0.16	0.00	0.017

1. Calculate the initial and final momentum of each cart in each Part 1 trial. Record the values for Cart 1 into Table 1 and the values for Cart 2 into Table 2.

Calculation using sample data for Trial 1, Cart 1:

$\vec{p} = m\vec{v}$

$\vec{p}_{i} = 0.249\text{ kg} \times 0.36\text{ m/s} = 0.90\text{ kg} \cdot \text{m/s}$

$\vec{p}_{f} = 0.249\text{ kg} \times 0.00\text{ m/s} = 0.00\text{ kg} \cdot \text{m/s}$

2. Calculate the initial and final kinetic energy of each cart in each Part 1 trial. Record the values for Cart 1 into Table 1 and the values for Cart 2 into Table 2.

Calculation using sample data for Trial 1, Cart 1:

$K = \frac{1}{2}mv^2$

$K_{1i} = \frac{1}{2} \times 0.249\text{ kg} \times \left(0.36\text{ m/s}\right)^2 = 0.016\text{ J}$

$K_{1f} = \frac{1}{2} \times 0.249\text{ kg} \times \left(0.00\text{ m/s}\right)^2 = 0.00\text{ J}$

3. Calculate the total momentum and total kinetic energy of the two-cart system before and after each Part 1 collision. Record the results into Table 3 below.

Calculation using sample data for Trial 1:

$\vec{p}_{1i} + \vec{p}_{2i} = 0.90\text{ kg} \cdot \text{m/s} + 0.00\text{ kg} \cdot \text{m/s} = 0.90\text{ kg} \cdot \text{m/s}$

$\vec{p}_{1f} + \vec{p}_{2f} = 0.00\text{ kg} \cdot \text{m/s} + 0.85\text{ kg} \cdot \text{m/s} = 0.85\text{ kg} \cdot \text{m/s}$

$K_{1i} + K_{2i} = 0.016\text{ J} + 0.00\text{ J} = 0.016\text{ J}$

$K_{1f} + K_{2f} = 0.00\text{ J} + 0.014\text{ J} = 0.014\text{ J}$

Table 3: Total system momentum and kinetic energy before and after the elastic collision

Trial	Initial Momentum of System (kg·m/s)	Final Momentum of System (kg·m/s)	Initial Kinetic Energy of System (J)	Final Kinetic Energy of System (J)
1	0.090	0.085	0.016	0.014
2	0.12	0.13	0.031	0.029
3	0.11	0.10	0.025	0.024

PART 2 – INELASTIC COLLISION

Student Part 2 graphs of velocity versus time for both carts undergoing an elastic collision will look similar to:

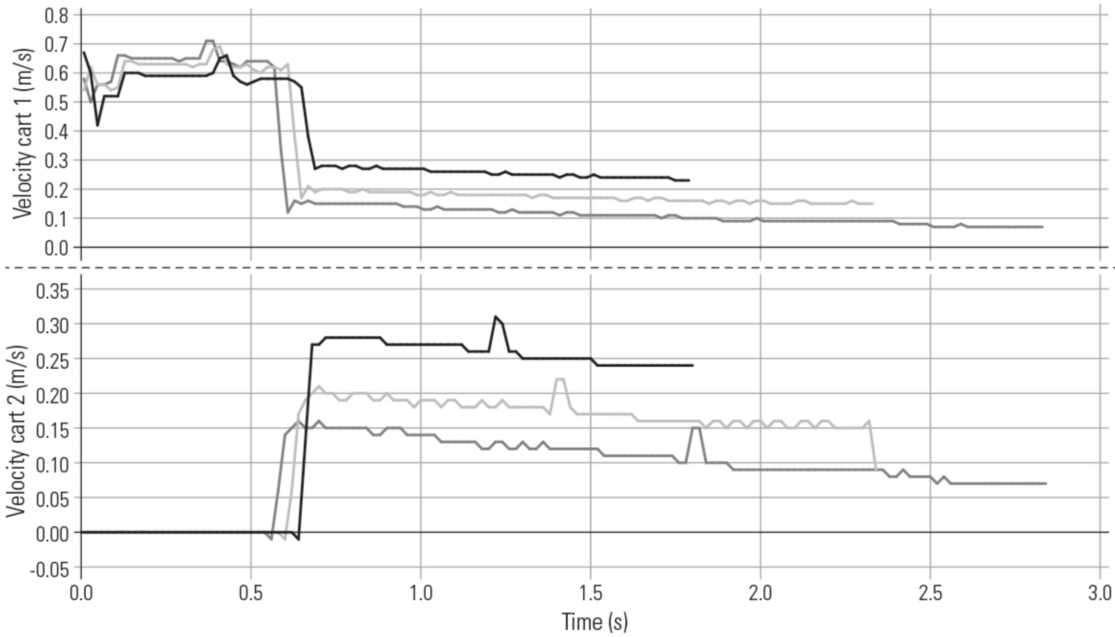


Table 4: Cart 1 Inelastic collision data

Trial	Mass (kg)	Initial Velocity (m/s)	Final Velocity (m/s)	Initial Momentum (kg·m/s)	Final Momentum (kg·m/s)	Initial Kinetic Energy (J)	Final Kinetic Energy (J)
4	0.249	0.58	0.26	0.14	0.065	0.042	0.0084
5	0.249	0.51	0.17	0.13	0.042	0.032	0.0036
6	0.249	0.59	0.15	0.15	0.037	0.043	0.0028

Table 5: Cart 2 Inelastic collision data

Trial	Mass (kg)	Initial Velocity (m/s)	Final Velocity (m/s)	Initial Momentum (kg·m/s)	Final Momentum (kg·m/s)	Initial Kinetic Energy (J)	Final Kinetic Energy (J)
4	0.257	0.00	0.27	0.00	0.069	0.00	0.0094
5	0.516	0.00	0.17	0.00	0.088	0.00	0.0075
6	0.775	0.00	0.14	0.00	0.11	0.00	0.0076

4. Calculate the initial and final momentum of each cart in each Part 2 trial. Record the values for Cart 1 into Table 4 and the values for Cart 2 into Table 5.

   Calculation using sample data for Trial 1, Cart 1:

   $\vec{p} = m\vec{v}$

   $\vec{p}_{\mathrm{i}} = 0.249\ \mathrm{kg} \times 0.58\ \mathrm{m/s} = 0.14\ \mathrm{kg \cdot m/s}$

   $\vec{p}_{\mathrm{f}} = 0.249\ \mathrm{kg} \times 0.26\ \mathrm{m/s} = 0.65\ \mathrm{kg \cdot m/s}$

5. Calculate the initial and final kinetic energy of each cart in each Part 2 trial. Record the values for Cart 1 into Table 4 and the values for Cart 2 into Table 5.

   Calculation using sample data for Trial 1, Cart 1:

   $$K = \frac{1}{2}mv^2$$

   $$K_{1i} = \frac{1}{2} \times 0.249\ \text{kg} \times (0.58\ \text{m/s})^2 = 0.042\ \text{J}$$

   $$K_{1f} = \frac{1}{2} \times 0.249\ \text{kg} \times (0.26\ \text{m/s})^2 = 0.0084\ \text{J}$$

6. Calculate the total momentum and total kinetic energy of the two-cart system before and after each Part 2 collision. Record the results into Table 6 below.

   Calculation using sample data for Trial 1:

   $$\vec{p}_{1i} + \vec{p}_{2i} = 0.14\ \text{kg} \cdot \text{m/s} + 0.00\ \text{kg} \cdot \text{m/s} = 0.14\ \text{kg} \cdot \text{m/s}$$

   $$\vec{p}_{1f} + \vec{p}_{2f} = 0.065\ \text{kg} \cdot \text{m/s} + 0.069\ \text{kg} \cdot \text{m/s} = 0.13\ \text{kg} \cdot \text{m/s}$$

   $$K_{1i} + K_{2i} = 0.042\ \text{J} + 0.00\ \text{J} = 0.042\ \text{J}$$

   $$K_{1f} + K_{2f} = 0.0084\ \text{J} + 0.0094\ \text{J} = 0.018\ \text{J}$$

Table 6: Total system momentum and kinetic energy before and after inelastic collision

Trial	Initial Momentum of System (kg·m/s)	Final Momentum of System (kg·m/s)	Initial Kinetic Energy of System (J)	Final Kinetic Energy of System (J)
4	0.14	0.13	0.042	0.018
5	0.13	0.13	0.032	0.011
6	0.15	0.15	0.043	0.010

Guided Inquiry Questions

Below are sample responses to the Guiding Questions found in the Guided Inquiry version of this lab activity.

❓ 1. When choosing two objects to collide together, should you be concerned about whether the two objects are rigid? Explain your answer.

   An elastic collision is one in which energy is not transferred via operations like the deformation of the objects colliding. It is essential that the objects chosen to perform an elastic collision do not change shape permanently or stick together. If the objects do either, it would be considered an inelastic collision, and kinetic energy will not be a conserved quantity.

❓ 2. Should the objects you choose have the same size and mass? Explain why this is or isn't important.

   The size and mass of the objects being collided do not have to be the same. Momentum is conserved regardless of the initial size, mass, and velocity of either object. In fact, mass could theoretically be transferred from one object to the other and momentum would still be a conserved quantity in an inelastic collision. The only requirement is that the total system mass remains the same before and after any collision.

❓ 3. Which will help simplify the experiment, a one-dimensional elastic and inelastic collision or a two-dimensional elastic and inelastic collision and why?

   A one-dimensional collision will be much simpler to implement and analyze than a two-dimensional collision. Students should only choose to perform this activity using one-dimensional collisions. A two dimensional collision will require that students calculate the component momentums of each object before and after the collision and show that each is conserved, thus increasing the amount of work needed to demonstrate the concept.

❓ 4. A perfectly elastic collision is nearly impossible to achieve in real life. How will you produce as perfect an elastic collision as possible between your two objects?

Answers will depend on the type of objects chosen to collide. Generally, students will produce a nearly perfect elastic collision using any very rigid objects that will not deform in the collision and having surfaces with minimal friction. Students can also use carts with magnetic or spring bumpers designed to produce elastic collisions, similar to the PASCO PAScar dynamics carts and track.

❓ 5. How will you produce an inelastic collision between your two objects?

Inelastic collisions are most easily produced by having the two objects deform or stick together upon colliding. Tape, clay, Velcro or other sticky surfaces on the two objects will achieve best results.

❓ 6. How do you plan to determine the momentum of the system before and after each collision: what quantities can you measure and what tools will you use to measure them?

Momentum is not a quantity that can be measured directly using conventional methods; however, students can measure the mass of each object and each object's velocity before and after the collision, and then use those values to calculate the total system momentum before and after the collision. Velocity can be measured using several different methods, including obtaining direct measurements from a motion sensor or photogate. Students must explicitly measure only the velocities immediately before and immediately after the collision. These velocities are more representative of the velocities associated with the collision. Velocities measured earlier or later may not be the actual velocity of the objects at the point the collision occurs.

❓ 7. How do plan to determine the kinetic energy of the system before and after the collision: what quantities can you measure and what tools will you use to measure them?

Kinetic energy is not a quantity that can be measured directly using conventional methods; however, students can measure the mass of each object and each object's velocity before and after the collision, and then use those values to calculate the total kinetic energy of the system before and after the collision. Again, students must explicitly measure only the velocities immediately before and immediately after the collision. These velocities are more representative of the velocities associated with the collision. Velocities measured earlier or later may not be the actual velocity of the objects at the point the collision occurs.

❓ 8. Will your experiment setup and procedure be the same for both elastic and inelastic collisions? Justify your answer.

Aside from the objects used in each collision, or a mechanism to produce an inelastic collision (see the sample response to the Guided Inquiry Question above, "How will you produce an inelastic collision…"), most experiment setups will be identical for elastic and inelastic collisions.

Assessment Questions: Sample Responses

Sample responses to the Analysis and Synthesis questions found in each version of the lab activity:

Analysis Questions

❓ 1. What experimental evidence do you have showing that momentum is conserved in inelastic and elastic collisions?

Answers will vary. A superior argument includes evidence showing that the total momentum of the two-object system is the same before and after each collision (both elastic and inelastic). Following the procedure outlined in the structured version of this lab activity, student data will show (within a small variance) that the sum of the momentums of the two carts involved in both collision types, regardless of their starting masses, is the same immediately before a collision as it is immediately after a collision.

❓ 2. How does your data support that kinetic energy is conserved in elastic collisions?

Supporting evidence should include calculated values of each object's kinetic energy before and after each elastic collision, and should show that the sum of the objects' initial kinetic energies just before a collision is equal to the sum of the objects' kinetic energies just after the collision (the more perfectly elastic the collision, the more likely that student data will show the same kinetic energy before and after an elastic collision).

3. How does your data support that kinetic energy is NOT conserved in inelastic collisions?

   Supporting evidence should include calculated values of each object's kinetic energy before and after each inelastic collision, and should show that the sum of the objects' initial kinetic energies just before a collision is much greater than the sum of the objects' kinetic energies just after the collision.

4. Why is kinetic energy not conserved in inelastic collisions? Where is the energy lost?

   In an inelastic collision, the system experiences a loss of kinetic energy due to other changes in the system such as the objects sticking fully or partially to each other, or the deformation of the objects in the collision.

Synthesis Questions

1. Two locomotives, each weighing 100,000 kg and having a speed of 100 km/hr, race toward each other and have a completely inelastic collision. What is the final momentum of the system? Justify your answer.

   The final momentum is 0 kg·m/s. Although the question specifies that the collision is perfectly inelastic, it is irrelevant considering that the total momentum of the system is conserved regardless of the collision type. This conservation law requires that the momentum of the two-locomotive system be equal before and after the collision. The system momentum is zero before the collision (the locomotives' momentums are equal and opposite; their sum equals zero), therefore the momentum of the system after the collision is zero.

2. A 10.0-kg bowling ball sliding across a frictionless surface, with a velocity of 3.00 m/s, collides head-on with a stationary 9.00-kg bowling ball. The collision is perfectly elastic, sending the 9.00-kg ball sliding away and leaving the 10.0-kg ball with a velocity of 0.156 m/s. What is the speed of the 9.00-kg ball after the collision? What is the total kinetic energy of the system after the collision?

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$

$$m_1\vec{v}_{1i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

$$\vec{v}_{2f} = \frac{m_1}{m_2}\left(\vec{v}_{1i} - \vec{v}_{1f}\right)$$

$$\vec{v}_{2f} = \frac{10.0\text{ kg}}{9.00\text{ kg}}\left(3.00\text{ m/s} - 0.156\text{ m/s}\right) = 3.16\text{ m/s}$$

$$K_f = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$$

$$K_f = \frac{1}{2}(10.0\text{ kg})(0.156\text{ m/s})^2 + \frac{1}{2}(9.00\text{ kg})(3.16\text{ m/s})^2 = 45.1\text{ J}$$

3. A mother (mass 60.0 kg) skates across an ice rink with negligible friction toward her child (mass 20.0 kg), who is standing still on the ice. If the mother moves at 4.0 m/s before she picks up her child, what is her new speed after she picks up her child and holds onto him? What is the total energy of the mother-child system after she picks up the child?

$$m_1\vec{v}_{1i} = (m_1 + m_2)\vec{v}_{1f}$$

$$\vec{v}_{1f} = \frac{m_1}{(m_1 + m_2)}\vec{v}_{1i}$$

$$\vec{v}_{1f} = \frac{60\text{ kg}}{(60.0\text{ kg} + 20.0\text{ kg})}(4.0\text{ m/s}) = 3.0\text{ m/s}$$

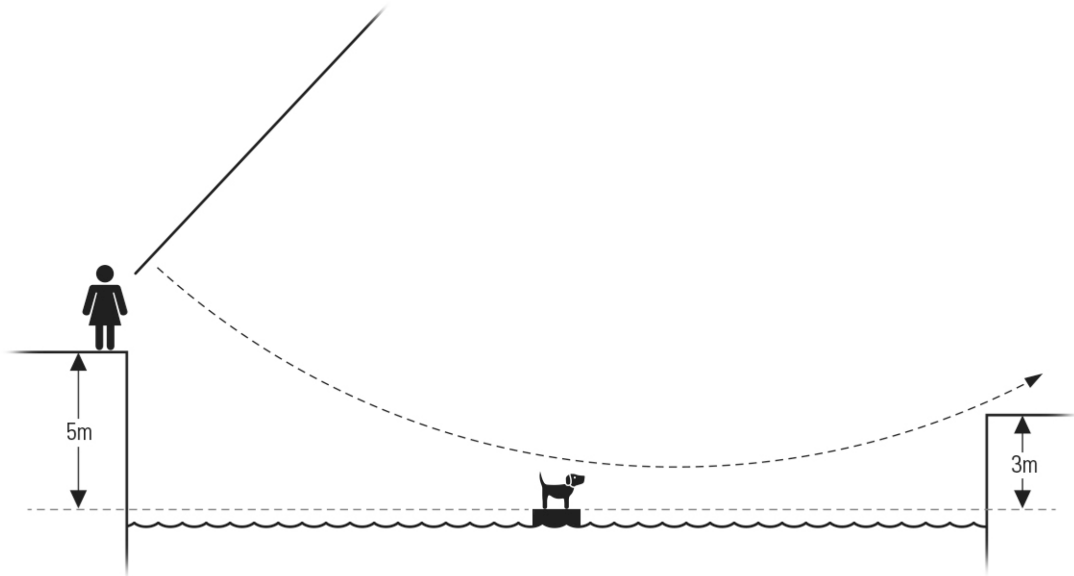
   Because the collision is inelastic, kinetic energy is not conserved:

$$K_f \neq K_i$$

$$K_f = \frac{1}{2}(m_1 + m_2)v_{1f}^2$$

$$K_f = \frac{1}{2}(60.0\text{ kg} + 20.0\text{ kg})(3.0\text{ m/s})^2 = 360\text{ J}$$

❷ 4. A 25.0-kg dog is trapped on a rock in the middle of a narrow river. A 66.0-kg rescuer has assembled a swing with negligible mass that she will use to swing down and catch the trapped dog at the bottom of her swing, and then continue swinging to the other side of the river. The ledge that the rescuer swings from is 5.0 m above the rock, which is not high enough so the rescuer and dog together can reach the other side of the river, which is 3.0 m above the rock. However, the rescuer can use a ladder to increase the height from which she swings. What is the minimum height of the ladder the rescuer must use so both dog and rescuer make it to the other side of the river? Assume that friction and air resistance are negligible.



This problem is best divided into three parts and then combined. The first part is the rescuer's trip from the ledge to the rock, the second part is the collision between the rescuer and the dog, and the third part is the rescuer and dog's trip from the rock to the other side of the river.

Part 1 – Rescuer's trip from the ledge to the rock: use conservation of energy to determine the rescuer's speed just before the collision:

$$m_{\text{rescuer}} g y_1 = \frac{1}{2} m_{\text{rescuer}} v_2^2$$

$$v_2 = \sqrt{2gy_1}$$

Part 2 – Collision between the rescuer and dog: use conservation of momentum to determine the rescuer and dog's speed just after the collision:

$$m_{\text{rescuer}} v_2 = \left(m_{\text{rescuer}} + m_{\text{dog}}\right) v_2'$$

$$v_2' = \frac{m_{\text{rescuer}} v_2}{\left(m_{\text{rescuer}} + m_{\text{dog}}\right)}$$

$$v_2' = \frac{m_{\text{rescuer}} \sqrt{2gy_1}}{\left(m_{\text{rescuer}} + m_{\text{dog}}\right)}$$

Part 3 –Rescuer's and dog's trip from the rock to the other side of the river: use conservation of energy to determine the final height of their swing:

$$\left(m_{\text{rescuer}} + m_{\text{dog}}\right) g y_3 = \frac{1}{2}\left(m_{\text{rescuer}} + m_{\text{dog}}\right) v_2'^2$$

$$v_2' = \sqrt{2gy_3}$$

Combine equations for v_2':

(continued on next page)

$$\sqrt{2gy_3} = \frac{m_{\text{rescuer}}\sqrt{2gy_1}}{\left(m_{\text{rescuer}} + m_{\text{dog}}\right)}$$

$$y_1 = \frac{\left(m_{\text{rescuer}} + m_{\text{dog}}\right)^2}{m_{\text{rescuer}}^2} y_3 = \frac{\left(66.0\text{ kg} + 25.0\text{ kg}\right)^2}{\left(66.0\text{ kg}\right)^2}\left(3.0\text{ m}\right) = 5.7\text{ m}$$

$$5.7\text{ m} - 5.0\text{ m} = 0.7\text{ m}$$

The minimum height of the ladder needed is 0.7 m.

Extended Inquiry Suggestions

A logical extension for this lab is Conservation of Momentum in more than one dimension of motion. Demonstrate 2-D momentum principles using several simple demonstrations, including air hockey tables, billiards, and projectiles launched from the same height, but with different horizontal velocities after they collide and fall to the ground.

Sports analogies can make a rich classroom discussion (or outside the class). A kicked soccer ball goes up in the air if the athlete kicks the bottom of the ball instead of the middle or top half of the ball. Football players running down the field and hit from the side will move in which direction after the collision? What causes a baseball player to hit a ground ball or a pop fly instead of a line drive? When a professional golfer uses a sloped club (like a 9-iron), how will it affect the ball's trajectory after collision?

NAME PERIOD DATE

8. CONSERVATION OF MOMENTUM

STRUCTURED

Driving Question | Objective

How is the total linear momentum and kinetic energy of a two-object system affected by a collision? Experimentally demonstrate that linear momentum and kinetic energy are conserved in an elastic collision, and that linear momentum is conserved but kinetic energy is not conserved in an inelastic collision.

Materials and Equipment

- Data collection system
- PASCO Motion Sensor (2)[1]
- PASCO Dynamics Cart with magnetic and Velcro® bumpers (2)[2]
- PASCO Dynamics Track with feet[3]
- 250-g cart mass (2)[2]
- Balance, 0.1-g resolution, 2,000-g capacity (1 per class)

[1]www.pasco.com/ap18 PASCO Motion Sensor

[2]www.pasco.com/ap07 PASCO PAScar

[3]www.pasco.com/ap08 PASCO PAStrack

Background

The momentum $\vec{p}$ of an object is equal to the product of its mass m and velocity $\vec{v}$:

$$\vec{p} = m\vec{v} \qquad (1)$$

For a linear system that is not influenced by outside forces, the total momentum of the system is conserved. This extends to objects experiencing two types of collisions: elastic and inelastic.

Elastic collisions occur when two objects bounce off each other perfectly (without the loss of kinetic energy), like two billiards balls colliding. Linear momentum is transferred from one object to the next, and if the two objects are the same mass, all of the momentum of the first is transferred to the second.

Inelastic collisions occur when two objects don't bounce off each other perfectly, such as two objects sticking fully or partially to each other, like two clay balls colliding and then moving as one object; or two objects colliding and deforming as a result of the collision, like a two-car accident.

In this activity, you will use an experimental procedure to demonstrate that the total linear momentum of a system consisting of two carts on a flat track is conserved in both elastic and inelastic collisions (implying that the total momentum of the system does not change in either collision type), but the total kinetic energy of the system is only constant in elastic collisions.

RELEVANT EQUATIONS

$$\vec{p} = m\vec{v} \qquad (1)$$

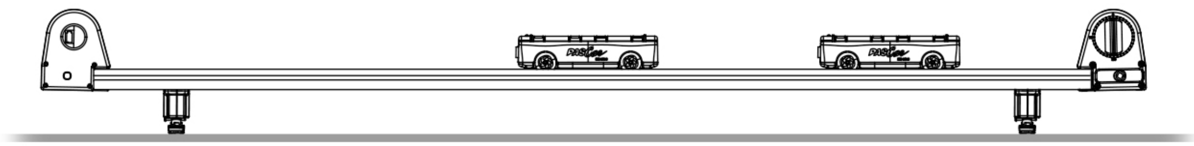
$$K = \frac{1}{2}mv^2 \qquad (2)$$

Procedure

Part 1 – Elastic Collision

SET UP

1. Set the track on a level surface with one motion sensor attached to each end as in the diagram below. Adjust the track feet to make sure the track is as level as possible.



2. Connect both motion sensors to the data collection system.

NOTE: The motion sensors are pointed in opposite directions. In this orientation, one sensor by default will measure positive velocity for an object moving left to right, while the other will measure negative velocity for the same object. This must be corrected within your data collection system so that both sensors report the same velocity for the same object at the same time. The following step addresses this.

3. In the data collection system, use the built-in "Change Sign" function, or create the calculation "Velocity2 = –[Velocity (m/s)]" (where [Velocity (m/s)] is velocity data measured by one of the two motion sensors) to change the sign of the velocity measurements made by one of the two motion sensors.

4. Create two graph displays: one graph display of velocity versus time from one motion sensor, and a second graph display of the corrected velocity measurement from the other motion sensor versus time.

5. Make sure that the sampling rate is set to at least 20 samples per second for each motion sensor. The motion sensors have a selector switch; make sure that they are in the cart mode.

COLLECT DATA

6. Measure the mass of each cart. Record the mass of Cart 1 into Table 1, and the mass of Cart 2 into Table 2 in the Data Analysis section below.

7. Place Cart 2 in the middle of the track and Cart 1 near one end of the track (just over 15 cm from the motion sensor) such that their magnetic bumpers are facing each other.

8. Make sure someone is at the other end of the track to catch the cart before it hits the motion sensor. You will be pushing the cart closest to the motion sensor (Cart 1) toward the cart in the middle of the track (Cart 2).

9. Start recording data.

10. While keeping hands out of the way of the motion sensor, gently push and release Cart 1 toward Cart 2, allowing them to collide.

11. Catch both carts before they hit either motion sensor, and then stop recording data.

12. Repeat the data collection steps two additional times, each time adding one of the 250-g cart masses to Cart 2.

13. Use the tools on your data collection system to find the velocity of Cart 1 on the graph just before the collision (initial velocity) and just after the collision (final velocity) in each trial. Enter the values in Table 1 in the Data Analysis section.

14. Use the tools on your data collection system to find the velocity of Cart 2 on the graph just before the collision (initial velocity) and just after the collision (final velocity) in each trial. Enter the values in Table 2 in the Data Analysis section.

Part 2 – Inelastic Collision

SET UP

15. Remove the cart masses from Cart 2.

COLLECT DATA

16. Repeat the same data collection steps from Part 1. However, in this part you will orient the carts so that they will hit Velcro bumper to Velcro bumper and stick together, resulting in an inelastic collision.

17. Perform three trials, each time adding one of the 250-g cart masses to Cart 2.

18. Record Cart 1's mass, initial velocity, and final velocity for each trial into Table 4.

19. Record Cart 2's mass, initial velocity, and final velocity for each trial into Table 5.

Data Analysis

Part 1 – Elastic Collision

Table 1: Cart 1 Elastic collision data

Trial	Mass (kg)	Initial Velocity (m/s)	Final Velocity (m/s)	Initial Momentum (kg·m/s)	Final Momentum (kg·m/s)	Initial Kinetic Energy (J)	Final Kinetic Energy (J)
1							
2							
3							

Table 2: Cart 2 Elastic collision data

Trial	Mass (kg)	Initial Velocity (m/s)	Final Velocity (m/s)	Initial Momentum (kg·m/s)	Final Momentum (kg·m/s)	Initial Kinetic Energy (J)	Final Kinetic Energy (J)
1							
2							
3							

1. Calculate the initial and final momentum of each cart in each Part 1 trial. Record the values for Cart 1 into Table 1 and the values for Cart 2 into Table 2.

2. Calculate the initial and final kinetic energy of each cart in each Part 1 trial. Record the values for Cart 1 into Table 1 and the values for Cart 2 into Table 2.

3. Calculate the total momentum and total kinetic energy of the two-cart system before and after each Part 1 collision. Record the results into Table 3 below.

Table 3: Total system momentum and kinetic energy before and after the elastic collision

Trial	Initial Momentum of System (kg·m/s)	Final Momentum of System (kg·m/s)	Initial Kinetic Energy of System (J)	Final Kinetic Energy of System (J)
1				
2				
3				

Part 2 – Inelastic Collision

Table 4: Cart 1 Inelastic collision data

Trial	Mass (kg)	Initial Velocity (m/s)	Final Velocity (m/s)	Initial Momentum (kg·m/s)	Final Momentum (kg·m/s)	Initial Kinetic Energy (J)	Final Kinetic Energy (J)
4							
5							
6							

Table 5: Cart 2 Inelastic collision data

Trial	Mass (kg)	Initial Velocity (m/s)	Final Velocity (m/s)	Initial Momentum (kg·m/s)	Final Momentum (kg·m/s)	Initial Kinetic Energy (J)	Final Kinetic Energy (J)
4							
5							
6							

4. Calculate the initial and final momentum of each cart in each Part 2 trial. Record the values for Cart 1 into Table 4 and the values for Cart 2 into Table 5.

5. Calculate the initial and final kinetic energy of each cart in each Part 2 trial. Record the values for Cart 1 into Table 4 and the values for Cart 2 into Table 5.

6. Calculate the total momentum and total kinetic energy of the two-cart system before and after each Part 2 collision. Record the results into Table 6 below.

Table 6: Total system momentum and kinetic energy before and after inelastic collision

Trial	Initial Momentum of System (kg·m/s)	Final Momentum of System (kg·m/s)	Initial Kinetic Energy of System (J)	Final Kinetic Energy of System (J)
4				
5				
6				

Analysis Questions

1. What experimental evidence do you have showing that momentum is conserved in inelastic and elastic collisions?

2. How does your data support that kinetic energy is conserved in elastic collisions?

3. How does your data support that kinetic energy is NOT conserved in inelastic collisions?

4. Why is kinetic energy not conserved in inelastic collisions? Where is the energy lost?

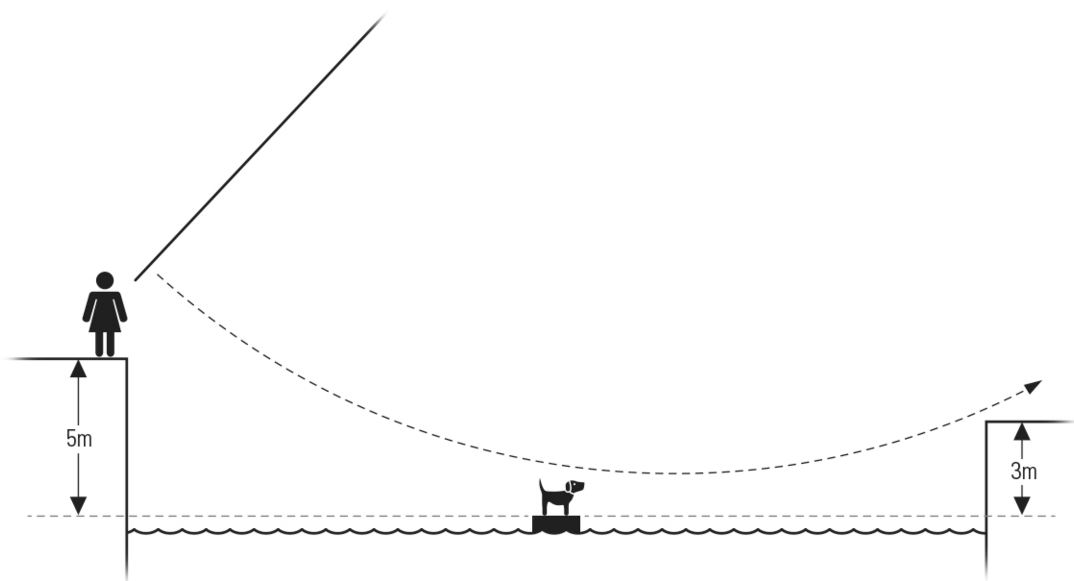
Synthesis Questions

1. Two locomotives, each weighing 100,000 kg and having a speed of 100 km/hr, race toward each other and have a completely inelastic collision. What is the final momentum of the system? Justify your answer.

2. A 10.0-kg bowling ball sliding across a frictionless surface, with a velocity of 3.00 m/s, collides head-on with a stationary 9.00-kg bowling ball. The collision is perfectly elastic, sending the 9.00-kg ball sliding away and leaving the 10.0-kg ball with a velocity of 0.156 m/s. What is the speed of the 9.00-kg ball after the collision? What is the total kinetic energy of the system after the collision?

❓ 3. A mother (mass 60.0 kg) skates across an ice rink with negligible friction toward her child (mass 20.0 kg), who is standing still on the ice. If the mother moves at 4.0 m/s before she picks up her child, what is her new speed after she picks up her child and holds onto him? What is the total energy of the mother-child system after she picks up the child?

❓ 4. A 25.0-kg dog is trapped on a rock in the middle of a narrow river. A 66.0-kg rescuer has assembled a swing with negligible mass that she will use to swing down and catch the trapped dog at the bottom of her swing, and then continue swinging to the other side of the river. The ledge that the rescuer swings from is 5.0 m above the rock, which is not high enough so the rescuer and dog together can reach the other side of the river, which is 3.0 m above the rock. However, the rescuer can use a ladder to increase the height from which she swings. What is the minimum height of the ladder the rescuer must use so both dog and rescuer make it to the other side of the river? Assume that friction and air resistance are negligible.



9. MOMENTUM AND IMPULSE

Connections to the AP® Physics 1 Curriculum*

The lab activity correlates to the following pieces of the AP® Physics 1 framework:

Big Idea	3	Enduring Understanding	D	Essential Knowledge	2

Learning Objective 3: The student is able analyze data to characterize the change in momentum of an object from the average force exerted on the object and the interval of time which the force is exerted.
Science Practices: 5.1

Learning Objective 4: The student is able to design a plan for collecting data to investigate the relationship between changes in momentum and the average force exerted on an object over time.
Science Practices: 4.2

Time Requirement

Preparation Time: 10 minutes

Lab Activity: 50 minutes

Prerequisites

Students should be familiar with the following concepts:

- Momentum is a vector quantity defined as the product of an object's mass and its velocity. The momentum of an object can only be changed when a non-zero net force is imparted to the object.
- Impulse is a vector quantity. For constant forces acting on an object, impulse is equal to the product of the force acting on the object and the time interval during which the force is acting. For variable forces acting on an object, impulse is equal to the area under the force versus time curve representing the interaction, which can be simplified to the product of the average of the variable force and the time interval during which the force is acting.

Driving Question | Objective

How is the impulse imparted to an object in a collision related to the change in momentum of the object? Investigate the relationship between the change in momentum of a cart undergoing a collision and the impulse imparted to the cart to change its momentum. Establish a measurement-based relationship between the change in momentum and the impulse.

Procedural Overview

Students measure the following: the force imparted to a cart as it lightly collides with a spring bumper, the time interval during which the force is imparted, and the velocity of the cart before, during, and after the collision. The mass and the velocity of the cart just before and just after the collision are used to calculate the change in momentum of the cart as a result of the collision.

Students determine the average force and the time interval associated with the same collision using force versus time graphs and the statistical tools on their data collection system. The average force and the time interval are then used to calculate the impulse associated with the collision, and students compare their calculated impulse to their calculated change in momentum.

Students repeat this process 4 times and ascertain that the impulse associated with the collision of an object is equal to the change in momentum of the object.

Pre-Lab Discussion and Activity

The vector nature of velocity and momentum are important facets of this investigative activity. Students can practice their use of vector sign convention using the following pre-lab questions:

Pre-Lab Questions

❷ 1. How is the change in momentum of an object calculated?

Change in momentum is calculated by subtracting the initial momentum from the final momentum. Students must be careful to pay close attention to the sign of each momentum value as these will affect the magnitude of their result.

❷ 2. If a ball is moving to the left (defined as the negative direction) and is struck so that it then moves to the right (defined as the positive direction), will the ball's change in momentum be positive or negative? Justify your answer.

Positive. The change in momentum is calculated by subtracting the initial momentum from the final momentum. If the final momentum is positive and the initial momentum is negative, the resultant equation will be: positive number − (negative number) = positive number.

❷ 3. If a ball is moving to the right (the positive direction) and is struck so that it then moves to the left (the negative direction), will the ball's change in momentum be positive or negative? Justify your answer.

Negative. The change in momentum is calculated by subtracting the initial momentum from the final momentum. If the final momentum is negative and the initial momentum is positive, the resultant equation will be: negative number − (positive number) = negative number.

❷ 4. The same ball is dropped straight down from the same height twice. In the first drop, the ball bounces off the floor with half the speed it hit the floor with, and in the second drop, the ball bounces off the floor with one quarter the speed it hit the floor with. In which drop does the ball experience a greater absolute change in momentum? Justify your answer.

The ball experiences a greater absolute change in momentum in the first drop:

$$\Delta\vec{p} = \vec{v}_\mathrm{f} - \vec{v}_\mathrm{i}$$

$$\left|\Delta\vec{p}_1\right| = \left|-0.5\vec{v}_1 - \vec{v}_1\right| = 1.5v_1$$

$$\left|\Delta\vec{p}_2\right| = \left|-0.25\vec{v}_1 - \vec{v}_1\right| = 1.25v_1$$

Materials and Equipment

- Data collection system
- PASCO Motion Sensor[1]
- PASCO High Resolution Force Sensor[2]
- Table clamp or large base
- Support rod, 45-cm
- PASCO Dynamics Track[3]
- PASCO Dynamics Cart[4]
- PASCO Dynamics Track rod clamp[5]
- PASCO Discover Collision Bracket[6]
- Balance, 0.1-g resolution, 2,000-g capacity (1 per class)

Probeware Resources

Below are web-link and QR codes that will direct you to instructional video resources for individual pieces of PASCO probeware, sensors, and other hardware used in the lab activity. These same links and codes are provided to students in their activity handouts.

[1]www.pasco.com/ap18
PASCO Motion Sensor

[2]www.pasco.com/ap22
PASCO High Resolution Force Sensor

[3]www.pasco.com/ap08
PASCO PAStrack

[4]www.pasco.com/ap07
PASCO PAScar

[5]www.pasco.com/ap17
PASCO Dynamics Track Rod Clamp

[6]www.pasco.com/ap12
PASCO Discover Collision Bracket

Safety

Follow these important safety precautions in addition to your regular classroom procedures:

- Keep fingers away from the lower section of the ramp to avoid a collision with the cart.
- Keep track angles less than 10°. Track angles greater than 10° may cause the cart to collide too hard with the force sensor which may permanently damage the sensor.

Teacher Tips

Tip 1 – Graphs Must Use "Force Inverted"

- In the Structured version of this lab activity, students use force and motion sensors simultaneously. By default, the motion sensor measures velocity as positive in the direction leading away from the sensor, while the force sensor measures positive force in the direction pointed away from the front of the force sensor. Based on this and the setup outlined in the Structured version of this lab activity, the positive direction for each sensor opposes the other, making the sign convention inconsistent. To correct this, have students build their force versus time graph displays from force measurements as *pull positive*, or force "inverted."
- If students choose to mount the force sensor to the top of the cart rather than to their bumper or other barrier, students should build their force versus time graph displays using force measurements as *push positive*, or the default "force" measured by the high resolution force sensor.

Tip 2 – Keep Impact Force Under 50 N

- Students using force sensors should keep the impact force received by the force sensor under 50 N, the maximum sustainable impact force of a PASCO force sensor. Any forces higher than that may cause permanent damage to the sensing element within the sensor.

Tip 3 – Dealing with Inconsistent Data

- If students following the procedure outlined in the Structured version of this lab activity have data that does not show an equality between impulse and change in momentum, have them check the following:
  - The force sensor should be zeroed before recording data.
  - The motion sensor should not be too far away from the cart—very high sample rates used with the motion sensor require that the target object not exceed a maximum distance (data will become very "noisy"). Do not exceed a maximum distance of approximately 70 cm for sample rates of 100 Hz or greater.

Tip 4 – Maximize the Amount of Data Collected in each Collision

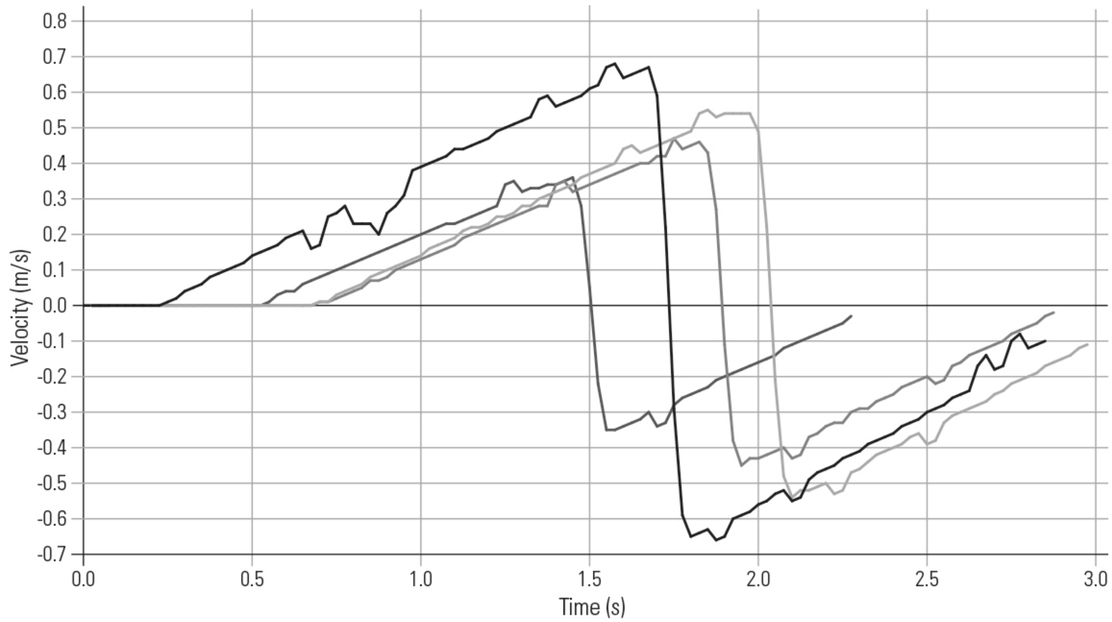
- When measuring the forces associated with impacts, students should attempt to capture as much force data during the collision as possible. More data helps provide more accurate and representative values for determining average force and for the area under a force versus time curve, especially when the impact happens quickly. To do this, students should use high sample rates (250–1,000 Hz) on their data collection system for the force sensor.
- Depending on the data collection system being used, students may be limited to sample rates of 250 Hz or less when using the force and motion sensors simultaneously (some data collection systems do not allow independent sample rate control of simultaneously connected sensors). Although the motion sensor can sample at 250 Hz, data can be erratic at this rate and the maximum distance between the cart and the motion sensor must be limited. In this case, students should set the sample rate to 100 Hz and use slow impact speeds and soft spring bumpers. This will help maximize the amount of data displayed during the collision.

Sample Data

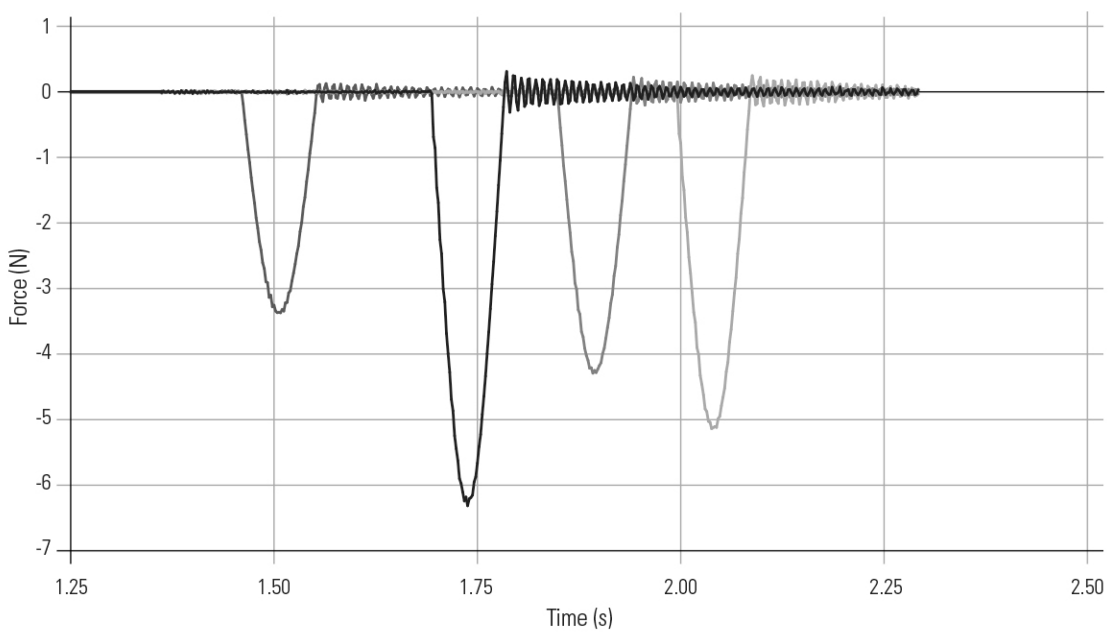
Below are sample data, acquired using the experimental setup and procedure outlined in the Structured version of the lab activity, and answers to questions in the Data Analysis section.

Data Analysis

Sample student graph of velocity versus time at varying release distances on the track:



Sample student graph of force versus time at varying release distances on the track



Mass of cart (kg) = 0.249

1. Using the tools on your data collection system, determine the velocity of the cart just before (initial velocity $\vec{v}_i$) and just after (final velocity $\vec{v}_f$) the collision for each trial, and then record each value in Table 1.

   NOTE: Make sure the signs of the velocity are correct

2. Using the tools on your data collection system, determine the time interval and average force of the collision in each trial. Record each value in Table 1.

Table 1: Impulse and momentum of a cart undergoing a collision

Trial	Initial Velocity (m/s)	Final Velocity (m/s)	Average Force (N)	Time Interval (s)	Impulse (N·s)	Change in Momentum (kg·m/s)
1	0.67	–0.65	–3.556	0.094	–0.33	–0.33
2	0.54	–0.54	–2.900	0.094	–0.27	–0.27
3	0.46	–0.45	–2.511	0.094	–0.24	–0.23
4	0.36	–0.35	–1.932	0.094	–0.18	–0.18

3. Calculate the impulse ($\vec{J} = \vec{F}_{ave}\Delta t$) for each trial and record the results in Table 1.

   Calculation using sample data for Trial 1:

   $$\vec{J} = -3.556\ \text{N} \times 0.094\ \text{s} = -0.33\ \text{N} \cdot \text{s}$$

4. Calculate the change in momentum ($\Delta\vec{p} = m\vec{v}_f - m\vec{v}_i$) for each trial and record the results in Table 1.

   Calculation using sample data for Trial 1:

   $$\Delta\vec{p} = \left(0.249\ \text{kg} \times -0.65\ \frac{\text{m}}{\text{s}}\right) - \left(0.249\ \text{kg} \times 0.67\ \frac{\text{m}}{\text{s}}\right) = -0.33\ \frac{\text{kg} \cdot \text{m}}{\text{s}}$$

Guided Inquiry Questions

Below are sample responses to the Guiding Questions found in the Guided Inquiry version of this lab activity.

1. Describe how you would construct an apparatus to reproduce the same collision between a cart and a barrier for multiple trials.

   Students are encouraged to use a cart and track system that allows them to keep variables like direction and speed constant in their investigation. Please refer to the setup outlined in the structured version of this lab activity as an example.

2. Based on the lab Objective, what values should you measure in your investigation?

   The goal for students is to identify values that will help them measure and calculate the impulse and the change in momentum of a cart in a collision. Essentially, students will need to directly or indirectly measure the mass of their cart, the velocity of their cart immediately before and after a collision, the average force experienced by the cart in a collision, and the time elapsed during a collision. Alternately, students may measure impulse by measuring the area under a force versus time graph.

3. Can these values be measured directly? If yes, what tools will you use to measure them? If no, how will you indirectly measure them and what tools will you use?

   The mass of the cart can be measured directly using a balance. The velocity of the cart can be measured directly using a motion sensor, or indirectly using conservation of energy arguments. Average force can be measured directly using a force sensor mounted to either the cart or the barrier with which the cart is colliding. Elapsed time can be measured from either velocity versus time or force versus time graphs. Measuring elapsed time will be difficult, if not impossible, using any other method, as the collisions happen in such a very short time.

4. How will you configure your data collection system to display your measured values? What sort of a display (graph, table, et cetera) will you use for each and why?

   Students may choose any display type, but must include good reasoning in their response as to why they chose the displays they did. It is recommended that students use graph displays of velocity and force versus time as this will allow them the freedom to extrapolate values for force, velocity, and elapsed time within the same display.

5. What sources of error in your measurements do you expect, and how do you plan to avoid or minimize these?

   Students will experience error in their measurements associated with the resolution of the measuring devices they are using as well as operator error. One potential source of accepted error in this investigation is the assumption that the magnitude of an impulse is equal to the time-averaged force of impact multiplied by the elapsed time of the impact. However, this is an approximation when dealing with variable forces.

Assessment Questions: Sample Responses

Sample responses to the Analysis and Synthesis questions found in each version of the lab activity:

Analysis Questions

1. Based on your measurements, how does the impulse compare to the change in momentum?

   Student data should show that the change in momentum of a cart experiencing a collision is equal to the product of the average force imparted to the cart in the collision and the time interval over which that force is imparted—the impulse.

2. What were some unexpected factors that may have caused error in your measured values, and how could these have been avoided?

   Students will experience error in their measurements associated with the resolution of the measuring devices they are using as well as due to operator error. One potential source of accepted error in this investigation is the assumption that the magnitude of the impulse is equal to the time-averaged force of impact multiplied by the elapse time of the impact; however, this is an approximation when dealing with variable forces.

3. How is the momentum of the cart before and after a collision related? Was there energy lost in the collision? If so, where did the energy go?

   Students using spring bumpers, or other Hookean-style bumpers, will record final momentum values nearly equal and opposite to the initial momentum values, indicating that nearly all of the cart energy was conserved in the collision. Students using soft rubber, clay, or other non-Hookean bumpers will find the final momentum opposite in sign, but much smaller in magnitude to the initial momentum. This indicates that some of the cart energy was converted into other forms (such as sound, heat, and permanent deformation of the bumper) during the collision.

4. How do the units associated with impulse compare to the units associated with change in momentum?

   Impulse is often specified in units of newton seconds (N·s) while momentum is specified in units of kilogram meter per second (kg·m/s). Expanding and simplify impulse units,

   $$\mathrm{N \cdot s} = \frac{\mathrm{kg \cdot m}}{\mathrm{s^2}} \cdot \mathrm{s} = \mathrm{kg} \cdot \frac{\mathrm{m}}{\mathrm{s}}$$

   Students will see that the units for change in momentum and impulse are equal.

5. In one sentence, describe what you believe the mathematical relationship is between change in momentum and impulse. Be specific, and use terms like “proportional to,” “equal to,” and “inversely proportional to.”

   Using their measurement-based relationship, students should ascertain that the impulse imparted to a cart in a collision is equal to the change in momentum of that cart.

Synthesis Questions

1. If an object experiencing a collision was to hit a soft cushion rather than a rigid surface, would the impulse be different? Assume the object's initial and final velocities are the same in both collisions. Justify your answer. If possible, try this with your lab setup.

   Based on the relationship between the change in momentum and the impulse, and because the change in momentum is the same in each collision, the impulse will also be the same.

2. Near the exits of many highways are yellow plastic barrels filled with sand called "Fitch barriers" after the inventor, race car driver John Fitch. How do these barrels reduce injury to vehicle occupants if their car crashes into them on exiting the freeway?

   Fitch barriers are designed to keep the impulse imparted to the car the same, but extend the elapsed time of the impact , lowering the absolute maximum force experienced by drivers and passengers.

3. A 50-kg rock climber accidentally falls from the side of a rock and free falls until she is stopped by her 9-m safety rope. Assuming that the rope stops her completely, with no rebound, what is the impulse imparted on her body by the rope? Show your work.

$$\vec{J} = \Delta\vec{p}$$

$$\Delta\vec{p} = m\vec{v}_f - m\vec{v}_i = -m\vec{v}_i$$

$$v_i^2 = v_0^2 + 2a(x - x_0)$$

$$v_i^2 = 2\left(9.8\ \text{m/s}^2\right)\left(9\ \text{m}\right) = 176.4\ \text{m}^2/\text{s}^2$$

$$v_i = \pm 13.28\ \text{m/s}$$

$$\Delta\vec{p} = -m\vec{v}_i = -\left(50\ \text{kg}\right)\left(-13.28\ \text{m/s}\right) = 664.0\ \text{kg}\cdot\text{m/s}$$

4. Passengers on a commercial jet experience changes in horizontal force as the jet accelerates and decelerates in mid-flight without changing altitude. If the 90,000.0-kg jet experiences an average force of 37,500 N over 60.0 seconds as it accelerates from an initial velocity of 187.8 m/s to its final cruising velocity, what is the final cruising velocity of the jet? Show your work.

$$\Delta\vec{p} = \vec{J} = \vec{F}_{ave}\Delta t$$

$$m\left(\vec{v}_f - \vec{v}_i\right) = \vec{F}_{ave}\Delta t$$

$$\vec{v}_f = \frac{\vec{F}_{ave}\Delta t}{m} + v_i = \frac{\left(37{,}500\ \text{N}\right)\left(60.0\ \text{s}\right)}{90{,}000.0\ \text{kg}} + 187.8\ \text{m/s} = 213\ \text{m/s}$$

Extended Inquiry Suggestions

Extend this topic with a group engineering project in which students use the basis of the impulse-momentum relationship to inform the construction of a collision-cart bumper. Challenge students to design and construct a bumper that can be attached to either the front of a cart or the force sensor that will minimize the maximum impact force experienced in a collision.

Before construction, establish a set of materials to be used and design constraints (rules) that students must follow when developing their bumpers. Many common household materials work well: cardboard, tin foil, cotton balls, paper, et cetera. Some examples of design constraints include: balloons may not be used, the bumper must hit the force sensor directly, and bumpers must be designed to attach to the front of the cart readily.

Students will find that reducing the maximum impact force involves reducing the impulse imparted to the cart in the collision, which can be done in many different ways; however, the basis for reducing the impulse is to extend the impact time, reduce the final velocity of the cart after the impact, or both of these.

9. MOMENTUM AND IMPULSE

STRUCTURED

Driving Question | Objective

How is the impulse imparted to an object in a collision related to the change in momentum of the object? Investigate the relationship between the change in momentum of a cart undergoing a collision and the impulse imparted to the cart to change its momentum. Establish a measurement-based relationship between the change in momentum and the impulse.

Materials and Equipment

- Data collection system
- PASCO Motion Sensor[1]
- PASCO High Resolution Force Sensor[2]
- Table clamp or large base
- Support rod, 45-cm
- PASCO Dynamics Track[3]
- PASCO Dynamics Cart[4]
- PASCO Dynamics Track rod clamp[5]
- PASCO Discover Collision Bracket[6]
- Balance, 0.1-g resolution, 2,000-g capacity (1 per class)

[1]www.pasco.com/ap18
PASCO Motion Sensor

[2]www.pasco.com/ap22
PASCO High Resolution Force Sensor

[3]www.pasco.com/ap08
PASCO PAStrack

[4]www.pasco.com/ap07
PASCO PAScar

[5]www.pasco.com/ap17
PASCO Dynamics Track Rod Clamp

[6]www.pasco.com/ap12
PASCO Discover Collision Bracket

Background

Vehicle airbags and the large yellow barrels full of sand called Fitch barriers near highway exits are credited with saving thousands of lives and reducing injuries to car occupants. Two important physics phenomena that occur during a collision are the change in momentum and the impulse imparted on the vehicle and its occupants.

The change in momentum of an object experiencing a collision can be calculated by subtracting the momentum before the collision from the momentum after the collision:

$$\Delta\vec{p} = m\vec{v}_{\mathrm{f}} - m\vec{v}_{\mathrm{i}} \qquad (1)$$

where $\Delta\vec{p}$ is the object's change in momentum, m is its mass, $\vec{v}_{\mathrm{f}}$ is its final velocity, and $\vec{v}_{\mathrm{i}}$ is its initial velocity.

For constant forces, impulse $\vec{J}$ is defined as the product of the force $\vec{F}$ acting on an object (the force responsible for changing the object's momentum) and the time interval Δt over which the force is acting:

$$\vec{J} = \vec{F}\Delta t \qquad (2)$$

For variable forces, impulse is defined as the area under a force versus time curve during the time in which the collision occurred.

$$\vec{J} = \text{Area Under } \vec{F} \text{ versus } t \text{ Curve} \quad (3)$$

For our purposes, the equation for the impulse associated with a variable force can be simplified to the product of the average force $\vec{F}_{\text{ave}}$ and the time interval during which the force was acting:

$$\vec{J} = \vec{F}_{\text{ave}}\Delta t \quad (4)$$

RELEVANT EQUATIONS

$$\Delta\vec{p} = m\vec{v}_{\text{f}} - m\vec{v}_{\text{i}} \quad (1)$$

$$\vec{J} = \vec{F}_{\text{ave}}\Delta t \quad (4)$$

Safety

Follow these important safety precautions in addition to your regular classroom procedures:

- Keep fingers away from the lower section of the ramp to avoid a collision with the cart.
- Track angles greater than 10° may cause the cart to collide too hard with the force sensor which may permanently damage the sensor.

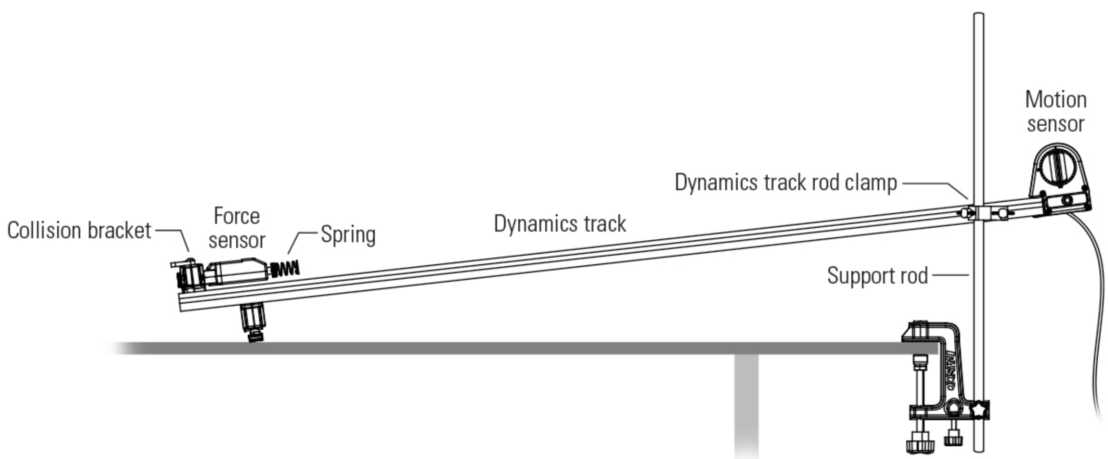
Procedure

SET UP

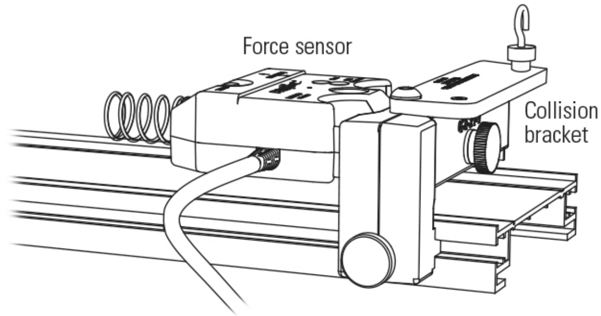
1. Attach one end of the dynamics track to the support rod using the dynamics track rod clamp, inclining the track just slightly (less than or equal to 10°).

   WARNING: Angles greater than 10° may cause the cart to collide too hard with the force sensor and permanently damage the sensor.

2. Mount the motion sensor to the raised end of the track with the front of the sensor pointing down the track. Make certain the switch on the top of the sensor is set to the cart icon.



3. Screw the softest spring bumper from the collision bracket to the front of the force sensor, and then attach the force sensor to the bracket using the large thumbscrew on the bracket.



4. Mount the force sensor with collision bracket combo to the lower end of the track with the force sensor pointed up the track.

5. Connect both sensors to the data collection system and then create two graph displays: one graph display of force (inverted) versus time, and the second graph display of velocity versus time.

6. Change the force sensor sample rate to 500 Hz and the motion sensor sample rate to 40 Hz.

COLLECT DATA

7. Measure the mass of the cart and record the value in the Data Analysis section below.

8. Press the Zero button on the top of the force sensor.

9. Place the cart on the track (plunger end pointing away from the collision bracket) approximately 15 cm in front of the motion sensor holding it in place.

10. Begin recording data, and then release the cart to roll freely down the track.

11. After the cart collides with the bumper on the force sensor, stop recording data.

12. Repeat the same data collection steps three more times, releasing the cart at three different distances from the force sensor.

Data Analysis

Mass of cart (kg) = ______________________

1. Using the tools on your data collection system, determine the velocity of the cart just before (initial velocity $\vec{v}_i$) and just after (final velocity $\vec{v}_f$) the collision for each trial, and then record each value in Table 1.

   NOTE: Make sure the signs of the velocities are correct

2. Using the tools on your data collection system, determine the time interval and average force of the collision in each trial. Record each value in Table 1.

Table 1: Impulse and momentum of a cart undergoing a collision

Trial	Initial Velocity (m/s)	Final Velocity (m/s)	Average Force (N)	Time Interval (s)	Impulse (N·s)	Change in Momentum (kg·m/s)
1						
2						
3						
4						

3. Calculate the impulse ($\vec{J} = \vec{F}_{ave}\Delta t$) for each trial and record the results in Table 1.

4. Calculate the change in momentum ($\Delta\vec{p} = m\vec{v}_f - m\vec{v}_i$) for each trial and record the results in Table 1.

Analysis Questions

1. Based on your measurements, how does the impulse compare to the change in momentum?

2. What were some unexpected factors that may have caused error in your measured values, and how could these have been avoided?

3. How is the momentum of the cart before and after a collision related? Was there energy lost in the collision? If so, where did the energy go?

4. How do the units associated with impulse compare to the units associated with change in momentum?

5. In one sentence, describe what you believe the mathematical relationship is between change in momentum and impulse. Be specific, and use terms like “proportional to,” “equal to,” and “inversely proportional to.”

Synthesis Questions

1. If an object experiencing a collision was to hit a soft cushion rather than a rigid surface, would the impulse be different? Assume the object's initial and final velocities are the same in both collisions. Justify your answer. If possible, try this with your lab setup.

2. Near the exits of many highways are yellow plastic barrels filled with sand called "Fitch barriers" after the inventor, race car driver John Fitch. How do these barrels reduce injury to vehicle occupants if their car crashes into them on exiting the freeway?

3. A 50-kg rock climber accidentally falls from the side of a rock and free falls until she is stopped by her 9-m safety rope. Assuming that the rope stops her completely, with no rebound, what is the impulse imparted on her body by the rope? Show your work.

4. Passengers on a commercial jet experience changes in horizontal force as the jet accelerates and decelerates in mid-flight without changing altitude. If the 90,000.0-kg jet experiences an average force of 37,500 N over 60.0 seconds as it accelerates from an initial velocity of 187.8 m/s to its final cruising velocity, what is the final cruising velocity of the jet? Show your work.

10. ROTATIONAL DYNAMICS

Connections to the AP® Physics 1 Curriculum*

The lab activity correlates to the following pieces of the AP® Physics 1 framework:

Big Idea	3	Enduring Understanding	F	Essential Knowledge	2

Learning Objective 1: The student is able to make predictions about the change in the angular velocity about an axis for an object when forces exerted on the object cause a torque about that axis.
Science Practices: 6.4

Learning Objective 2: The student is able to plan data collection and analysis strategies designed to test the relationship between a torque exerted on an object and the change in angular velocity of that object about an axis.
Science Practices: 4.1, 4.2, 5.1

Time Requirement

Preparation Time: 10 minutes

Lab Activity: 50 minutes

Prerequisites

Students should be familiar with the following concepts:

- Torque is equal to the product of the perpendicular distance from the axis of rotation to an applied force and the magnitude of the applied force.
- The rotational inertia of point masses increases with increasing distance from the axis of rotation.
- The total rotational inertia of a combination of rigid bodies, mechanically connected, is equal to the sum of their individual rotational inertias assuming the axis of rotation is the same for all.

Driving Question | Objective

How do net torque and rotational inertia affect the angular acceleration of a rotating object? Experimentally determine the mathematical relationship between net torque, rotational inertia, and angular acceleration of a rotating object.

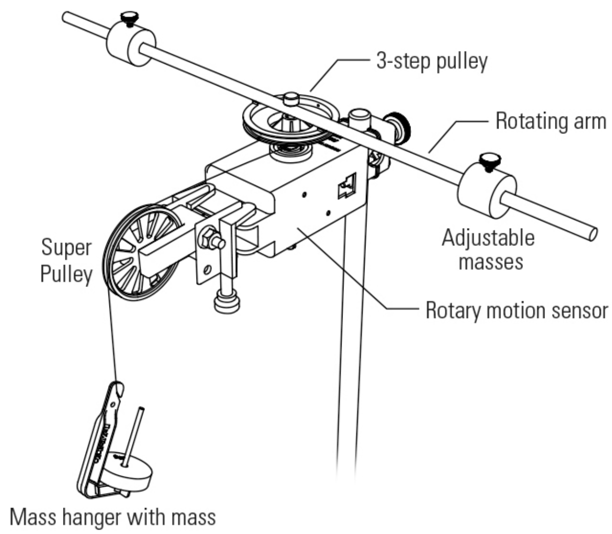
Procedural Overview

The Structured version of this lab activity is divided into two parts:

Part 1 – Students use hanging masses to apply torque to a rotating arm, oriented horizontally (see figure below). In each Part 1 trial, students keep the rotational inertia of the arm constant and measure its angular acceleration while increasing the amount of applied torque by adding more hanging mass. A plot of angular acceleration versus net torque will show a straight line, and from that, students are expected to recognize the proportional relationship:

$$\alpha \propto \tau_{\text{net}}$$

Part 2 – Students use a fixed hanging mass to apply a constant non-zero net torque to the same rotating arm used in Part 1 while measuring the angular acceleration of the arm; however, in each Part 2 trial they keep the applied torque (amount of hanging mass) constant and vary the rotational inertia of the rotating arm by adjusting the position of two masses fixed to the rotating arm. Rotational inertia is increased as the masses are positioned farther from the axis of rotation. Students calculate the rotational inertia of the arm in each trial and then calculate the inverse rotational inertia (1/rotational inertia). A plot of angular acceleration versus the inverse of rotational inertia will show a straight line, and from that, students are expected to recognize the proportional relationship:



$$\alpha \propto \frac{1}{I}$$

Students are then expected to combine the relationships outlined in each part into one mathematical expression relating the three variables:

$$\alpha \propto \frac{\tau_{\text{net}}}{I}$$

Pre-Lab Discussion and Activity

The preliminary discussion could take place the day before the lab (recommended for classes of 60 minutes or less), or the same day as the lab (recommended for 90-minute class periods).

Begin a discussion about rotational inertia (an object's resistance to changes in how it spins), which can lead into a discussion about torque:

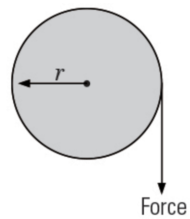
There are many common demonstrations that show the relative difficulty of spinning an object when its mass is redistributed farther from the axis of rotation. A simple demonstration would be swinging a baseball bat 2 different ways: by gripping it at its handle and by gripping it around the heavy end. Which way is it easier to swing and why?

Another common demonstration is spinning a student in a chair while holding weights in his or her hands. Have one student spin the chair while the other student holds the masses far from the axis of rotation, then repeat with the masses near the axis of rotation. Which is easier?

NOTE: This lab does not cover the conservation of angular momentum, so you need NOT include that topic here; this is better suited to a later lecture after students have grasped the simpler concepts in this lab first.

Ask students to then relate the concept to common activities. Ask them what ice skaters do to go into a very fast spin? (They pull their arms in.) Conclusion: Objects spin more easily when their mass is distributed closer to the axis of rotation. Objects with high rotational inertia are harder to get spinning.

What made the skater spin in the first place? What provided that spinning force? Torque, or more precisely, a "net" or "unbalanced" torque. (This will make a valuable connection to Newton's Second Law involving net forces). Students should already know what torque is, but a reminder here will help with the lab: *Torque* is a tangential force applied to an object at a distance from the axis of rotation. Point out that the direction of the force must be at 90 degrees from the lever arm (tangential) to properly calculate it, or either the force or lever arm must be broken into components. (You can use a socket wrench and a hex-head bolt to demonstrate this or simply draw a diagram on the board).



For a pulley and string, the string always pulls at 90 degrees, and students should consider what happens to the lever arm as the string unwraps. Is the lever arm really constant if the string is wrapped on top of itself? (Does a spool of kite string stay the same size as the kite rises high into the air? Ask students if these effects are negligible for this lab.

NOTE: They probably are, but the discussion provokes critical thinking and error analysis.

Why did the student in the spinning chair not spin forever? Friction on the spinning chair's axle bearings and air drag provide their own torque opposite to the motion, so he or she slowed down. A net torque causes a change in how an object spins. If an object is at rest (not spinning at all), and a net torque is applied, it will start spinning, which means angular acceleration occurred.

Pre-Lab Questions

1. What two variables determine the torque applied to a rotating object?

   Tangential force and lever arm.

2. How will you change these two variables in the lab?

   We will change tangential force by using different masses, pulled by gravity. We will change the lever arm by using pulleys with different diameters.

3. What properties of the rotating arm affect its ability to spin faster or slower?

   The rotational inertia of the arm determines how quickly it will accelerate from rest. We can change the rotational inertia by moving masses to different positions on the rotating arm.

4. For a given net torque and rotational inertia, would you expect the angular acceleration to be constant?

   Since net torque and rotational inertia are both constant, we would expect angular acceleration to be constant within a reasonable margin of error for this experiment.

Materials and Equipment

- Data collection system
- PASCO Rotary Motion Sensor[1]
- PASCO Pendulum Accessory[2]
- PASCO Super Pulley with Clamp[3]
- Stainless steel caliper
- Balance, 0.1-g resolution, 2,000-g capacity (1 per class)
- Table clamp or large base
- Support rod, 60-cm
- Mass and hanger set, 0.5-g resolution
- Scissors
- Meter stick
- Thread

Probeware Resources

Below are web-link and QR codes that will direct you to instructional video resources for individual pieces of PASCO probeware, sensors, and other hardware used in the lab activity. These same links and codes are provided to students in their activity handouts.

[1]www.pasco.com/ap20
PASCO Rotary Motion Sensor

[2]www.pasco.com/ap01
PASCO Pendulum Accessory

[3]www.pasco.com/ap13
PASCO Super Pulley With Clamp

Safety

Follow these important safety precautions in addition to your regular classroom procedures:

- Keep hands and heads clear of the spinning platform while it is moving and use caution when stopping the rotating arm.
- Be sure objects on the spinning platform are attached or they will fly off.

Teacher Tips

Tip 1 – Reading Stainless Steel Calipers

- Students should know how to read a set of stainless steel calipers. If they do not, showing them how could take 10 to 15 minutes of instruction. An overhead camera is helpful for this, and several websites now have good demonstrations on how to read such a caliper (watching one of these demonstrations could be given as homework the day before).

Tip 2 – Attaching the String to the Rotary Motion Sensor Pulley

- Students may have difficulty tying the string to the pulley wheels on the rotary motion sensor. Have them take the rotating arm off to provide easier access to the pulley. On the inside of the middle and large pulley grooves are small holes. Pass the string through either of the holes, and then tie a knot big enough so the string does not slip back through the hole when pulled.

Tip 3 – Other Ways to Increase or Decrease Net Torque

- In the Structured version of this lab activity, students change net torque by adjusting the amount of hanging mass attached to the smallest pulley groove on the PASCO Rotary Motion Sensor. Students could also adjust the amount of applied torque by winding the thread around either of the two larger pulley grooves. Using the larger pulley grooves also helps eliminate error when measuring the distance from the axis of rotation to the applied force: Students must consider the small added distance to the lever arm due to string wraps. Using a larger radius pulley minimizes the relative effect.

Tip 4 – Other Ways to Increase or Decrease Rotational Inertia

- In the Structured version of this lab activity, students change rotational inertia by adjusting the position of two fixed masses on a rotating arm. When using the Guided Inquiry or Student Designed version of this lab activity, students may choose to increase or decrease the rotational inertia of their setup by adjusting the size of the fixed masses rather than their position, or by using rotating objects (in place of the rotating arm) with different geometries (disc, hoop, solid rod, et cetera).

  However, students should be careful to choose alternative rotating objects with simple geometries so their rotational inertia calculations remain simple. The PASCO CI-6691 Mini Rotational Accessory can be used as a replacement for the PASCO Pendulum Accessory and comes with an additional aluminum disc and solid iron hoop with convenient geometries for easy rotational inertia calculations.

Tip 5 – Building Your Own Rotating Arm

- If you do not have access to the PASCO Pendulum Accessory, or prefer to use a rotating arm with a simpler geometry, custom rotating arms can easily be made using any solid metal rod. Simply cut the rod to any desirable length and drill a hole through its center to mount the rod to the PASCO Rotary Motion Sensor using a thumbscrew with #6-32 threading. To vary rotational inertia, rods with varying length and similar density, or rods with similar length but varying mass (different material), or both, can be used.

Sample Data

Below are sample data, acquired using the experimental setup and procedure outlined in the Structured version of the lab activity, and answers to questions in the Data Analysis section.

Data Analysis

Mass of rotating arm (kg): m_{arm} = 0.0284

Mass of sliding masses (kg): M = 0.150

Length of rotating arm (m): l = 0.381

PART 1 – VARYING NET TORQUE BY CHANGING FORCE

Student Part 1 graphs of angular velocity versus time with varying net torque will look similar to:

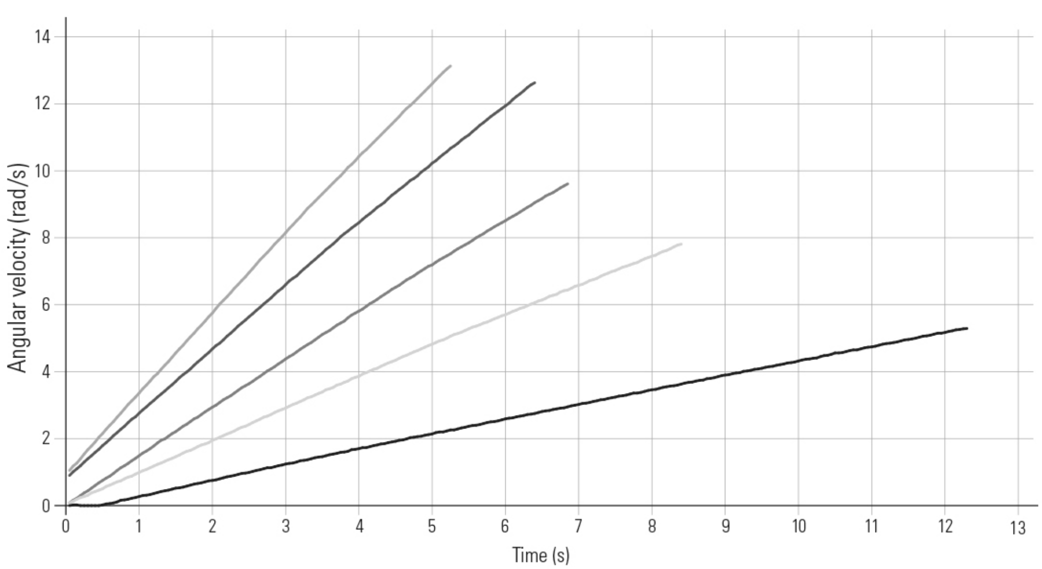


Table 1: Varying net torque with constant rotational inertia

Trial	Hanging Mass (kg)	Net Applied Force (N)	Net Applied Torque (N·m)	Angular Acceleration (rad/s²)
1	0.050	0.49	0.0024	0.447
2	0.100	0.98	0.0047	0.926
3	0.150	1.5	0.0072	1.402
4	0.200	2.0	0.0096	1.849
5	0.250	2.5	0.012	2.324

Pulley groove radius (m) = 0.0048

1. Calculate the magnitude of the net force applied to the pulley $\Sigma F_{\text{applied}}$ for each Part 1 trial, keeping in mind that the magnitude of the net force applied to the pulley was not equal to the weight of the hanging mass, but rather, to the tension T in the string. Use the following equation:

$$\sum F_{\text{applied}} = T = mg - m(\alpha r)$$

where m is the Hanging Mass value in each trial, α is the Angular Acceleration in each trial, r is the radius of the pulley groove, and g is earth's gravitational acceleration 9.8 m/s². Record the result for each trial in Table 1 in units of newtons (N).

For the sample data:

$$\sum F_{\text{applied}} = T = 0.050 \text{ kg} \times 9.8 \text{ m/s}^2 - 0.050 \text{ kg} (0.447 \text{ rad/s}^2 \times 0.0048 \text{ m}) = 0.49 \text{ N}$$

2. Calculate the net applied torque in each Part 1 trial: Multiply the net applied force for each trial in Table 1 by the radius of the smallest pulley groove. Record the result for each trial in Table 1 in units of newton-meters (N·m).

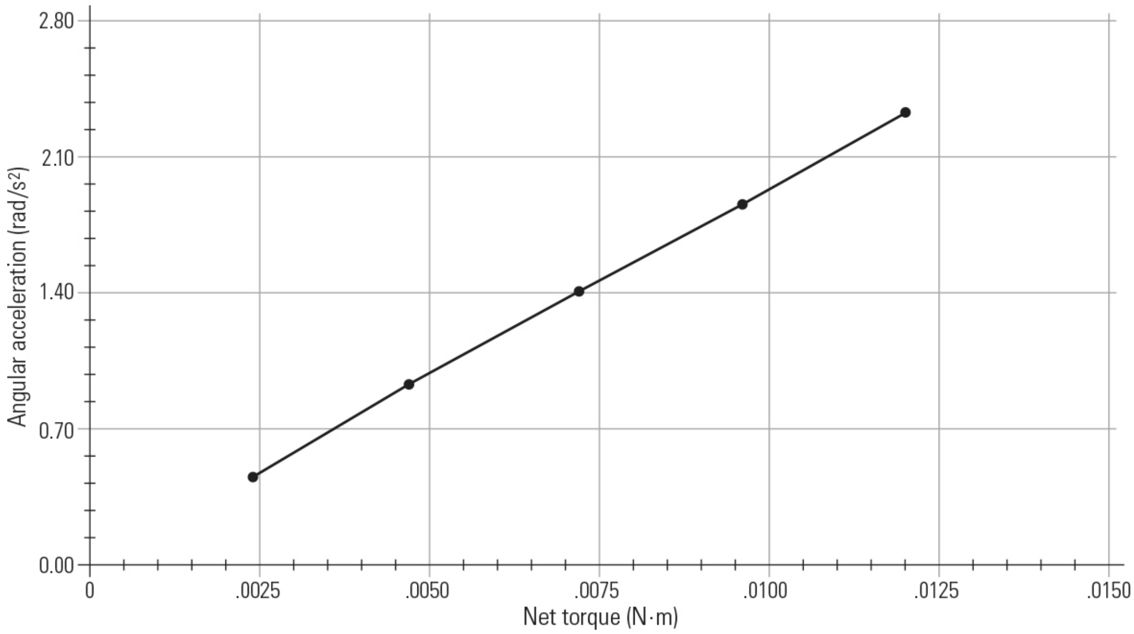
Calculation using sample data for Trial 1:

$$\tau_{\text{net}} = r \sum F_{\text{applied}}$$

$$\tau_{\text{net}} = 0.0048 \text{ m} \times 0.49 \text{ N} = 0.0024 \text{ N} \cdot \text{m}$$

3. Plot a graph of *angular acceleration* versus *net torque* in Graph 1. Be sure to label both axes with the correct scale and units.

Graph 1: Angular acceleration versus net torque with constant rotational inertia



PART 2 – VARYING ROTATIONAL INERTIA

Student Part 2 graphs of angular velocity versus time with varying rotational inertia will look similar to:

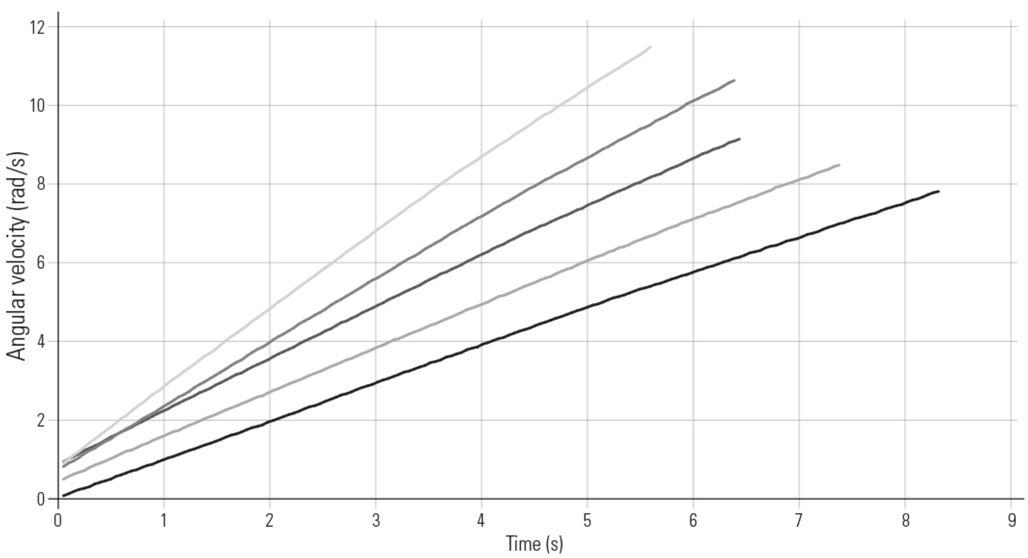


Table 2: Varying rotational inertia with constant net torque

Trial	Rotational Arm Length (m)	Rotational Inertia ($kg \cdot m^2$)	1/Rotational Inertia ($kg^{-1} \cdot m^{-2}$)	Angular Acceleration (rad/s^2)
1	0.120	0.00250	400	1.885
2	0.135	0.00308	325	1.537
3	0.150	0.00372	269	1.271
4	0.165	0.00443	226	1.078
5	0.180	0.00520	192	0.926

4. In Part 2 it was assumed that net torque was held constant in each trial; however, just as in Part 1, the applied force changes as the angular acceleration of the rotating arm changes. Why is it safe to assume that the applied force is equal to mg in Part 2?

   The difference between mg and $mg - m(\alpha r)$ is so small using the setup and procedure outlined above, that it can be considered negligible. For setups using larger pulleys, this may not be the case.

5. Use the following equations to calculate the total rotational inertia I_{total} of the rotating arm for each Part 2 trial:

$$I_{\text{total}} \approx I_{\text{arm}} + I_{\text{masses}}$$
$$I_{\text{arm}} \approx \frac{1}{12} m_{\text{arm}} l^2$$
$$I_{\text{masses}} \approx MR^2$$

   where R is the rotational arm length. Record the result for each trial in Table 2 in units of kilogram-meters-squared.

   Calculation using sample data for Trial 1:

$$I_{\text{masses}} \approx MR^2 = 0.150 \text{ kg} \times (0.120 \text{ m})^2 = 0.00216 \text{ kg} \cdot \text{m}^2$$

$$I_{\text{arm}} \approx \frac{1}{12} m_{\text{arm}} l^2 = \frac{1}{12} \times 0.0284 \text{ kg} \times (0.381 \text{ m})^2 = 0.000344 \text{ kg} \cdot \text{m}^2$$

$$I_{\text{total}} \approx I_{\text{arm}} + I_{\text{masses}} = 0.00216 \text{ kg} \cdot \text{m}^2 + 0.000344 \text{ kg} \cdot \text{m}^2 = 0.00250 \text{ kg} \cdot \text{m}^2$$

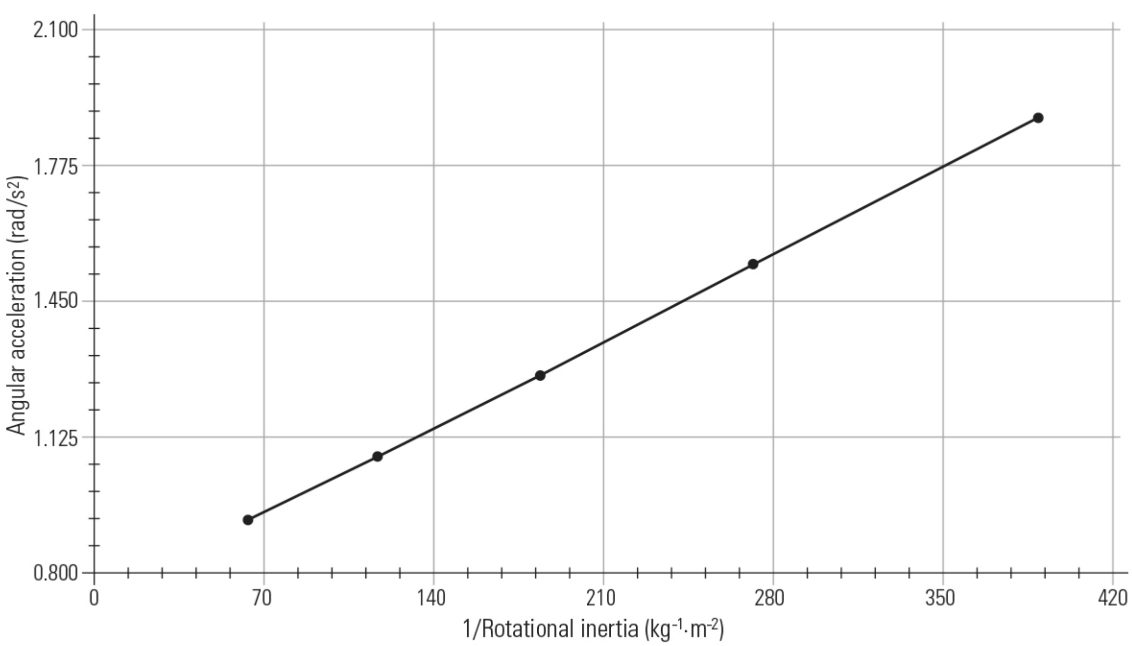
6. Calculate the inverse rotational inertia (1/rotational inertia) for each Part 2 trial. Record the result for each trial in Table 2 in units of inverse kilogram-meters-squared ($\text{kg}^{-1} \cdot \text{m}^{-2}$).

   Calculation using sample data for Trial 1:

   $1/0.00250 \text{ kg} \cdot \text{m}^2 = 400 \text{ kg}^{-1} \cdot \text{m}^{-2}$

7. Plot a graph of *angular acceleration* versus *1/rotational inertia* in Graph 2. Be sure to label both axes with the correct scale and units.

Graph 2: Angular acceleration versus inverse rotational inertia with constant net torque



Guided Inquiry Questions

Below are sample responses to the Guiding Questions found in the Guided Inquiry version of this lab activity.

❓ 1. If the objective of your experiment is to experimentally determine how net torque and rotational inertia affect the angular acceleration of a rotating object, what should the dependent and independent variables be in your experiment?

The objective statement indicates three variables: angular acceleration, rotational inertia, and net torque. Students will explore how net torque and rotational inertia affect the angular acceleration of an object, so angular acceleration should be the dependent variable and rotational inertia and net torque should be the independent variables.

❓ 2. What equipment do you have at your disposal to measure each variable, and how can you set up this equipment to measure each variable?

Angular acceleration can be measured using any of (but is not limited to) the following tools: a rotary motion sensor, a linear motion sensor measuring the acceleration of the hanging mass (you will need to convert from linear to angular: $\alpha = a/r$), a stopwatch, or video-motion analysis software and a camera. Students may choose to indirectly measure angular acceleration by using the slope of an angular velocity versus time curve, as outlined in the structured version of this lab activity.

❓ 3. How will you change each independent variable while collecting data? What steps will you take, and should you change more than one variable at a time? Justify your answer.

Each independent variable should be changed one at a time while the others are held constant: measure angular acceleration and vary net torque while holding rotational inertia constant; measure angular acceleration and vary rotational inertia while net torque is held constant. See Teacher Tip 3 below for suggestions on how to vary the net torque in this experiment. See Teacher Tip 4 below for suggestions on how to vary the rotational inertia in this experiment.

❓ 4. Should you measure all of the variables in your experiment at the same time? Justify your answer.

Only the dependent and independent variable need to be measured and recorded in each trial. If students choose to draw graphs on *x*–*y* axes, only two variables can be represented at a time. All other controlled (constant) variables need only be measured once as long as students are confident that they remain constant in each trial.

❓ 5. What other variables in your experiment must remain unchanged to help isolate each variable of interest? Justify your answer.

The measuring devices and techniques that students use should remain constant throughout the experiment. This helps maintain consistency in resolution and accuracy in measurements. Students measuring angular acceleration as their dependent variable must avoid any variable changes, other than those to the independent variable being investigated, that may affect the rotational inertia or net applied torque within the system such as (but not limited to) the diameter of the pulley groove used, excessive friction or drag in the rotational system, impedances to the path of the rotating body, and rotations outside the plane parallel to earth's surface (gravitational influences).

Assessment Questions: Sample Responses

Sample responses to the Analysis and Synthesis questions found in each version of the lab activity:

Analysis Questions

❓ 1. For each part of your experiment, list each variable involved and state whether it was held constant, increased, or decreased.

Answers for the Structured version of this activity:

Part 1 – Independent variable: net torque (increased); Dependent variable: angular acceleration (increased)

Part 2 – Independent variable: rotational inertia (increased); Dependent variable: angular acceleration (decreased)

Answers for the Guided Inquiry and Student Designed versions of this activity will vary depending on the independent variables tested.

2. How did changing the rotational inertia of the system affect its angular acceleration?

   As the moment of inertia increased, the angular acceleration decreased with constant net torque.

3. How did changing the net torque applied to the system affect its angular acceleration?

   As net torque increased, the angular acceleration increased with constant moment of inertia.

4. Based on your data, what is the mathematical relationship between angular acceleration and rotational inertia (proportional, inverse, squared, et cetera)? Justify your answer.

   Angular acceleration and rotational inertia are inversely proportional because the graph of angular acceleration versus inverse moment of inertia is a straight line (proportional).

5. What is the mathematical relationship between angular acceleration and torque (proportional, inverse, squared, et cetera)? Justify your answer.

   Angular acceleration and rotational inertia are inversely proportional because the graph of angular acceleration versus torque is a straight line (proportional).

6. From your results, can you predict a mathematical relationship between net torque τ_{net}, rotational inertia I, and angular acceleration α? Explain how your data supports your prediction.

   Our data showed that angular acceleration is inversely proportional to moment of inertia and proportional to torque. Combining the two relationships results in the equation:

$$\text{Angular acceleration} = \frac{\text{Net torque}}{\text{Rotational inertia}}$$

$$\alpha = \frac{\tau_{net}}{I}$$

Synthesis Questions

1. What angular acceleration would you expect from a rotating object with rotational inertia of $0.0655\ \text{kg}\cdot\text{m}^2$ that was subjected to a net torque of $4.25\ \text{N}\cdot\text{m}$?

$$\alpha = \frac{\tau_{net}}{I}$$

$$\alpha = \frac{4.25\ \text{N}\cdot\text{m}}{0.0655\ \text{kg}\cdot\text{m}^2}$$

$$\alpha = 64.9\ \text{rad/s}^2$$

2. If gravity is used to produce a torque applied to a rotating platform, do you expect the angular acceleration to be constant? Justify your answer.

   Angular acceleration is proportional to torque and inversely proportional to rotational inertia. If torque and rotational inertia are constant, then angular acceleration is constant. Because gravitational force is constant, then torque is constant, and thus angular acceleration is constant.

3. Would frictional errors affect this lab more or less if you had used a rotating system with a much larger rotational inertia? Justify your answer.

   Assuming that friction in the system is constant regardless of the rotational inertia, the relative error due to friction can be shown using the equation:

$$\alpha = \frac{\tau_{net}}{I} = \frac{\tau_{force} - \tau_{friction}}{I} = \frac{\tau_{force}}{I} - \frac{\tau_{friction}}{I}$$

   where $\frac{\tau_{friction}}{I}$ represents the error in our angular acceleration measurements due to friction. As rotational inertia I increases, this term goes to zero, which indicates that the relative error from friction also goes to zero.

4. Compare the angular quantities you just examined to linear quantities you have measured in the past. Fill in the blanks below with the angular (spinning) quantities that best correspond to their linear motion counterparts. Be sure to include the variable used to denote the quantity, and the SI units used to measure it.

Table 3: Angular quantities that correspond to linear motion quantities

Linear Motion	→	Angular Motion
Position x (m)	→	Angle θ (rad)
Velocity v (m/s)	→	Angular velocity ω (rad/s)
Acceleration a (m/s^2)	→	Angular acceleration α (rad/s)
Force F (N)	→	Torque τ (N·m)
Mass m (kg)	→	Rotational inertia I ($kg{\cdot}m^2$)

5. Kinematics equations are commonly used for 1-dimensional and 2-dimensional motion problems with constant acceleration. Use your completed table above to "translate" and rewrite the kinematics equations for angular motion (rotational kinematics).

Table 4: Kinematics equations translated to angular motion equations

Kinematics	→	Rotational Kinematics
$\Delta x = v_0 t + \frac{1}{2}at^2$	→	$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
$v = v_0 + at$	→	$\omega = \omega_0 + \alpha t$
$v^2 = v_0^2 + 2a\Delta x$	→	$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$

Extended Inquiry Suggestions

The tables that compare angular properties to their linear motion counterparts (see Synthesis Questions 4 and 5) is a great way of connecting this lab's main concepts back to familiar properties and equations from linear kinematics. Teachers are strongly recommended to build this connection to help students eventually solve rotational problems using the kinematics equations for angular motion.

The concepts of this lab naturally lead to angular applications of momentum and kinetic energy, although these are not covered here. Provide a demonstration of the conservation of angular momentum by spinning students who move weights away from and closer to their axis of rotation. Have students calculate angular kinetic energy. They should find that angular energy is *not* conserved in the above demonstration because the student is doing work by moving the weights.

10. ROTATIONAL DYNAMICS

STRUCTURED

Driving Question | Objective

How do net torque and rotational inertia affect the angular acceleration of a rotating object? Experimentally determine the mathematical relationship between net torque, rotational inertia, and angular acceleration of a rotating object.

Materials and Equipment

- Data collection system
- PASCO Rotary Motion Sensor[1]
- PASCO Pendulum Accessory[2]
- PASCO Super Pulley with Clamp[3]
- Stainless steel caliper
- Balance, 0.1-g resolution, 2,000-g capacity (1 per class)
- Table clamp or large base
- Support rod, 60-cm
- Mass and hanger set, 0.5-g resolution
- Scissors
- Meter stick
- Thread

[1]www.pasco.com/ap20
PASCO Rotary Motion Sensor

[2]www.pasco.com/ap01
PASCO Pendulum Accessory

[3]www.pasco.com/ap13
PASCO Super Pulley With Clamp

Background

Spinning and rotating objects are all around us. Bike tires, your car's drive shaft, and the earth itself, are all examples of spinning objects. How they spin, what makes them spin, and what factors will change the way they spin, are all relevant questions answered by the physics of rotational dynamics.

As an object rotates, it has angular velocity ω (the rate at which it rotates). The only way to cause objects to rotate is to apply a non-zero net torque τ: a non-zero tangential force F applied at some distance r from the axis of rotation. As more or different torque is applied to a rotating object, the object's angular velocity changes, which implies that it experiences an angular acceleration α (the rate at which angular velocity changes). More simply, changing the net torque applied to a rotating object affects the angular acceleration of the object.

Also affecting an object's angular acceleration is the rotational inertia I of an object. Derived from the object's mass and geometry, rotational inertia is best defined as an object's resistance to changes in angular velocity. Just like changing mass affects the linear acceleration of an object, changing rotational inertia affects the angular acceleration of a rotating object.

RELEVANT EQUATIONS

$$a = \alpha r \qquad \tau = r_{\perp}F \qquad \vec{F}_{\text{net}} = m\vec{a} \qquad I_{\text{total}} = \sum I$$

$I_{\text{rod}} = \frac{1}{12}ml^2$ Rotational inertia of a thin rod of mass m and length l rotated about its center.

$I_{\text{point mass}} = mr^2$ Rotational inertia of a point mass m a distance r from the axis of rotation.

Procedure

Part 1 – Varying Net Torque by Changing Force

SET UP

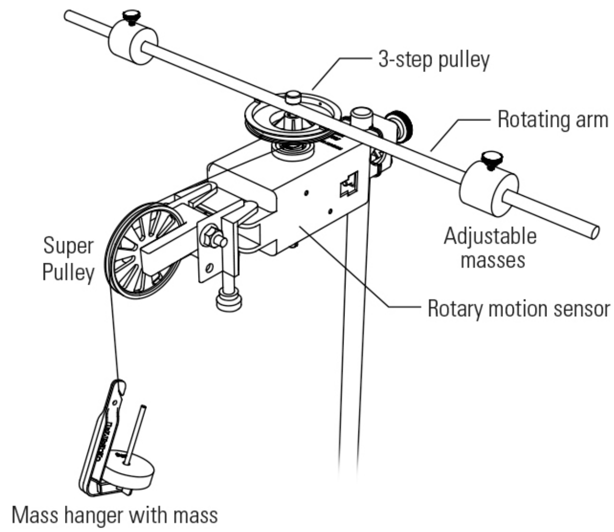
1. Measure the mass of the rotating arm *m* by itself, the combined mass of both sliding masses *M* by themselves, and the length *l* of the rotating arm. Record these values at the top of the Data Analysis section below.

2. Attach the rotary motion sensor to the top of the support rod using the rod clamp on the top of the sensor.

3. Mount the Super Pulley to the opposite end of the rotary motion sensor (cable end) using the Super Pulley clamp.

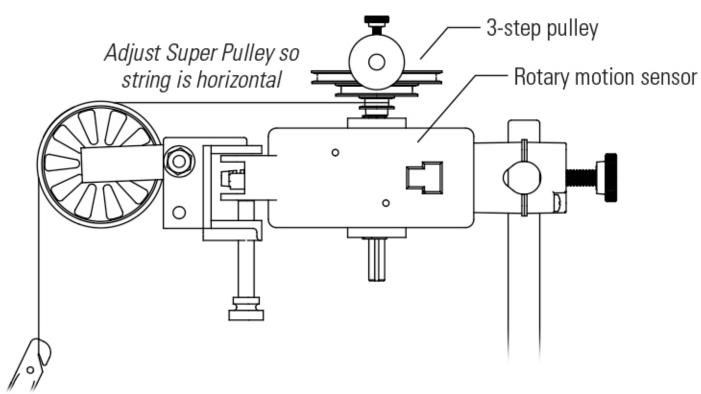
4. Mount the rotating arm to the rotary motion sensor.

   NOTE: Invert the pulley on the rotary motion sensor before mounting the rotating arm.



5. Cut a 50-cm section of thread, and then attach one end of the thread to the smallest pulley groove on the 3-step pulley and the other end to the mass hanger. Run the string from the 3-step pulley over the Super Pulley, allowing the mass hanger to hang freely.

6. Adjust the height of the Super Pulley so that the section of string between the top of the Super Pulley and the point at which it meets the 3-step pulley is horizontal.



7. Adjust the sliding masses along the rotating arm so the distance between the center of each mass and the axis of rotation is 18.0 cm.

8. Connect the rotary motion sensor to the data collection system.

9. Create a graph display of Angular Velocity on the *y*-axis with Time on the *x*-axis.

10. Add 45 g of mass to the 5 g mass hanger, resulting in a total of 50 g of mass hanging from the string.

11. Wind the string 6 or 7 times around the smallest pulley groove on the 3-step pulley and hold the rotating arm in place.

COLLECT DATA

12. Begin data recording.

13. Release the rotating arm (from rest), and collect data until the arm has completed 5 full revolutions.

14. Once the arm has completed 5 full revolutions, stop data recording and then carefully stop the rotating arm.

15. Change the amount of mass on the hanger: Repeat the previous steps four more times using an additional 50 g each time.

16. Record the hanging mass used in each trial into Table 1 in units of kilograms.

17. Use the tools on your data collection system to determine the slope of a linear fit to your Angular Velocity versus Time data during the time when the arm was rotating freely. Record this as the *angular acceleration* of the rotating arm in Table 1 for each trial.

18. Use the stainless steel caliper to measure the radius of the smallest groove on the 3-step pulley. Be sure to measure the part of the pulley the string encircles, not the outside edge of the pulley. Record this value in units of meters in the Data Analysis section under Table 1.

Part 2 – Varying Rotational Inertia

SET UP

19. Remove mass from the mass hanger until you have a total of 100 g of mass attached to the string.

20. Move the sliding masses on the rotating arm so the center of each is 12.0 cm from the axis of rotation.

21. Wind the string 6 or 7 times around the smallest pulley groove on the 3-step pulley and hold rotating arm in place.

COLLECT DATA

22. Repeat the data collection steps from Part 1. However, in this part, keep the hanging mass constant (constant torque) and vary the rotational arm length (rotational inertia changes).

    NOTE: Rotational arm length is equal to the distance between the axis of rotation and the center of either sliding mass.

23. Repeat the collect data steps three additional times, moving the center of each sliding mass 1.5 cm further from the axis of rotation each time.

    NOTE: There is no need to repeat the procedure with the sliding masses at 18 cm since data for that setup was recorded during Part 1 (Table 1, Trial 2).

24. Record the rotational arm length from each trial in Table 2 in units of meters.

25. Use the tools on your data collection system to determine the angular acceleration for each trial, and then record the results in Table 2.

Data Analysis

Mass of rotating arm (kg): m_{arm} = ______________________

Mass of sliding masses (kg): M = ______________________

Length of rotating arm (m): l = ______________________

Part 1 – Varying Net Torque by Changing Force

Table 1: Varying net torque with constant rotational inertia

Trial	Hanging Mass (kg)	Net Applied Force (N)	Net Applied Torque (N·m)	Angular Acceleration (rad/s^2)
1				
2				
3				
4				
5				

Pulley groove radius (m) = ______________________

1. Calculate the magnitude of the net force applied to the pulley $\Sigma F_{applied}$ for each Part 1 trial, keeping in mind that the magnitude of the net force applied to the pulley was not equal to the weight of the hanging mass, but rather, to the tension T in the string. Use the following equation:

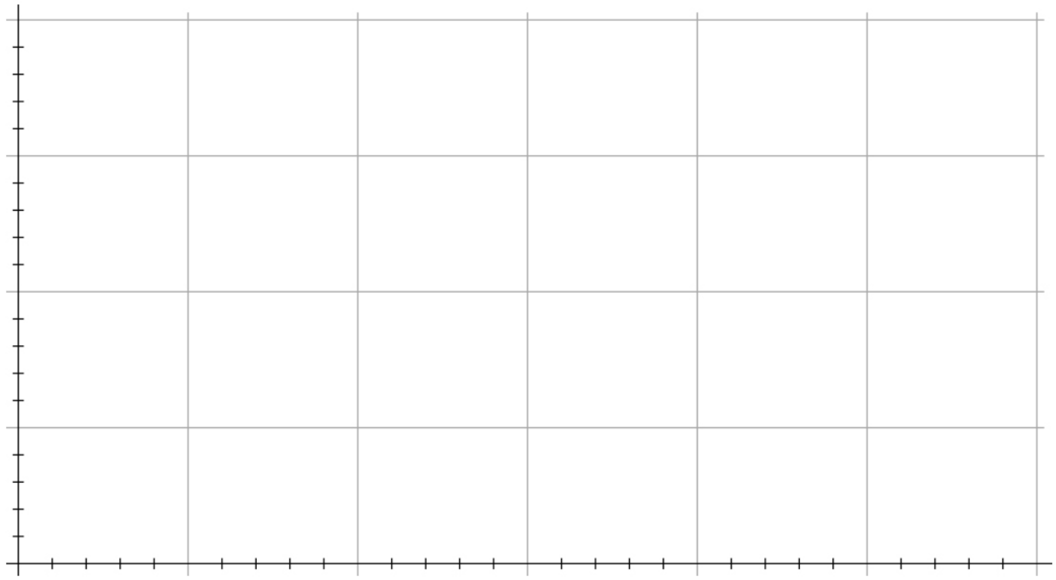
   $$\sum F_{applied} = T = mg - m(\alpha r)$$

   where m is the Hanging Mass value in each trial, α is the Angular Acceleration in each trial, r is the radius of the pulley groove, and g is earth's gravitational acceleration 9.8 m/s^2. Record the result for each trial in Table 1 in units of newtons (N).

2. Calculate the net applied torque in each Part 1 trial: Multiply the net applied force for each trial in Table 1 by the radius of the smallest pulley groove. Record the result for each trial in Table 1 in units of newton-meters (N·m).

3. Plot a graph of *angular acceleration* versus *net torque* in Graph 1. Be sure to label both axes with the correct scale and units.

Graph 1: Angular acceleration versus net torque with constant rotational inertia



Part 2 – Varying Rotational Inertia

Table 2: Varying rotational inertia with constant net torque

Trial	Rotational Arm Length (m)	Rotational Inertia ($kg \cdot m^2$)	1/Rotational Inertia ($kg^{-1} \cdot m^{-2}$)	Angular Acceleration (rad/s^2)
1				
2				
3				
4				
5				

4. In Part 2 it was assumed that net torque was held constant in each trial; however, just as in Part 1, the applied force changes as the angular acceleration of the rotating arm changes. Why is it safe to assume that the applied force is equal to mg in Part 2?

____

____

____

5. Use the following equations to calculate the total rotational inertia I_{total} of the rotating arm for each Part 2 trial:

$$I_{total} \approx I_{arm} + I_{masses}$$

$$I_{arm} \approx \frac{1}{12} m_{arm} l^2$$

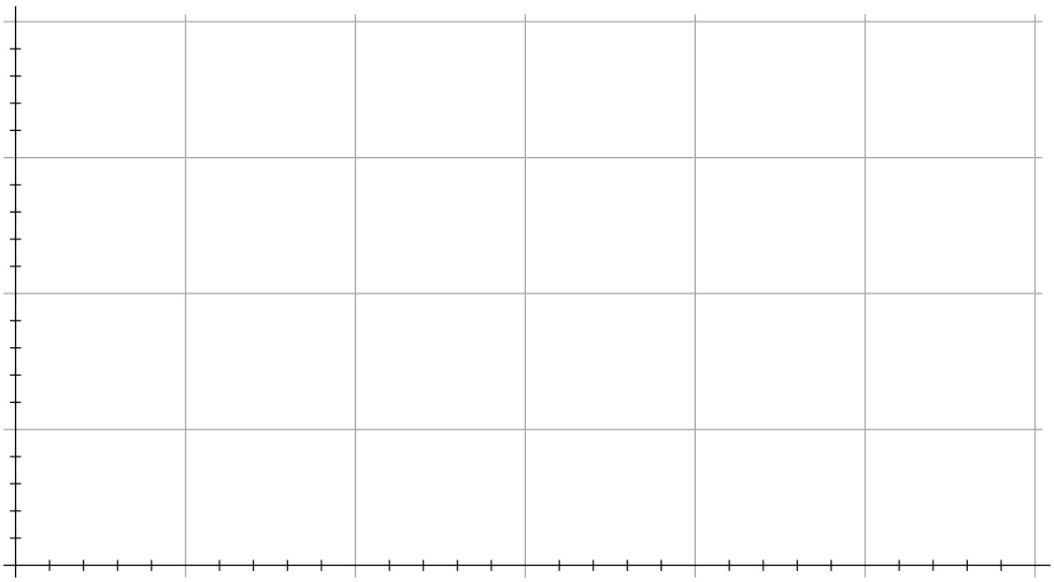
$$I_{masses} \approx MR^2$$

where R is the rotational arm length. Record the result for each trial in Table 2 in units of kilogram-meters-squared.

6. Calculate the inverse rotational inertia (1/rotational inertia) for each Part 2 trial. Record the result for each trial in Table 2 in units of inverse kilogram-meters-squared ($kg^{-1} \cdot m^{-2}$).

7. Plot a graph of *angular acceleration* versus *1/rotational inertia* in Graph 2. Be sure to label both axes with the correct scale and units.

Graph 2: Angular acceleration versus inverse rotational inertia with constant net torque



Analysis Questions

1. For each part of your experiment, list each variable involved and state whether it was held constant, increased, or decreased.

2. How did changing the rotational inertia of the system affect its angular acceleration?

3. How did changing the net torque applied to the system affect its angular acceleration?

❓ 4. Based on your data, what is the mathematical relationship between angular acceleration and rotational inertia (proportional, inverse, squared, et cetera)? How do you know?

❓ 5. What is the mathematical relationship between angular acceleration and net torque (proportional, inverse, squared, et cetera)? How do you know?

❓ 6. From your results, can you predict a mathematical relationship between net torque τ_{net}, rotational inertia I, and angular acceleration α? Explain how your data supports your prediction.

Synthesis Questions

❓ 1. What angular acceleration would you expect from a rotating object with rotational inertia of $0.0655\ \text{kg}\cdot\text{m}^2$ that was subjected to a net torque of $4.25\ \text{N}\cdot\text{m}$?

❓ 2. If gravity is used to produce a torque applied to a rotating platform, do you expect the angular acceleration to be constant? Why or why not?

❓ 3. Would frictional errors affect this lab more or less if you had used a rotating system with a much larger rotational inertia? Explain your answer?

4. Compare the angular quantities you just examined to linear quantities you have measured in the past. Fill in the blanks below with the angular (spinning) quantities that best correspond to their linear motion counterparts. Be sure to include the variable used to denote the quantity, and the SI units used to measure it.

Table 3: Angular quantities that correspond to linear motion quantities

Linear Motion	→	Angular Motion
Position x (m)	→	
Velocity v (m/s)	→	
Acceleration a (m/s^2)	→	
Force F (N)	→	
Mass m (kg)	→	

5. Kinematics equations are commonly used for 1-dimensional and 2-dimensional motion problems with constant acceleration. Use your completed table above to "translate" and rewrite the kinematics equations for angular motion (rotational kinematics).

Table 4: Kinematics equations translated to angular motion equations

Kinematics	→	Rotational Kinematics
$\Delta x = v_0 t + \frac{1}{2}at^2$	→	
$v = v_0 + at$	→	
$v^2 = v_0^2 + 2a\Delta x$	→	

11. ROTATIONAL STATICS

Connections to the AP® Physics 1 Curriculum*

The lab activity correlates to the following pieces of the AP® Physics 1 framework:

Big Idea	3	Enduring Understanding	F	Essential Knowledge	1

Learning Objective 1: The student is able to use representations of the relationship between force and torque.
Science Practice: 1.4

Learning Objective 2: The student is able to compare the torques on an object caused by various forces.
Science Practice: 1.4

Learning Objective 3: The student is able to estimate the torque on an object caused by various forces in comparison to other situations.
Science Practice: 2.3

Learning Objective 4: The student is able to design an experiment and analyze data testing a question about torques in a balanced rigid system.
Science Practices: 4.1, 4.2, 5.1

Learning Objective 5: The student is able to calculate torques on a two-dimensional system in static equilibrium by examining a representation or model (such as a diagram or physical construction).
Science Practices: 1.4, 2.2

Time Requirement

Preparation Time: 10 minutes

Lab Activity: 50 minutes

Prerequisites

Students should be familiar with the following concepts:

- A rigid body or system is an extended object whose shape and size remain constant when it moves.
- Drawing free-body force diagrams.
- Newton's Second Law for translational motion: $\Sigma\vec{F} = m\vec{a}$, and its rotational equivalent: $\Sigma\vec{\tau} = I\vec{\alpha}$
- Torque is the rotational equivalent of force. Torque must be applied to an object to start the object rotating. For calculations involving torque, the entire weight of an object can be considered to act at a single point, the center of gravity or center of mass of the object. Torque is a vector quantity with direction parallel to the axis of rotation. A torque is positive when it is applied in the counterclockwise direction and negative when it is applied in the clockwise direction.

Driving Question | Objective

What must the net force and net torque on an object be if the object is in static equilibrium (translational and rotational)? Experimentally demonstrate that the sum of the forces acting on an object in static translational equilibrium is equal to zero, and the sum of the torques acting on an object in static rotational equilibrium is equal to zero.

Procedural Overview

The Structured version of this lab activity is divided into two parts:

Part 1 – Students suspend a 500-g mass from two lengths of thread and use two PASCO Tension Protractors to measure the tension and angle applied to the mass by each thread. Students then calculate the component forces acting on the mass as it hangs motionlessly, and sum each set of component forces. This process is repeated for two additional trials; each trial with a different thread length configuration. Student data for each trial will demonstrate that the sums of the component forces acting on the mass in static equilibrium are equal to zero (or nearly zero) for both the x- and y-directions.

Part 2 – Students balance a meter stick on a fulcrum (directly under its center), and then place a 100-g mass at different distances from the fulcrum (pivot point) while applying a force, using a force sensor, to the meter stick on the opposite side of the pivot. When the applied force is enough to balance the meter stick and mass (static equilibrium), students record the force applied by the force sensor, the distance from the pivot to the force sensor, and the distance from the pivot to the mass. Students then calculate the torques applied to the meter stick by the mass and the force sensor and demonstrate that when the meter stick is in static equilibrium, the sum of the torques is zero, regardless of where the mass and force sensor are positioned.

Pre-Lab Discussion and Activity

This is a pre-lab suggestion and activity.

Review center of mass and center of gravity. Remind students that for a symmetrical object, the center of gravity is at the center of the object. It is recommended to show students an example of an object that is asymmetrical and free to rotate, such as a cardboard cut-out of your state or the continental United States. Show students that when such an object is supported at a pivot point, it will rotate unless its center of gravity is directly below the pivot point. This is a demonstration of the effect of gravitational torque on an asymmetrical object.

Review the equation for finding the balance point between two masses: $m_1r_1 = m_2r_2$ where m is the mass and r is the distance the mass is from the axis of rotation, or the fulcrum, in the case of a lever.

PRE-LAB QUESTIONS

❓ 1. If an object is suspended by two threads at two different angles (refer to the illustration in the Procedural Overview section) which thread has the greater tension?

The thread that makes a larger angle with the horizontal will have the greater tension because the sine of the angle formed by the thread and the horizontal is at a maximum when the angle is ninety degrees.

❓ 2. List some examples of common objects or systems in static equilibrium.

Parked vehicles, motorcycles, bicycles, aircraft; artistic mobiles (in the absence of air or other fluid currents), tree houses, ships at rest, athletes holding poses, such as football players at the line of scrimmage, open doors, and fan blades switched off, are a few.

❓ 3. Consider a helium balloon float held tightly at rest by its handlers at a famous Thanksgiving Day Parade. List any physical variables in this system that you think could affect its equilibrium.

Student answers may include: gravity, the buoyant force of the air displaced by the balloon, wind or air currents pushing the balloon, people pulling down on the ropes, releasing the ropes, or moving horizontally while holding the ropes.

Materials and Equipment

- Data collection system
- PASCO High Resolution Force Sensor with rubber bumper[1]
- PASCO Tension Protractor (2)[2]
- Table clamp or large base (2)
- Support rod, 60-cm or longer (2)
- Support rod, 90-cm or longer
- Hooked mass set
- AA-cell battery or similar cylindrical object
- Right angle clamp (2)
- Thread
- Meter stick
- Tape
- Scissors

Probeware Resources

Below are web-link and QR codes that will direct you to instructional video resources for individual pieces of PASCO probeware, sensors, and other hardware used in the lab activity. These same links and codes are provided to students in their activity handouts.

[1]www.pasco.com/ap22

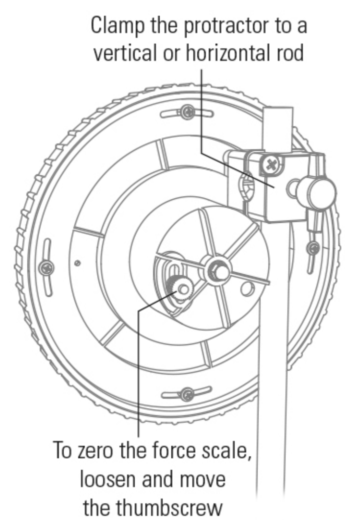
PASCO High Resolution Force Sensor

[2]www.pasco.com/ap06

PASCO Tension Protractor

Teacher Tips

Tip 1 – Tension Protractors



1. Students may need help zeroing the force and angle scales on the tension protractors after they have attached them to the rod. Follow these steps to ensure that the red pointer indicates 0 N when no force is applied.

    a. Allow the string to hang freely with no force applied.

    b. Loosen the thumb screw on the back of the tension protractor, as shown in the diagram.

    c. Move the screw up or down to set the red pointer to zero on the face of the tension protractor.

    d. Tighten the thumb screw.

2. Read the *magnitude of force* on the inner red scale as indicated by the red pointer. Read the *angle* on the outer scale where the string crosses it.

Tip 2 – Physics String

- The tension protractors use PASCO Braided Physics String to connect a wire hook to the rotating part of the protractor. This string is wrapped twice around a small pulley. If this portion of string needs to be replaced, using string of a different diameter will introduce error into the force measurements. Although PASCO Braided Physics String is recommended, any non-stretch string may be used ass replacement, as long as the string diameter is comparable (0.4 mm). Examples include: nylon or Dacron® non-stretch string, available at craft or hobby stores; metallic "beading" string or wire, available in very fine gauge.

Tip 3 – Reading the Tension Protractor

- Remind students to read the scales on the tension protractors with their eyes at the level of the scale to minimize parallax error. This is similar to the need to read the volume of a graduated cylinder at eye level to accurately see the bottom of the meniscus.
- Make sure students are aware which angle the tension protractors are measuring so they use the correct angle in their calculations and not the complement of the angle they need. For example, if the tension protractors are set up to read an angle from the horizontal, students will need to use the sine of the angle to compute the vertical components of the force vectors, and the cosine of the angle to compute the horizontal components of the force vectors.

Sample Data

Below are sample data, acquired using the experimental setup and procedure outlined in the Structured version of the lab activity, and answers to questions in the Data Analysis section.

Data Analysis

PART 1 – NET FORCE ON AN OBJECT IN STATIC EQUILIBRIUM

Table 1: Tension components on the mass from the left thread

Configuration	Left Thread Tension (N)	Left Thread Angle (°)	T_x (N)	T_y (°)
1	4.59	31.5	−3.91	2.40
2	4.00	58.5	−2.09	3.41
3	4.68	84.8	−0.424	4.66

Table 2: Tension components on the mass from the right thread

Configuration	Right Thread Tension (N)	Right Thread Angle (°)	T_x (N)	T_y (°)
1	4.59	31.5	3.91	2.40
2	2.55	35.8	2.07	1.49
3	0.50	32.0	0.424	0.265

1. Calculate the x-component of the tension force T_x from each thread on the suspended mass ($T_x = T\cos\theta$). Record the results into Tables 1 and 2.

   NOTE: If you choose the positive x-direction to be directed toward the right, the leftward pointing tension force will be negative.

   Using the sample data from Configuration 2:

   $$T_{x,\text{left string}} = T\cos\theta = (-4.00\text{ N})\cos\ 58.5° = -2.09\text{ N}$$

   $$T_{x,\text{right string}} = T\cos\theta = (2.55\text{ N})\cos\ 35.8° = 2.07\text{ N}$$

2. Sum the component tension forces T_x from both threads to determine the net force ΣF_x in the x-direction. Record this net force in Table 3 for each configuration.

   Using the sample data from Configuration 2:

   $$\sum F_x = T_{x,\text{left string}} + T_{x,\text{right string}} = -2.09\text{ N} + 2.07\text{ N} = -0.02\text{ N}$$

3. Calculate the downward force from the suspended 500-g mass and record it here:

   Force from hanging mass (N): −4.9

   Sample calculation:

   $F_g = mg = (0.500 \text{ kg})(-9.8 \text{ m/s}^2) = -4.9 \text{ N}$

4. Calculate the y-component of the tension force T_y from each thread on the suspended mass ($T_y = T \sin\theta$). Record the results into Tables 1 and 2.

   Using the sample data from Configuration 2:

   $T_{y,\text{left string}} = T\sin\theta = (4.00 \text{ N})\sin 58.5° = 3.41 \text{ N}$

   $T_{y,\text{right string}} = T\sin\theta = (2.55 \text{ N})\sin 35.8° = 1.49 \text{ N}$

5. Sum the component tension forces T_y from both threads, plus the force from the hanging mass, to determine the net force ΣF_y in the y-direction. Record this net force in Table 3 for each configuration.

   Using the sample data from Configuration 2:

   $\sum F_y = T_{y,\text{left string}} + T_{y,\text{right string}} + F_g = 3.41 \text{ N} + 1.49 \text{ N} + (-4.9 \text{ N}) = 0.00 \text{ N}$

Table 3: Net force on the mass

Configuration	Net Force, *x*-direction (N)	Net Force, *y*-direction (N)
1	0.00	−0.10
2	−0.02	0.00
3	0.000	−0.025

PART 2 – NET TORQUE ON AN OBJECT IN STATIC EQUILIBRIUM

Horizontal distance from the force sensor contact point to the pivot point (m): 0.350

Table 4: Torques due to the mass added on one side of the pivot point

Trial	Horizontal Distance from Mass to Pivot (m)	Force Applied by Sensor (N)	Torque Applied by Sensor (N·m)	Torque Applied by Mass (N·m)	Net Torque (N·m)
1	0.100	0.28	0.098	−0.098	0.000
2	0.200	0.56	0.20	−0.20	0.00
3	0.300	0.84	0.29	−0.29	0.00
4	0.400	1.12	0.392	−0.39	0.002
5	0.500	1.40	0.490	−0.49	0.00

6. Calculate the torque τ applied by the force sensor in each trial using the equation

$$\tau = r_{\perp} F \qquad (3)$$

where $r_{\perp}$ is the horizontal distance from the force sensor contact to the pivot point, and F is the force applied by the sensor. Record your results for each trial into Table 3.

NOTE: Torques applied in a clockwise direction about the pivot point are negative, while torques applied in a counterclockwise direction are positive.

Using the sample data from Trial 1:

$$\tau_{\text{sensor}} = r_{\perp} F_{\text{sensor}} = (0.350\text{ m})(0.28\text{ N}) = 0.098\text{ N}\cdot\text{m}$$

7. Use Equation 3 to calculate the torque applied by the mass in each trial, where $r_{\perp}$ is equal to the horizontal distance from the mass to the pivot point, and F is the weight (in N) of the 100-g mass. Record your results for each trial in Table 3.

Using the sample data from Trial 1:

$$\tau_{\text{mass}} = r_{\perp} F_g = r_{\perp} mg = (0.100\text{ m})(0.100\text{ kg})\left(9.8\text{ m/s}^2\right) = -0.098\text{ N}\cdot\text{m}$$

8. Calculate the sum of the torques (the net torque) in each trial. Record your results into Table 3.

Using the sample data from Trial 1:

$$\sum \tau = \tau_{\text{sensor}} + \tau_{\text{mass}} = 0.098\text{ N}\cdot\text{m} + (-0.098\text{ N}\cdot\text{m}) = 0.000\text{ N}\cdot\text{m}$$

Guided Inquiry Questions

Below are sample responses to the Guiding Questions found in the Guided Inquiry version of this lab activity.

1. Explain how you would assemble the equipment from the materials provided to measure the tension force on an object suspended with two threads. Describe how you would use these measurements to determine the vertical and horizontal components of the forces.

   Students should set up a framework that accommodates both tension protractors. The mass should be suspended so that it is attached to the wire hook of one tension protractor with one length of thread, and attached to the other tension protractor with the other length of thread. Students may suspend the mass so that each thread is the same length, giving them congruent angles and equal tensions, or they may choose to have different lengths of thread, and therefore different angles and tensions. Students should explain that they will resolve the thread tension force vectors into vertical and horizontal components.

2. If you hang a mass from two threads, how can you configure your setup to ensure that each thread has a different tension force?

   The threads used to connect the mass should be of different lengths, since the thread forms the hypotenuse of the triangle whose base and height form the vertical and horizontal components of the tension force vector.

3. How will you collect data that will demonstrate that the sum of forces acting on an object in static translational equilibrium is equal to zero?

   Students can measure force using several different methods that will be dependent on the setup chosen by each group. In most cases, students will need to measure (either directly or indirectly) the component (*x* and *y*) forces acting on the object under investigation; students must use component forces when calculating the sum of forces. Groups using a setup similar to that outlined in the Structured version of this lab activity (a mass hung asymmetrically from two PASCO Tension Protractors) will use two tension protractors to measure the tension force delivered by each thread to a hanging mass.

   Students will need to make measurements of both tension force and the angle of each force relative to the horizontal so as to determine each *x* and *y*-component force. Students may also substitute the tension protractor with a force sensor or other spring scale to measure tension force.

4. Explain how you would assemble the equipment from the materials provided to demonstrate a system that could rotate, but does not because the torques applied to it are balanced.

   Students may set up their equipment similar to the configuration shown in the Set Up section of Part 2 in the Structured version of this lab activity. They may also substitute any of the rods as a lever arm in place of the meter stick. Students may use one of the rods as a fulcrum. Students may choose not to balance the lever arm portion of their system, but instead hold it in equilibrium with a force applied by the force sensor.

5. How would you manipulate the system described in the previous question to change the direction or magnitude, or both, of the torques being applied to it, but still show that the sum of torques is zero if the system is in static equilibrium?

   In most cases, students will choose to manipulate the torques on their static object by either changing the magnitude of applied force on their system, or the distance from the pivot point to the force(s) being applied. Students may choose to measure the force required to hold the system in static equilibrium as the distance of each force from the fulcrum increases or decreases. Some students may choose to change the mass while holding the distance constant.

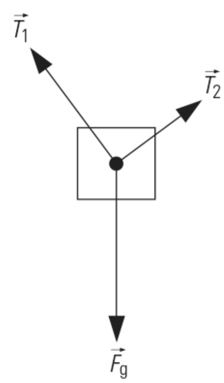
Assessment Questions: Sample Responses

Sample responses to the Analysis and Synthesis questions found in each version of the lab activity:

Analysis Questions

1. Draw a free-body force diagram of the setup you used to demonstrate that the net force acting on an object in static translational equilibrium is zero. Indicate the direction and magnitude of the forces involved in your experiment.

   Diagrams will vary, depending on the setup chosen. This diagram refers to the setup described in the Structured version of this lab activity, Part 1, Configuration 2.



2. Explain how your data demonstrates that the net force acting on an object in static equilibrium is zero.

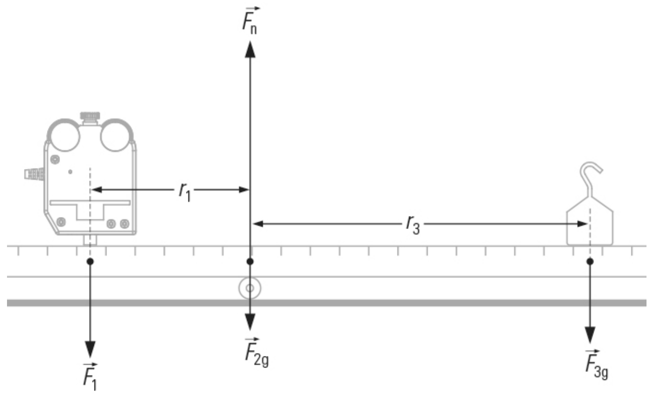
   Explanations should point to data that shows a net force equal to zero for any object at rest with no changes to its translational motion. Data should include component forces in the *x* and *y*-directions that sum to zero.

3. How would your experiment have been different had you used an object that was three times as massive? Justify your answer.

   Regardless of the setup chosen, the effect of having an object that is three times as massive would only be on the magnitude, and possibly direction, of forces acting on the object; however, if the object (regardless of its mass) is in static equilibrium, the sum of the torques and forces will always be zero.

4. Draw a free-body force diagram of the setup you used to experimentally demonstrate that the net torque acting on an object in static rotational equilibrium is zero. Indicate the direction and magnitude of the forces involved in your experiment.

Diagrams will vary depending on the setup chosen. This diagram refers to the setup described in Part 2 of the Structured version of this lab activity.



$$\sum F_x = 0$$
$$\sum F_y = F_n - F_{2g} - F_{3g} - F_1 = 0$$
$$\sum \tau = F_1 r_1 - F_{3g} r_3 = 0$$

5. Explain how your data demonstrates that the net torque acting on an object in static equilibrium is zero.

Explanations should point to data that shows a net torque equal to zero for any object at rest with no changes to its rotational motion. For this to be true, students must include directionality in the torques that are being investigated: positive for counterclockwise torques; negative for clockwise torques.

Synthesis Questions

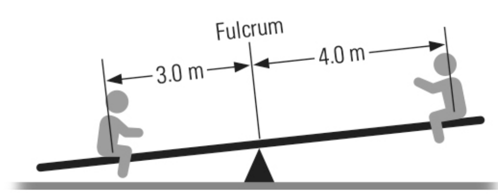
1. Describe the relationship between static equilibrium and net torque.

When an object is in static equilibrium, there is no net torque on the object. For any axis of rotation, the vector sum of the torques is zero.

2. Can you assume that a larger force always produces a greater torque? Why? Justify your answer.

A larger force does not always produce a greater torque. Distance from the fulcrum must be considered. A force of 20 N applied at a distance of 0.05 m from the axis of rotation produces a torque of 1.0 N·m, while a force of 5 N applied at a distance of 0.4 m produces a torque of 2.0 N·m, or double the torque with one-fourth of the force.

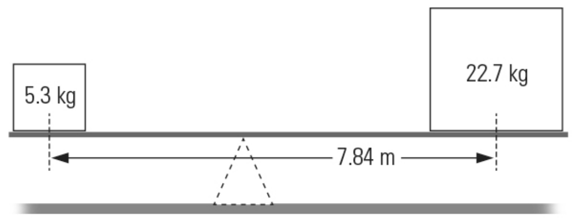
3. Two children are sitting on a seesaw. If the child on the left has a mass of 31.0 kg and sits 3.0 m from the fulcrum, and the child on the right has a mass of 25.0 kg and sits 4.0 m from the fulcrum, what is the net torque on the seesaw? If the seesaw is initially at rest, which way will it rotate? Assume that the fulcrum is directly below the center of mass of the seesaw. Show your work.



$$\tau = r_{\perp} F = dmg$$
$$\sum \tau = \tau_{\text{left child}} - \tau_{\text{right child}}$$
$$\sum \tau = d_1 m_1 g - d_2 m_2 g = (3.0\text{ m})(31.0\text{ kg})\left(9.8\text{ m/s}^2\right) - (4.0\text{ m})(25.0\text{ kg})\left(9.8\text{ m/s}^2\right) = -69\text{ N}\cdot\text{m}$$

Because the net torque is negative and the seesaw is initially at rest, the seesaw will rotate clockwise.

❷ 4. A student is trying to balance a long beam on a fulcrum so that the beam does not rotate. If the beam has two masses on it that are 7.84 m apart (a 5.3-kg mass on the left end, and a 22.7-kg mass at the opposite end), where should the student place the fulcrum so the system will be in static equilibrium? (Assume the beam is massless.) Show your work.



In this problem there are two unknowns: the distance from the left mass to the fulcrum, d_1, and the distance from the right mass to the fulcrum, d_2. To solve for the two unknowns, we must have two equations, and because the system is balanced (static equilibrium) we can use the following two equations:

$$\sum \tau = 0 = d_1 m_1 g - d_2 m_2 g$$

$d_1 + d_2 = l$ or $d_2 = l - d_1$

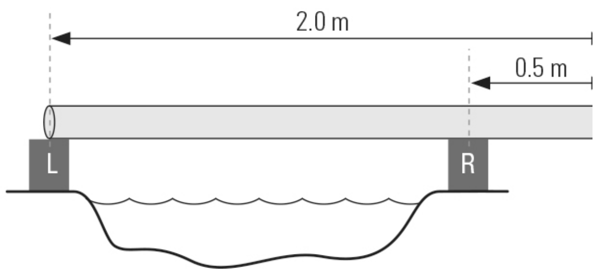
Substituting $l - d_1$ for d_2 in the first equation:

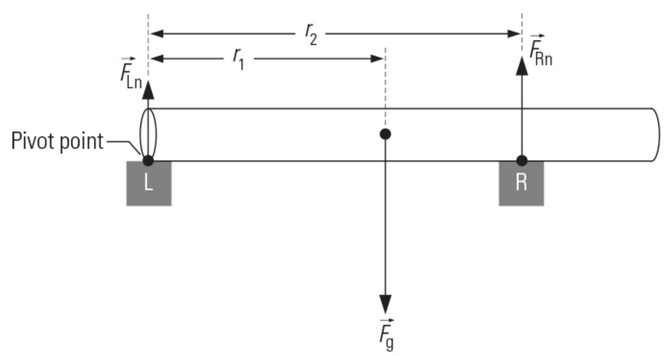
$$0 = d_1 m_1 g - (l - d_1) m_2 g$$

$$d_1 m_1 + d_1 m_2 = l m_2$$

$d_1 = l \dfrac{m_2}{m_1 + m_2} = (7.84\text{ m}) \dfrac{22.7\text{ kg}}{5.3\text{ kg} + 22.7\text{ kg}} = 6.36\text{ m}$; The fulcrum should be placed 6.36 m from the left mass.

❷ 5. A log weighing 510 N is laid across cinder block supports (L and R) on each bank of a stream to form a bridge, as shown in the diagram. The length of the log is 2.0 m, and 0.5 m of the log hangs past the R block to the right. What is the magnitude of the normal force on each block? Assume the log has uniform density.





Because the log is in static equilibrium, the sum of forces on the log is equal to zero and the sum of the torques on the log is equal to zero. Choosing the pivot point to be where the log contacts the left cinder block (positive forces are upward and positive torques are counterclockwise), the normal force can be calculated as follows:

$$\sum \tau = 0 = r_2 F_{Rn} - r_1 F_g$$

$$F_{Rn} = F_g \frac{r_1}{r_2} = F_g \frac{l/2}{l - 0.5\text{ m}} = (510\text{ N}) \frac{1.0\text{ m}}{1.5\text{ m}} = 340\text{ N}$$

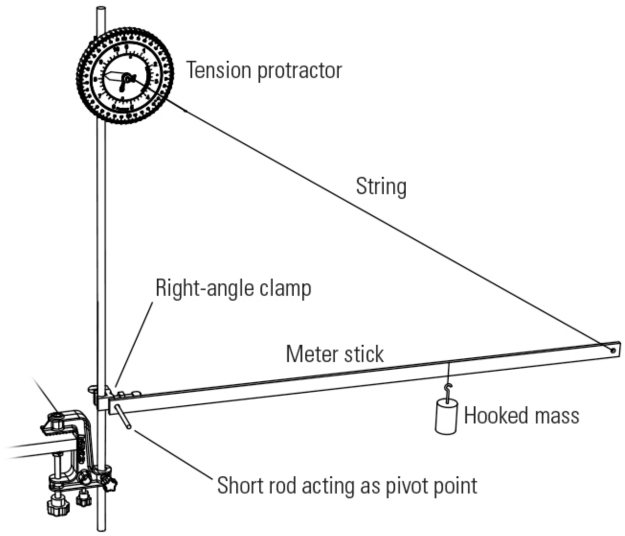
$$\sum F_x = 0$$

$$\sum F_y = 0 = F_{Ln} + F_{Rn} - F_g$$

$$F_{Ln} = F_g - F_{Rn} = 510\text{ N} - 340\text{ N} = 170\text{ N}$$

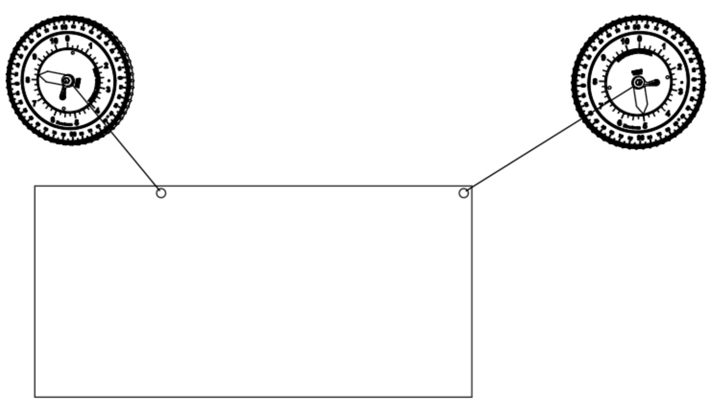
Extended Inquiry Suggestions

Students can investigate the forces and torques on a boom in static equilibrium using a setup similar to the one shown:



Students would make measurements to determine the clockwise torques about the pivot point caused by the hanging mass and the meter stick boom. They would then determine the counterclockwise torque due to the tension in the angled string supporting the right end of the boom. Students would compare the clockwise and counterclockwise torques acting on the boom in static equilibrium. In addition, students could calculate known horizontal and vertical forces on the boom system, and use those results to calculate the unknown forces acting on the pivot point. Students could then determine the magnitude and direction of the force acting at the pivot.

Students can also be challenged to hang a rectangular sign board from two different-length threads so that the sign is level. Students should attach the threads to the sign as shown, adjust the positions of the tension protractors to make the sign level, and demonstrate that the two requirements for equilibrium (zero net force; zero net torque) are satisfied to within an acceptable error.



11. ROTATIONAL STATICS

STRUCTURED

Driving Question | Objective

What must the net force and net torque on an object be if the object is in static equilibrium (translational and rotational)? Experimentally demonstrate that the sum of the forces acting on an object in static translational equilibrium is equal to zero, and the sum of the torques acting on an object in static rotational equilibrium is equal to zero.

Materials and Equipment

- Data collection system
- PASCO High Resolution Force Sensor with rubber bumper[1]
- PASCO Tension Protractor (2)[2]
- Table clamp or large base (2)
- Support rod, 60-cm or longer (2)
- Support rod, 90-cm or longer
- Hooked mass set
- AA-cell battery or similar cylindrical object
- Right angle clamp (2)
- Thread
- Meter stick
- Tape
- Scissors

[1]www.pasco.com/ap22

PASCO High Resolution Force Sensor

[2]www.pasco.com/ap06

PASCO Tension Protractor

Background

An object at rest, experiencing zero changes to its motion, is said to be in *static equilibrium*. This equilibrium describes both the translational and rotational motion of the object. For an object to remain in static equilibrium, the forces and torques acting on the object must satisfy certain conditions that are easily derived using Newton's Second Law.

If an object is at rest and experiencing zero changes to its translational motion, its acceleration $\vec{a}$ must equal zero, and according to Newton's Second Law:

$$\sum \vec{F} = \vec{F}_{\text{net}} = m\vec{a} = 0 \qquad (1)$$

the sum of the forces $\vec{F}_{\text{net}}$ (net force) acting on the object must also be zero.

If an object is at rest and experiencing zero changes to its rotational motion, its angular acceleration $\vec{\alpha}$ must equal zero, and according to the rotational equivalent of Newton's Second Law,

$$\sum \vec{\tau} = \vec{\tau}_{\text{net}} = I\vec{\alpha} = 0 \qquad (2)$$

the sum of the torques $\vec{\tau}_{net}$ (net torque) acting on the object must also be zero.

In this lab activity, you will investigate the forces acting on a suspended mass and demonstrate that static equilibrium in translational motion occurs when the sum of forces acting on the mass is zero. You will also investigate the forces and torques acting on a balanced rigid system, and demonstrate that the system experiences zero rotational motion if the sum of the torques acting on it is zero.

RELEVANT EQUATIONS

$$\sum \vec{F} = \vec{F}_{\text{net}} = m\vec{a} = 0 \qquad (1)$$

$$\sum \vec{\tau} = \vec{\tau}_{\text{net}} = I\vec{\alpha} = 0 \qquad (2)$$

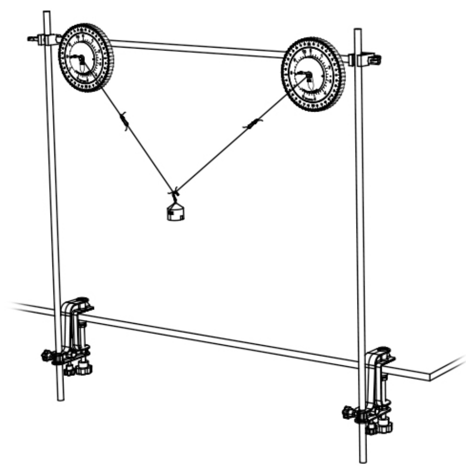
$$\tau = r_{\perp}F = r\left(F\sin\theta\right) \qquad (3)$$

Equation 3 indicates that torque is equal to the product of the lever arm $r_{\perp}$ (sometimes referred to as the moment arm) and the force F being applied, where the lever arm is defined as the perpendicular distance between the axis of rotation and the line of action of F. This equation simplifies to the product of the distance from the axis of rotation r to the point at which the force is applied and the component of force acting perpendicular to r: $F\sin\theta$.

Procedure

Part 1 – Net Force on an Object in Static Equilibrium

SET UP



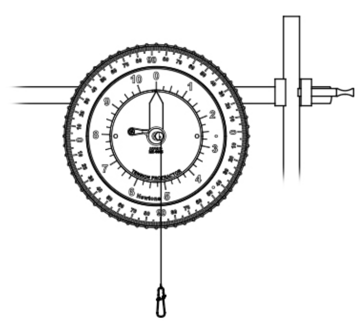
1. Set up the three support rods and two tension protractors similar to the diagram at right:

   a. Assemble the two shorter rods vertically using the table clamps or rod bases, and place them about 80 cm apart. Attach a right-angle clamp to the top of each rod.

   b. Slide the protractors onto the third rod using the rod clamp on the back of each, and then use the right-angle clamps to connect the third rod horizontally to the two vertical rods.

   c. Once the third rod is connected, slide each protractor along the horizontal rod so they are as far apart as possible, adjust each protractor so its face points out perpendicular to the top of the lab table, and then tighten the thumbscrew on the each protractor's clamp to lock it in place.

   d. Zero the force scale of each tension protractor: Without anything attached to the tension protractor string, adjust the thumb screw in the back until the force scale reads zero.

   e. Zero the angle scale of each tension protractor: Hang a small mass (10-g) from the hook so the string hangs straight down. Rotate the outer ring to align the 90° mark with the string (refer to the diagram).

2. Cut four lengths of thread: two 25-cm lengths, one 45-cm length, and one 65-cm length.

3. Use the two 25-cm pieces of thread to suspend a 500-g hooked mass from the wire hooks of the tension protractors: Tie one end of each thread to each protractor hook and tie the other end of each thread to the mass.

COLLECT DATA

4. With the mass hanging motionless, read the magnitude of tension T and the angle θ for each thread. Record the tension and angle for the left thread into Table 1 and the tension and angle for the right thread into Table 2, in the Data Analysis section.

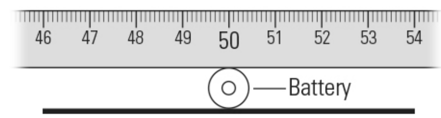
5. Remove one of the 25-cm threads and replace it first with the 45-cm thread, and then with the 65-cm thread, recording the tension and angle for each new thread configuration into Tables 1 and 2.

Part 2 – Net Torque on an Object in Static Equilibrium

SET UP

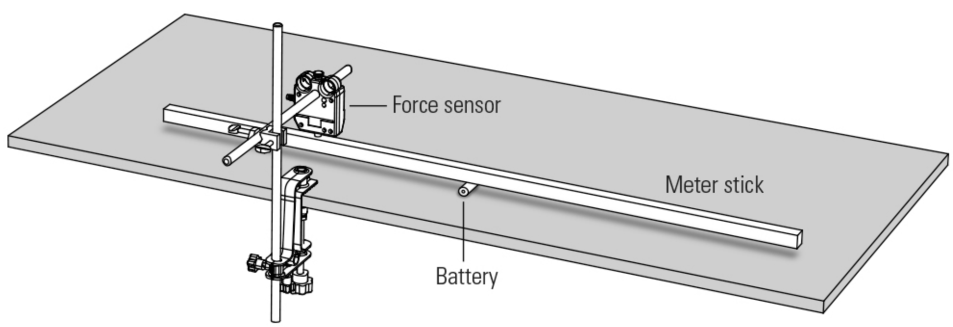
6. Place the battery, or other small cylindrical object, near the edge of the lab table and tape it in place so it does not roll.

7. Find the balance point of the meter stick by placing the center of the meter stick across the battery. Adjust the meter stick until it balances (or nearly balances).



8. Assemble the table clamp or base, two support rods, right angle clamp, and force sensor with bumper similar to the picture below. Adjust the position and height of the force sensor so that it is *just* in contact with the meter stick at some distance between the balance point, or *pivot point*, and the end of the meter stick.

   NOTE: In order to display the scale on the meter stick, the illustrations show the meter stick balancing on its narrow edge, but the wider side of the meter stick should be placed on the cylinder for contact with the force sensor, and so it can support the mass that will be set on it.



9. Connect the force sensor to the data collection system, and then press the Zero button on the top of the force sensor.

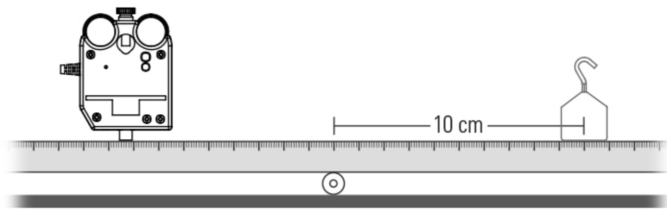
10. On the data collection system, create a digits display of force measured by the force sensor.

    NOTE: The force measured by the force sensor will be equal to the downward force applied by the sensor to the meter stick.

COLLECT DATA

11. Measure the distance between the pivot point and the point where the force sensor makes contact with the meter stick. Record this distance in the space above Table 4 in the Data Analysis section below.

12. Begin recording data, and then place a 100-g hooked mass on the meter stick so that its center is 0.10 m from the pivot point, opposite the force sensor.



13. When the force measurement has stabilized, record it and the horizontal distance from the mass to the pivot point into Table 4.

14. Repeat the same collect data steps four more times, moving the mass an additional 0.10 m from the balance point in each trial. Record the force and horizontal distance for each trial into Table 4.

15. Stop recording data after the fifth trial.

NOTE: If the meter stick becomes unbalanced at any time, stop data collection, re-establish the balance, and begin a new data set.

Data Analysis

Part 1 – Net Force on an Object in Static Equilibrium

Table 1: Tension components on the mass from the left thread

Configuration	Left Thread Tension (N)	Left Thread Angle (°)	T_x (N)	T_y (°)
1				
2				
3				

Table 2: Tension components on the mass from the right thread

Configuration	Right Thread Tension (N)	Right Thread Angle (°)	T_x (N)	T_y (°)
1				
2				
3				

1. Calculate the x-component of the tension force T_x from each thread on the suspended mass ($T_x = T\cos\theta$). Record the results into Tables 1 and 2.

   NOTE: If you choose the positive x-direction to be directed toward the right, the leftward pointing tension force will be negative.

2. Sum the component tension forces T_x from both threads to determine the net force ΣF_x in the x-direction. Record this net force in Table 3 for each configuration.

3. Calculate the downward force from the suspended 500-g mass and record it here:

   Force from hanging mass (N): ______________________________

4. Calculate the y-component of the tension force T_y from each thread on the suspended mass ($T_y = T\sin\theta$). Record the results into Tables 1 and 2.

5. Sum the component tension forces T_y from both threads, plus the force from the hanging mass, to determine the net force ΣF_y in the y-direction. Record this net force in Table 3 for each configuration.

Table 3: Net force on the mass

Configuration	Net Force, *x*-direction (N)	Net Force, *y*-direction (N)
1		
2		
3		

Part 2 – Net Torque on an Object in Static Equilibrium

Horizontal distance from the force sensor contact point to the pivot point (m): ____________________

Table 4: Torques due to the mass added on one side of the pivot point

Trial	Horizontal Distance from Mass to Pivot (m)	Force Applied by Sensor (N)	Torque Applied by Sensor (N·m)	Torque Applied by Mass (N·m)	Net Torque (N·m)
1					
2					
3					
4					
5					

6. Calculate the torque τ applied by the force sensor in each trial using the equation

$$\tau = r_{\perp}F \qquad (3)$$

where $r_{\perp}$ is the horizontal distance from the force sensor contact to the pivot point, and F is the force applied by the sensor. Record your results for each trial into Table 3.

NOTE: Torques applied in a clockwise direction about the pivot point are negative, while torques applied in a counterclockwise direction are positive.

7. Use Equation 3 to calculate the torque applied by the mass in each trial, where $r_{\perp}$ is equal to the horizontal distance from the mass to the pivot point, and F is the weight (in N) of the 100-g mass. Record your results for each trial in Table 3.

8. Calculate the sum of the torques (the net torque) in each trial. Record your results into Table 3.

Analysis Questions

1. Draw a free-body force diagram of the setup you used to demonstrate that the net force acting on an object in static translational equilibrium is zero. Indicate the direction and magnitude of the forces involved in your experiment.

2. Explain how your data demonstrates that the net force acting on an object in static equilibrium is zero.

3. How would your experiment have been different had you used an object that was three times as massive? Justify your answer.

4. Draw a free-body force diagram of the setup you used to experimentally demonstrate that the net torque acting on an object in static rotational equilibrium is zero. Indicate the direction and magnitude of the forces involved in your experiment.

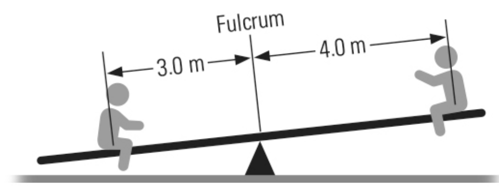
5. Explain how your data demonstrates that the net torque acting on an object in static equilibrium is zero.

Synthesis Questions

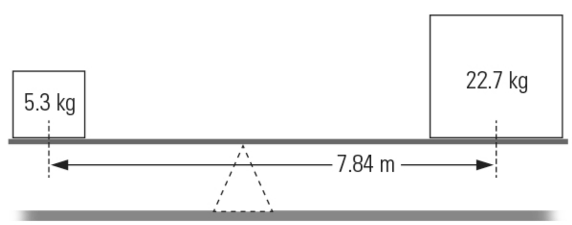
1. Describe the relationship between static equilibrium and net torque.

2. Can you assume that a larger force always produces a greater torque? Why? Justify your answer.

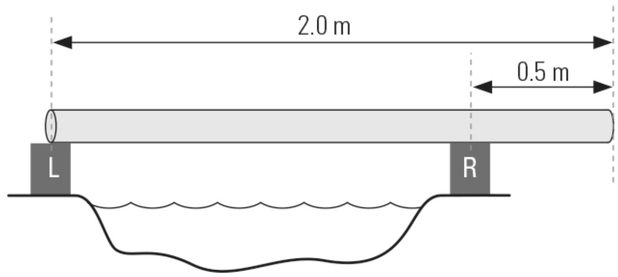
3. Two children are sitting on a seesaw. If the child on the left has a mass of 31.0 kg and sits 3.0 m from the fulcrum, and the child on the right has a mass of 25.0 kg and sits 4.0 m from the fulcrum, what is the net torque on the seesaw? If the seesaw is initially at rest, which way will it rotate? Assume that the fulcrum is directly below the center of mass of the seesaw. Show your work.



4. A student is trying to balance a long beam on a fulcrum so that the beam does not rotate. If the beam has two masses on it that are 7.84 m apart (a 5.3-kg mass on the left end, and a 22.7-kg mass at the opposite end), where should the student place the fulcrum so the system will be in static equilibrium? (Assume the beam is massless.) Show your work.



❷ 5. A log weighing 510 N is laid across cinder block supports (L and R) on each bank of a stream to form a bridge, as shown in the diagram. The length of the log is 2.0 m, and 0.5 m of the log hangs past the R block to the right. What is the magnitude of the normal force on each block? Assume the log has uniform density.



12. PERIODIC MOTION: MASS AND SPRING

Connections to the AP® Physics 1 Curriculum*

The lab activity correlates to the following pieces of the AP® Physics 1 framework:

Big Idea	3	Enduring Understanding	B	Essential Knowledge	3

Learning Objective 1: The student is able to predict which properties determine the motion of a simple harmonic oscillator and what the dependence of the motion is on these properties.
Science Practices: 6.4, 7.2

Learning Objective 2: The student is able to design a plan and collect data in order to ascertain the characteristics of the motion of a system undergoing oscillatory motion caused by a restoring force.
Science Practice: 4.2

Learning Objective 3: The student can analyze data to identify qualitative or quantitative relationships between given values and variables (i.e., force, displacement, acceleration, velocity, period of motion, frequency, spring constant, string length, mass) associated with objects in oscillatory motion to use that data to determine the value of an unknown.
Science Practice: 2.2, 5.1

Learning Objective 4: The student is able to construct a qualitative and/or quantitative explanation of oscillatory behavior given evidence of a restoring force.
Science Practices: 2.2, 6.2

Time Requirement

Preparation Time: 30 minutes

Lab Activity: 90 minutes

Prerequisites

Students should be familiar with the following concepts:

- Hooke's Law
- Newton's Second Law
- The graphical connection between position, velocity, and acceleration versus time graphs. Students should know how to sketch a velocity versus time graph based on the shape of a corresponding position versus time graph.

Driving Question | Objective

What variables affect the period of oscillation of a mass and spring system? Experimentally determine the physical properties of a hanging mass and spring system that affect its period of oscillation.

Procedural Overview

The Structured version of this lab activity is divided into four parts:

Part 1 – Students assemble a vertical mass and spring system and then displace the mass vertically to set the system into oscillatory motion. Students measure the period T_s of their system as it oscillates, while increasing the initial vertical displacement of the mass in each trial. They measure the time for ten cycles of motion in each trial and use this time to determine the average period of the system. Student data will show that the average period in each trial is identical, or nearly identical, prompting students to recognize that the period of a mass and spring system is unaffected by changes in initial vertical displacement.

Part 2 – Using the same setup as in Part 1, students measure the average period of two mass and spring systems with identical spring constant and hanging mass, but different length. Student data will show that the average period for each system is identical, or nearly identical, prompting students to recognize that the period of a mass and spring system is unaffected by the length of the system.

Part 3 – Using the same setup as in Part 1, students measure the average period of three mass and spring systems with identical length and mass, but different spring constant. Students calculate $1/\sqrt{\text{spring constant}}$ for each system, and then plot a graph of period versus $1/\sqrt{\text{spring constant}}$. The plot of period versus $1/\sqrt{\text{spring constant}}$ will show a linear (proportional) relationship. Students are expected to recognize that the period of a mass and spring system is proportional to the inverse square root of the spring constant.

Part 4 – Using the same setup as in Part 1, students measure the average period of a mass and spring system while increasing the amount of hanging mass in each trial. Students calculate $\sqrt{\text{mass}}$ for each trial, and plot a graph of period versus $\sqrt{\text{mass}}$. The plot of period versus $\sqrt{\text{mass}}$ will show a linear (proportional) relationship. Students are expected to recognize that the period of a mass and spring system is proportional to the square root of the hanging mass value.

Combining the results of all four parts of the lab activity, students use their data to support the actual mathematical relationship between period, mass, and spring constant for a mass and spring system:

$$T_s = 2\pi\sqrt{\frac{m}{k}}$$

Pre-Lab Discussion and Activity

This is a pre-lab suggestion and activity.

Review Hooke's Law and simple harmonic motion and its properties. Define what an equilibrium point is and remind students that the restoring force from a spring increases as the displacement grows, and that the restoring force has the opposite direction as displacement.

Ask students what the displacement of an oscillating mass and spring system is when the speed of the mass is maximum (answer: zero), and what the speed of the mass is when the displacement is maximum (also zero). Additional questions can be asked about the kinetic energy of the mass as it oscillates. It is recommended to begin these discussions describing a horizontal example of an oscillating mass and spring system on a frictionless surface, so that gravitational potential energy is not a factor in the equations. That discussion can come in a subsequent lecture.

Materials and Equipment

- Data collection system
- Table clamp or large base
- Support rod, 60-cm or taller
- Support rod, 45-cm
- Right angle clamp
- Meter stick
- Hooked mass set
- PASCO Motion Sensor[1]
- Springs of similar size (diameter and length), but varying spring constant (3), 1–15 N/m
- Springs with similar spring constant and diameter, but of varying length (2), 0.1–0.3 m
- Tape

Probeware Resources

Below are web-link and QR codes that will direct you to instructional video resources for individual pieces of PASCO probeware, sensors, and other hardware used in the lab activity. These same links and codes are provided to students in their activity handouts.

[1]www.pasco.com/ap18

PASCO Motion Sensor

Lab Preparation

These are the materials and equipment to set up prior to the lab:

1. Before students perform the Structured version of this lab activity, the teacher must determine the spring constant of each spring that will be used. This information will need to be provided to students when performing the activity. If the spring constant of each spring is already known, no further preparation is necessary.

   Although there are many methods for determining the spring constant of a spring, the procedure outlined here is simple and will help minimize preparation time:

   a. Hang the spring vertically and suspend 200 g of mass from the bottom of the spring.

   b. With the mass and spring system motionless, use a meter stick to note the vertical position of the spring where it attaches to the hanging mass. This is the equilibrium position of the system.

   c. Suspend an additional 200 g of mass from the spring, and measure the displacement d (in meters) of the motionless system from its equilibrium position.

   d. Use the following equation to calculate the spring constant k of the spring:

   $$k = \frac{1.96\ \text{N}}{|d|}$$

Teacher Tips

Tip 1 – Motion Sensor Tips for Clean Data

Students following the procedure outlined in the Structured version of this lab activity use a PASCO motion sensor to measure the motion of the mass and spring system. The motion sensor is aimed at the bottom of the hanging mass and continuously measures its position as it oscillates. If students find that their data is bad or noisy, they can use these tips to help eliminate this:

- Because the motion sensor uses ultrasound, it is best to use a hanging mass with a relatively rigid and flat bottom to cleanly reflect the sound waves. The better the sound waves from the motion sensor are reflected, the cleaner student position data will be. If you do not have hanging masses with a flat bottom, an index card or small flat piece of cardboard can be affixed to the bottom of the mass (with its flat surface facing the motion sensor) to more cleanly reflect sound waves.
- The front of the motion sensor should be placed directly beneath and aimed up at the bottom of the hanging mass. Position data that has smooth sections and then sections of noisy data is evidence that the motion sensor is not aimed directly at the bottom of the mass. Adjust the position of the sensor and use the rotating transceiver housing to make certain that the alignment and aim are correct.
- The space between the sensor and 15 cm in front of the sensor is a "dead zone" in which the sensor cannot measure position. If a student's hanging mass passes through this region, the sensor will not record data. If needed, readjust the height of the mass and spring so the hanging mass does not get closer than 15 cm to the motion sensor at maximum displacement.

Tip 2 – Using More Springs in the Structured Version

- In Part 3 of the procedure outlined in the Structured version of this lab activity, students test the effect of the spring constant on the period of three oscillating mass and spring systems having different spring constants. Students then plot the three data points from these tests to graphically determine that the period is proportional to the inverse square root of the spring constant. The number of Part 3 trials was limited to three to accommodate teachers using the PASCO Demonstration Spring Set (this set has only three springs with similar size and varying spring constant).

  Although these three tests should be sufficient for students to establish this graphical relationship, you may choose to have them test additional springs with similar size and varying spring constant. If you choose to have students perform more Part 3 tests, you will need to add rows to Table 3 in the Structured version of this activity to facilitate the additional trials. You will also need to alter the Part 3 Collect Data procedure to indicate the total number of trials to perform.

Tip 3 – Measuring Displacement

- Any student groups that are including measurements of displacement in any part of their experiment should understand that displacement is a relative measure of position change that their mass and spring system experiences. Displacement should be measured from the same point in each experiment or trial, for example, students adding hanging mass to their systems by hanging additional masses to the bottoms of already attached masses should measure displacement relative to a point *not* on the newly added masses. A better point of reference would be the point at which the spring attaches to the hanging masses.

Tip 4 – Keep Displacement to a Minimum

- For students to record consistent data, all trials should be conducted keeping the vertical displacement of their springs to no more than the length of the unstretched spring. Springs that are stretched too far may display non-hookean behavior manifesting itself as inconsistencies in the period measurements. Also, stretching the springs too far may permanently damage the springs.

Sample Data

Below are sample data, acquired using the experimental setup and procedure outlined in the Structured version of the lab activity, and answers to questions in the Data Analysis section.

Data Analysis

PART 1 – DISPLACEMENT AND PERIOD

Student Part 1 graphs of position versus time of an oscillating mass and spring system varying the displacement of the mass will look similar to:

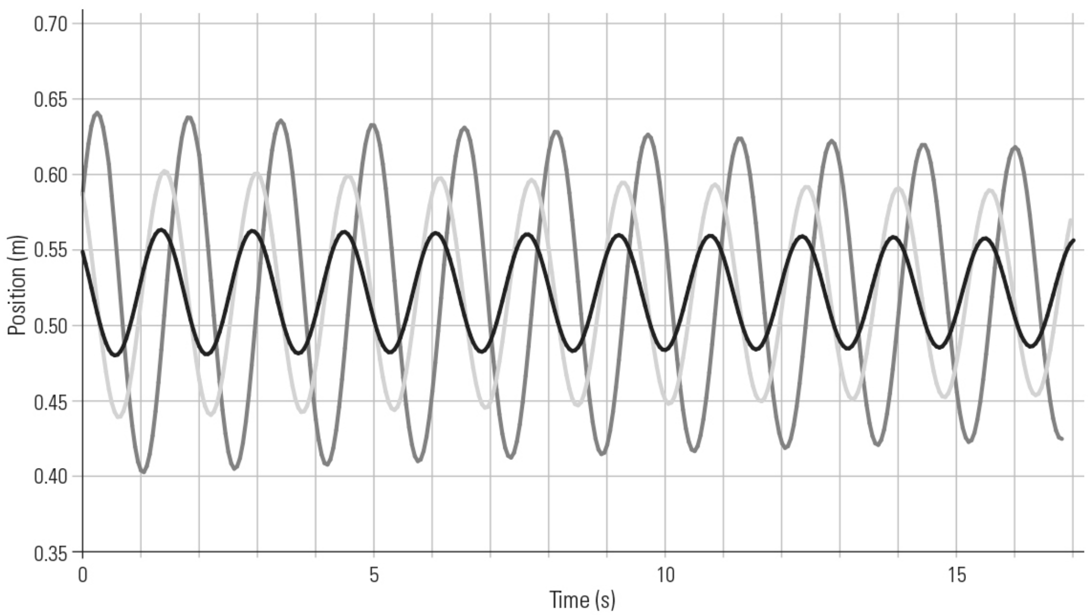


Table 1: Period and displacement data for a mass and spring system

Trial	Time for 10 Cycles (s)	Initial Vertical Displacement (cm)	Average Period (s)
1	15.7	4.0	1.57
2	15.7	8.0	1.57
3	15.7	12.0	1.57

1. Calculate the average period for each Part 1 trial in Table 1. Record your results for each trial in Table 1.

$$\text{Average period} = \frac{\text{Time for 10 cycles}}{10}$$

Calculation using sample data for Trial 1:

$$\text{Average period} = \frac{15.7\text{ s}}{10} = 1.57\text{ s}$$

2. Did changing the displacement of the mass affect the period of the mass and spring system? Justify your answer.

   Changing the displacement of the mass does not affect the period of an oscillating mass and spring system. Student data will show this, as the period for each displacement should be equal.

PART 2 – LENGTH AND PERIOD

Student Part 2 graphs of position versus time of two oscillating mass and spring systems having springs with the same spring constant but different spring lengths will look similar to:

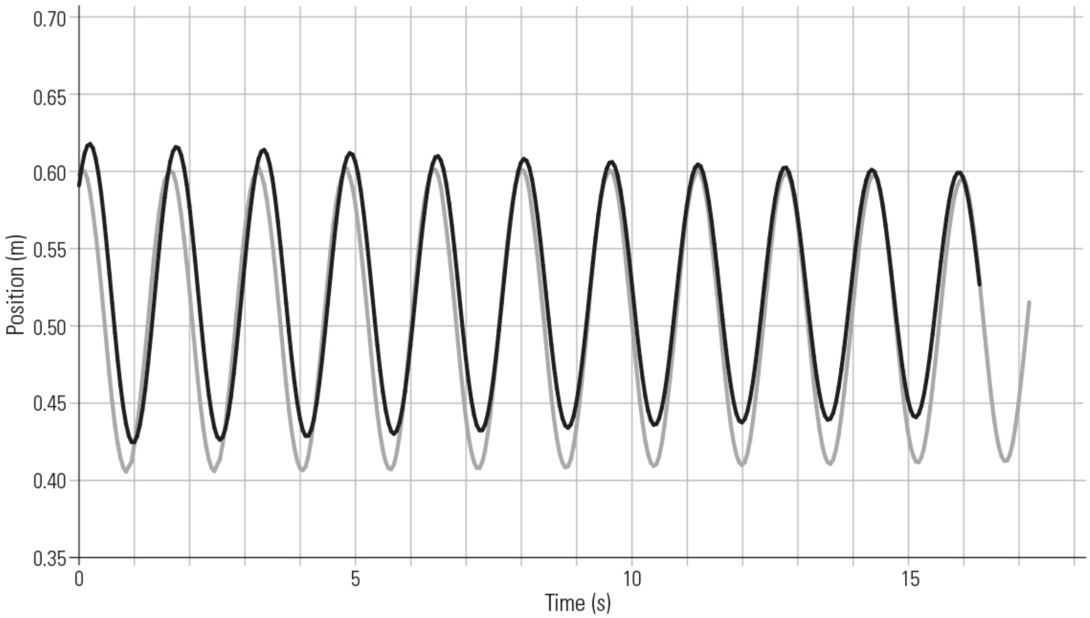


Table 2: Period of two springs with the same spring constant but different length

Spring	Time for 10 Cycles (s)	Spring Length (cm)	Average Period (s)
1	15.7	10.8	1.57
2	15.7	21.6	1.57

3. Calculate the average period for each spring. Record your results in Table 2.

   Calculation using sample data for Spring 1:

   $$\text{Average period} = \frac{15.7\ \text{s}}{10} = 1.57\ \text{s}$$

4. Did changing the length of the spring affect the period of the mass and spring system? Justify your answer.

   Changing the length of the spring does not affect the period of an oscillating mass and spring system. Student data will show this as the period for each spring length should be equal as long as the spring constant of each spring is equal.

PART 3 – SPRING CONSTANT AND PERIOD

Student Part 3 graphs of position versus time of three oscillating mass and spring systems having springs of the same length but with different spring constants will look similar to:

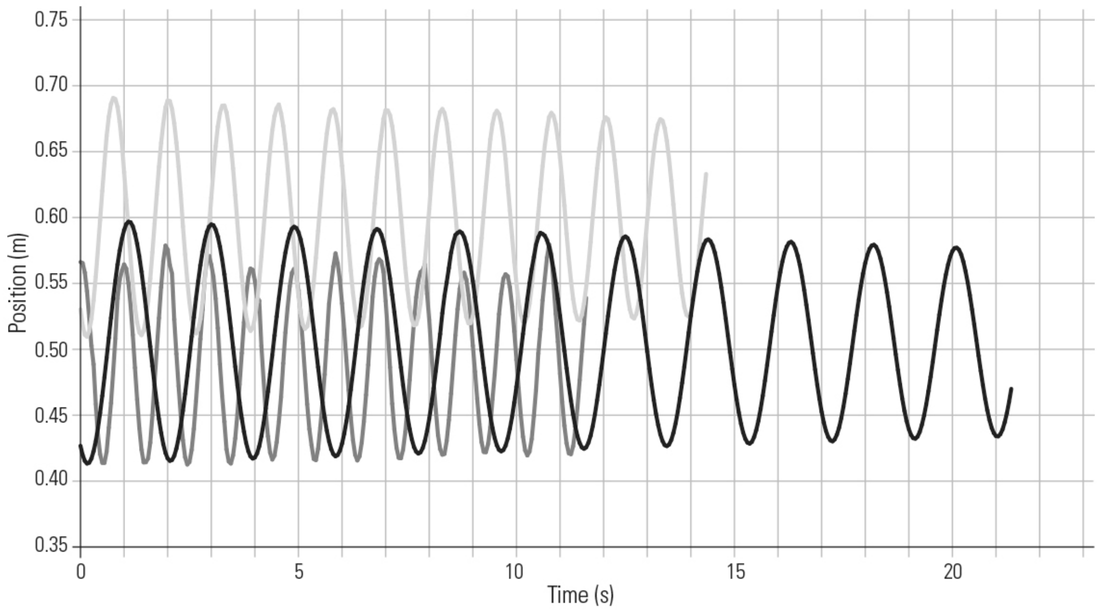


Table 3: Period of three springs with the same length but different spring constant

Spring	Time for 10 Cycles (s)	Spring Constant (N/m)	Average Period (s)	$\frac{1}{\sqrt{\text{Spring Constant}}}$ [(N/m)$^{-½}$]
1	19.0	3.33	1.90	0.548
2	12.6	7.91	1.26	0.356
3	9.7	13.5	0.97	0.272

5. Calculate the average period for each spring. Record your results for each trial in Table 3.

   Calculation using sample data for Spring 1:

   $$\text{Average period} = \frac{19.0\text{ s}}{10} = 1.90\text{ s}$$

6. Did changing the spring constant affect the period of the mass and spring system? Justify your answer.

   Student data will show that the period of an oscillating mass and spring system decreases as the spring constant decreases.

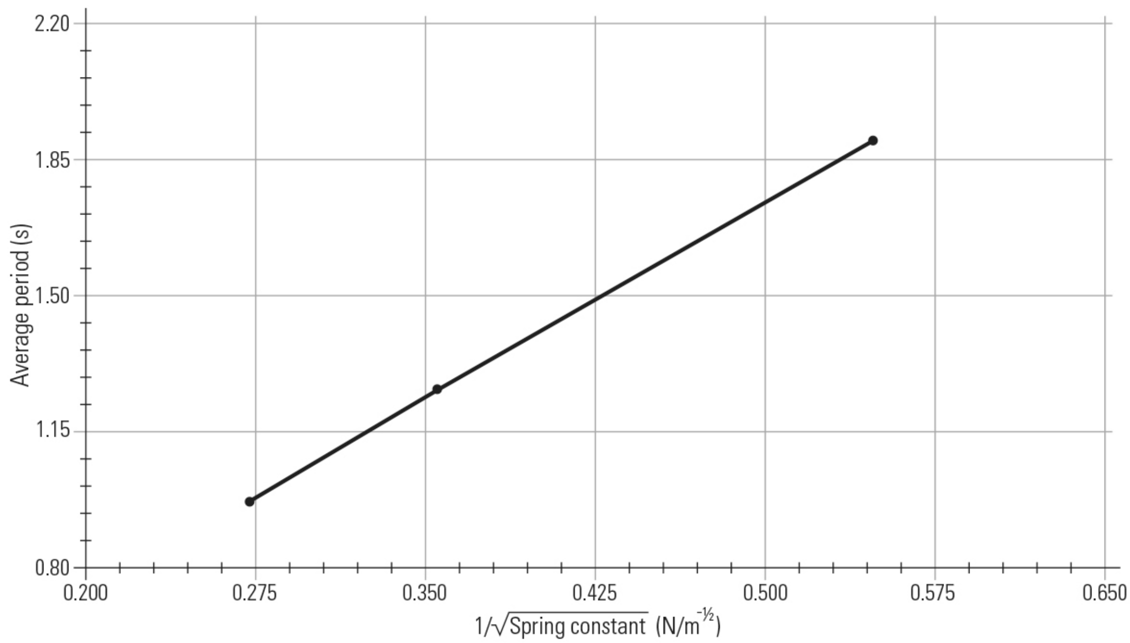
7. Calculate $1/\sqrt{\text{spring constant}}$ for each spring in Table 3. Record the results in Table 3.

   Calculation using sample data for Spring 1:

   $$\frac{1}{\sqrt{\text{spring constant}}} = \frac{1}{\sqrt{3.33\text{ N/m}}} = 0.548\ (\text{N/m})^{-½}$$

8. Plot a graph of *average period* versus $1/\sqrt{\textit{spring constant}}$ in the blank Graph 1 axes. Be sure to label both axes with the correct scale and units.

Graph 1: Average period versus $1/\sqrt{\text{spring constant}}$ for three mass and spring systems with constant spring length and mass



9. Based on Graph 1, what is the relationship between period and spring constant for an oscillating mass and spring system (proportional, inverse, squared, et cetera)? Justify your answer.

Students' graphs of period versus $1/\sqrt{\text{spring constant}}$ should be linear, which indicates that the period is proportional to the inverse square root of spring constant.

PART 4 – MASS AND PERIOD

Student Part 4 graphs of position versus time of five mass and spring systems with the same spring but varying mass will look similar to:

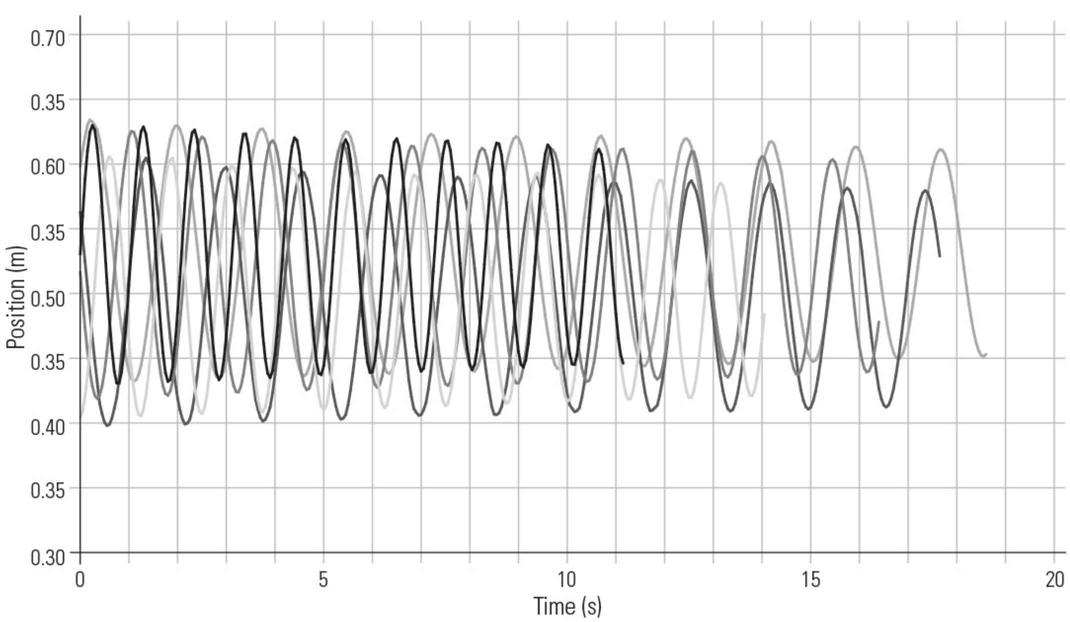


Table 4: Period of a mass and spring system with varying mass

Trial	Time for 10 Cycles (s)	Hanging Mass (kg)	Average Period (s)	$\sqrt{\text{Hanging Mass}}$ ($kg^{½}$)
1	10.4	0.200	1.04	0.447
2	12.6	0.300	1.26	0.548
3	14.4	0.400	1.44	0.632
4	16.0	0.500	1.60	0.707
5	17.5	0.600	1.75	0.775

10. Calculate the average period for each spring. Record your results for each trial in Table 4.

Calculation using sample data for Trial 1:

$$\text{Average period} = \frac{10.4\text{ s}}{10} = 1.04\text{ s}$$

11. Did changing the mass affect the period of the mass and spring system? Justify your answer.

Student data will show that the period of an oscillating mass and spring system increases as the amount of hanging mass increases.

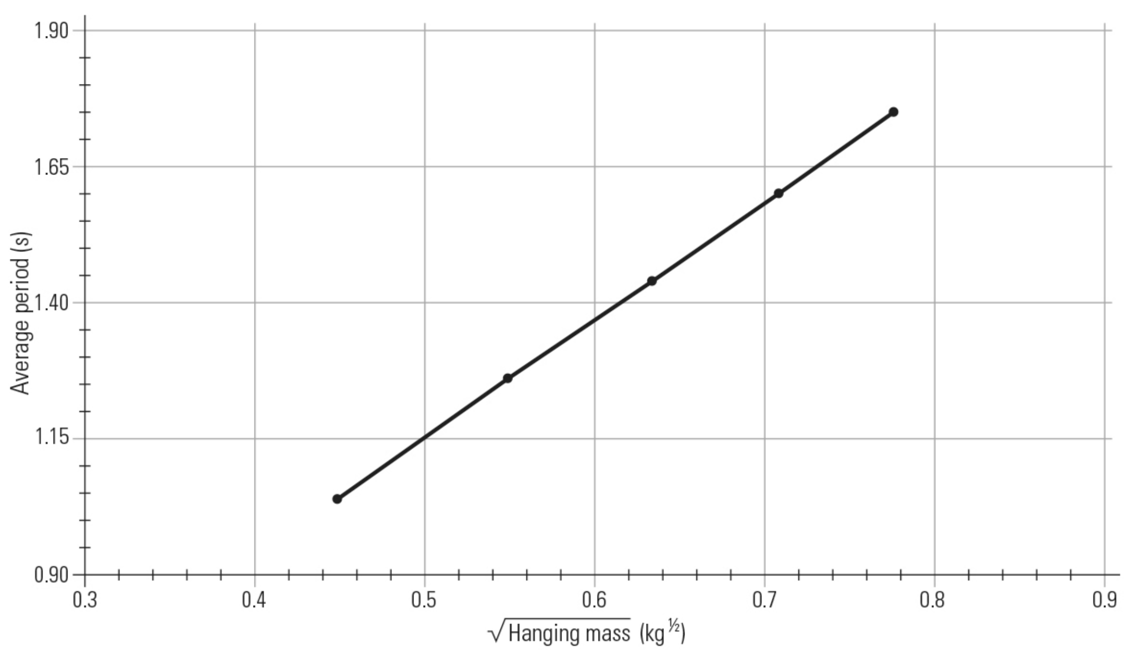
12. Calculate $\sqrt{\text{hanging mass}}$ for each trial in Table 4. Record the results in Table 4.

Calculation using sample data for Trial 1:

$$\sqrt{\text{hanging mass}} = \sqrt{0.200\text{ kg}} = 0.447\text{ kg}^{½}$$

13. Plot a graph of *average period* versus $\sqrt{\textit{hanging mass}}$ in Graph 2. Be sure to label both axes with the correct scale and units.

Graph 2: Average period versus $\sqrt{\text{hanging mass}}$ for a mass and spring system



14. Based on Graph 2, what is the relationship between period and mass for an oscillating mass and spring system (proportional, inverse, squared, et cetera)? Justify your answer.

Students' graphs of period versus $\sqrt{\text{hanging mass}}$ should be linear, which indicates that the period is proportional to the square root of the amount of hanging mass.

Guided Inquiry Questions

Below are sample responses to the Guiding Questions found in the Guided Inquiry version of this lab activity.

1. How do you plan on constructing your mass and spring system? Will the system be oriented horizontally or vertically? Explain your choice.

   Mass and spring systems should consist of a spring with one end fixed, and the other end attached to a mass that is free to move in one dimension that either compresses or extends the spring. These systems can be oriented horizontally or vertically, but a horizontal orientation is more likely to be subject to friction, which students should avoid. Many students will orient their systems vertically, which is perfectly acceptable; however, students must understand that their system equilibrium point will be in a different position than if it were oriented horizontally, due to the force from gravity acting on the hanging mass. The equilibrium position in this case will be the position at which the force from gravity is counteracted by the restoring force from the spring.

2. What are four physical properties of your mass and spring system that can be changed, that you believe will affect its oscillation period?

   Students' answers will vary. Below is a list of possible responses:

   - Amount of hanging mass
   - Diameter of spring
   - Length of spring
   - Compression versus extension of spring
   - Initial displacement of spring
   - Stiffness of spring (spring constant)

3. How do you plan on changing each of these properties within your experiment? Explain the process for changing each one.

   All descriptions should include details regarding how each property is varied either directly or indirectly. The independent variables chosen should be changed one at a time while all other variables are held constant: for example, vary the amount of hanging mass in each trial while measuring the period of oscillation, keeping the spring length, spring constant, and spring diameter constant in all trials.

4. How would you assemble the equipment from the materials provided in preparation for making measurements? Explain the important points regarding your setup and explain how you plan to measure the period of the system.

   Setups will vary. The period of oscillation can be measured using a PASCO Motion Sensor (as outlined in the Structured version of this lab activity) or other motion measuring device: students will measure the position of the system and then determine the time it takes the system to complete one cycle of motion (period). If students choose to use a stopwatch to make period measurements, they should pay close attention to the beginning and end of the system's motion cycle, being careful to start and stop at the exact moment the cycle starts and completes, respectively. To achieve greater accuracy in measurement when using a stopwatch, students can measure the time it takes to complete several or many motion cycles and then divide that time by the number of cycles.

5. The experimental process generally involves changing one variable, keeping others constant while recording data. For this experiment, which variables will you change and which variables will be held constant?

   Student responses will depend on which variables were chosen as independent variables in their experiment (refer to the Guided Inquiry Question above). Each independent variable chosen should be changed one at a time while all other variables are held constant. Examples of other variables that should be held constant are listed in the sample response above.

Assessment Questions: Sample Responses

Sample responses to the Analysis and Synthesis questions found in each version of the lab activity:

Analysis Questions

❷ 1. For each part of your experiment, list each variable involved and state whether it was held constant, increased, or decreased.

Answers for the Structured version of this activity:

Part 1—Independent variable: initial vertical displacement (increased); Dependent variable: system period (stayed constant); Variables held constant: spring length, hanging mass, spring constant.

Part 2—Independent variable: spring length (increased); Dependent variable: system period (stayed constant); Variables held constant: initial vertical displacement, hanging mass, and spring constant.

Part 3—Independent variable: spring constant (increased); Dependent variable: system period (decreased); Variables held constant: spring length, hanging mass, initial vertical displacement.

Part 4—Independent variable: hanging mass (increased); Dependent variable: system period (increased); Variables held constant: spring length, initial vertical displacement, spring constant.

Answers for the Guided Inquiry and Student Designed versions of this activity will vary depending on the independent variables tested.

❷ 2. In your experiment, what variables (physical properties) affected the period of a mass and spring system and how did they affect the period?

The only variables that affected the period of our mass and spring system were the amount of mass hung from the spring, and the spring constant. As the amount of hanging mass increased, so did the period of oscillation; as the spring constant increased, the period of oscillation decreased.

❷ 3. The mathematical equation describing the period T_s of a mass and spring system is

$$T_s = 2\pi\sqrt{\frac{m}{k}} \qquad (2)$$

where k is the spring constant of the spring, and m is the amount of hanging mass. Does your data support this mathematical relationship? Justify your answer.

Students' data should support this relationship. Graphs of period versus $1/\sqrt{\text{spring constant}}$ will show a linear relationship (proportional) that supports Equation 2; graphs of period versus $\sqrt{\text{mass}}$ will show a linear relationship (proportional) that also supports Equation 2.

Synthesis Questions

❷ 1. The motion of oscillating mass and spring systems follow cyclical patterns, so their motion is often described using sinusoidal functions with an angular velocity ω:

$$\omega = \frac{2\pi}{T_s} \qquad (3)$$

What is the period for a mass and spring system whose angular velocity is 6.28 rad/s? Show calculations and all work.

$$T_s = \frac{2\pi}{\omega} = \frac{2\pi}{6.28\text{ rad/s}} = 1.00\text{ s}$$

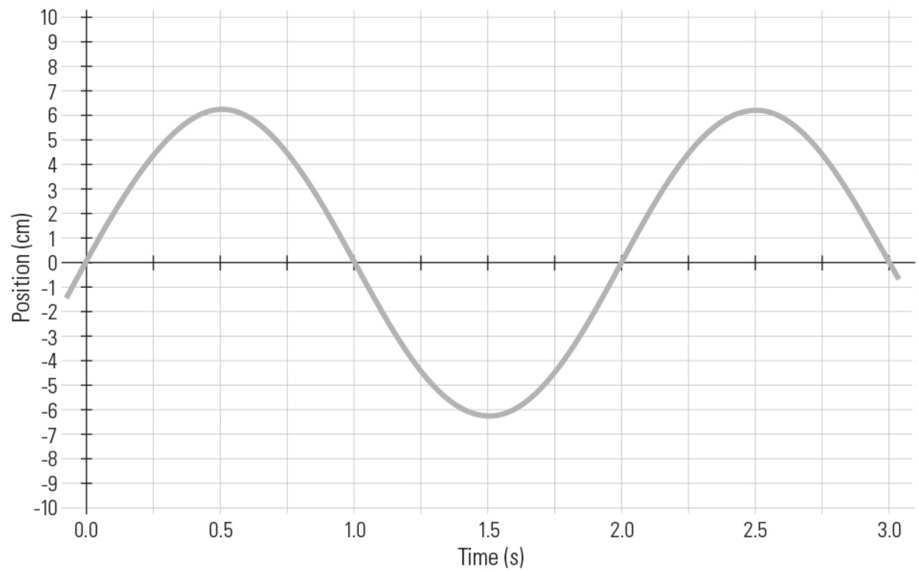
❷ 2. Use Equations 2 and 3 to derive a new expression for ω using just mass m and the spring constant k. Show your work here.

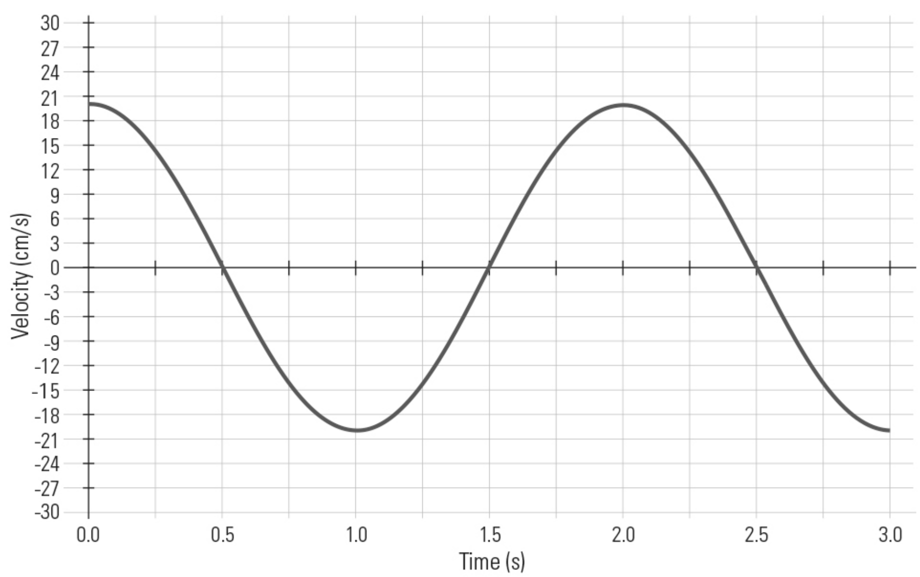
$\omega = \dfrac{2\pi}{T_s}$ and $T_s = 2\pi\sqrt{\dfrac{m}{k}}$ Combining and simplifying: $\omega = \dfrac{2\pi}{2\pi\sqrt{\dfrac{m}{k}}} = \sqrt{\dfrac{k}{m}}$

❷ 3. The position versus time graph below shows the motion of an oscillating mass and spring system. This graph can be described using the equation:

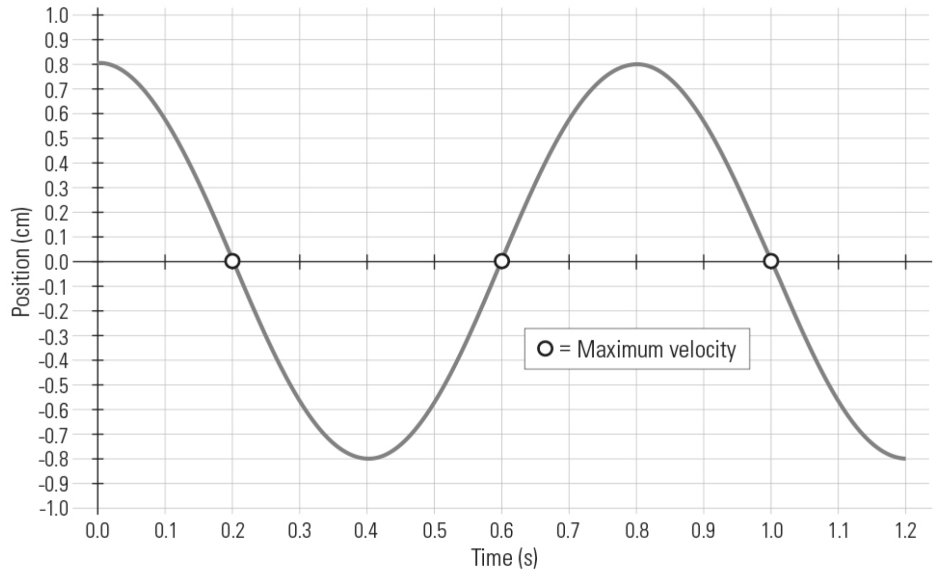
$$x(t) = A\sin\left(\frac{2\pi}{T_s}t\right)$$

where A is the maximum displacement of the mass from equilibrium (both positive and negative). Use your knowledge of the graphical connection between position versus time and velocity versus time graphs to sketch the system's corresponding velocity versus time graph in the blank axes below. Be sure to label both axes with a correct scale.





4. Sketch the position versus time graph for an oscillating mass and spring system whose position at time $t = 0$ is equal to its maximum displacement of 8.0 cm, and takes 0.80 seconds to complete one cycle of motion. Sketch as much of the graph that will fit in the blank axes below, and identify on your sketch the points at which the system has maximum velocity.



5. If the spring in the previous question has a spring constant of 25 N/m, what is the value of the mass? Show calculations and all work.

$T_s = 2\pi\sqrt{\frac{m}{k}}$ Square both sides: $T_s^2 = 4\pi^2\frac{m}{k}$

$m = \frac{kT_s^2}{4\pi^2} = \frac{(25\text{ N/m})(0.80\text{ s})^2}{4\pi^2} = 0.41\text{ kg}$

6. A given spring has a spring constant k and period T_s. If you doubled the mass, what would the new period T_s' be? (Show all work, and put T_s' in terms of T_s).

$T_s = 2\pi\sqrt{\frac{m}{k}}$

$T_s' = 2\pi\sqrt{\frac{2m}{k}} = 2\pi\sqrt{\frac{m}{k}}\cdot\left(\sqrt{2}\right) = T_s\sqrt{2}$

Extended Inquiry Suggestions

Have a discussion about the cyclical nature of objects undergoing simple harmonic motion (SHM). This links naturally with angular (circular) motion, a connection often made by imagining, or actually performing, a demonstration of an object going around on a turntable. If a bright light is projected horizontally on the object, what motion would the shadow of the object produce? (Answer: SHM; the same as a pendulum going back and forth in sync with the turntable.)

Another important link can be made to sinusoidal motion, which can be demonstrated with this example: Imagine a bucket full of sand hanging from a string and put into motion as a pendulum. If the bucket started leaking sand and you could slide a wide piece of paper at constant speed on the floor beneath the bucket, perpendicular to the plane of the pendulum, what pattern would the sand make? (Answer: sine curve). This can lead to making the link between sinusoidal motion and ω, as shown in Synthesis Questions 2 through 4.

12. PERIODIC MOTION: MASS AND SPRING

STRUCTURED

Driving Question | Objective

What variables affect the period of oscillation of a mass and spring system? Experimentally determine the physical properties of a hanging mass and spring system that affect its period of oscillation.

Materials and Equipment

- Data collection system
- Table clamp or large base
- Support rod, 60-cm or taller
- Support rod, 45-cm
- Right angle clamp
- Meter stick
- Hooked mass set
- PASCO Motion Sensor[1]
- Springs of similar size (diameter and length), but varying spring constant (3), 1–15 N/m
- Springs with similar spring constant and diameter, but of varying length (2), 0.1–0.3 m
- Tape

[1]www.pasco.com/ap18

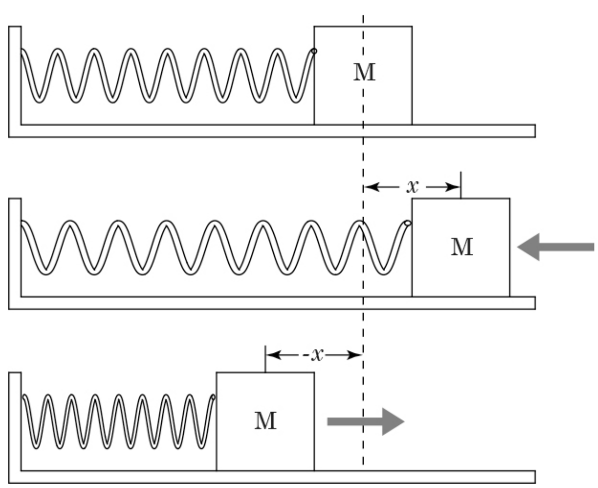
PASCO Motion Sensor

Background

It is easy to think of examples of oscillating objects in our physical world. A bungee jumper, the swinging pendulum of a clock, and the vibrating string of a guitar are all examples of oscillations around us. By definition, oscillating objects move back and forth in some form of repetitive motion, so they are *periodic,* which means that although the oscillations may be small or large, the time for each individual cycle of motion is the same.

As an object oscillates, it must pass through its original (or *undisturbed*) position, called the *equilibrium point*. The time for one complete cycle of oscillation is called the *period* and the maximum distance the oscillating object moves from the equilibrium point is referred to as the *amplitude*.

What makes these objects move in a repeating pattern? Let us look at an oscillating mass attached to a spring, sliding along a surface of negligible friction.



If the spring is not stretched, it does not apply any force to the mass. However, when the spring (with spring constant k) is stretched or compressed, displacing the mass some distance $\vec{x}$, it will apply a force $\vec{F}_s$ proportional to the displacement of the mass but in the opposite direction of that displacement (Hooke's Law):

$$\left|\vec{F}_s\right| = -k\left|\vec{x}\right| \tag{1}$$

After the mass is displaced, if left to move freely, it will accelerate toward its equilibrium point until it passes through that position, after which it will experience a restoring force applied by the spring, drawing it back toward the equilibrium position once again. This cycle repeats itself and the result is a periodic motion.

In this lab activity, you will explore what variables affect the period of oscillation of a mass and spring system; but unlike the example above, you will use a system oriented vertically to avoid external forces other than gravity acting on the system.

RELEVANT EQUATIONS

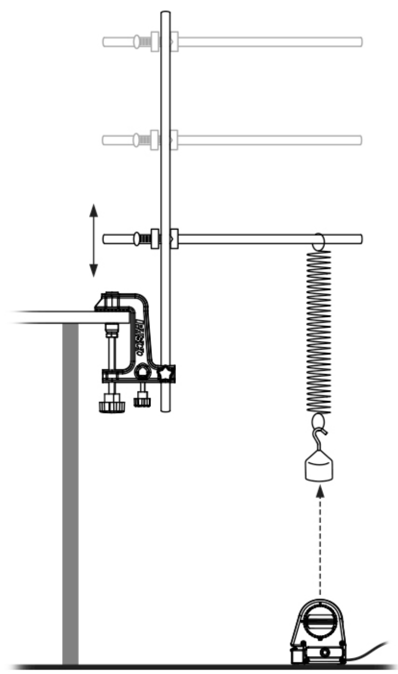
$$\left|\vec{F}_s\right| = -k\left|\vec{x}\right| \quad (1)$$

This equation states that a displaced spring will deliver a restoring spring force $\vec{F}_s$ proportional to the displacement $\vec{x}$ but in the opposite direction. The constant of proportionality k is known as the *spring constant*.

Procedure

Part 1 – Displacement and Period

SET UP



1. Mount the long support rod to the table clamp or large base, and then attach the smaller support rod at a right angle using the right angle clamp. The horizontal support rod should extend past the edge of the lab table.

2. Choose one spring and attach it to the horizontal support rod so it hangs beyond the edge of the lab table. Place a small piece of tape over the spring where it attaches to the rod to prevent it from sliding.

3. Hang 200 g of mass from the spring, and then place the motion sensor on the lab table or floor directly under the hanging mass with the front of the sensor aimed up at the bottom of the mass. Set the switch on the top of the sensor to the cart icon.

4. Adjust the height of the mass and spring so that the mass hangs, motionless, approximately 50 cm above the motion sensor.

5. Connect the motion sensor to the data collection system and then create a graph display of position versus time.

COLLECT DATA

6. Allow the mass to hang motionless from the spring, and then use the meter stick to note the height of the mass; this height is the equilibrium position of the mass and spring system.

7. Raise the mass 4 cm vertically from the equilibrium position and release it to oscillate freely.

8. After the mass has begun oscillating, start recording data.

9. Once the system has finished at least 10 complete oscillations, stop recording data.

10. Use the tools on your data collection system to determine the time needed to complete 10 oscillations. Record this value, as well as the initial vertical displacement, into Table 1 in the Data Analysis section.

11. Repeat the same data collection steps 2 more times, increasing the initial vertical displacement by an additional 4 cm each trial. Record the time for 10 complete cycles and the initial vertical displacement for each trial into Table 1.

Part 2 – Length and Period

SET UP

12. Choose two springs with the same spring constant but different lengths, and attach one to the horizontal support rod (after removing the spring used in Part 1) so it hangs beyond the edge of the lab table. Place a small piece of tape over the spring where it attaches to the rod to prevent it from sliding.

13. Hang 200 g of mass from the spring, and then adjust the position of the motion sensor so it is directly under the hanging mass with the front of the sensor aimed up at the bottom of the mass.

14. Adjust the height of the mass and spring so that the mass hangs, motionless, approximately 50 cm above the motion sensor.

COLLECT DATA

15. Allow the mass to hang motionless from the spring, and then use the meter stick to note the height of the mass; this height is the equilibrium position of the mass and spring system.

16. Raise the mass 10 cm vertically from the equilibrium position and release it to oscillate freely.

17. After the mass has begun oscillating, start recording data, and then stop after you have recorded at least 10 complete oscillation cycles.

18. Use the tools on your data collection system to determine the time needed to complete 10 oscillations. Record this value into Table 2 in the Data Analysis section.

19. Remove the mass from the spring and measure the unstretched length of the spring. Record this value into Table 2.

20. Repeat the same data collection steps with the second spring. Record the time for 10 complete cycles and the unstretched length of the second spring into Table 2.

Part 3 – Spring Constant and Period

SET UP

21. Choose three springs with the same length but different spring constants, and attach one to the horizontal support rod so it hangs beyond the edge of the lab table. Place a small piece of tape over the spring where it attaches to the rod to prevent it from sliding.

22. Hang 300 g of mass from the spring, and then adjust the position of the motion sensor so it is directly under the hanging mass with the front of the sensor aimed up at the bottom of the mass.

23. Adjust the height of the mass and spring so that the mass hangs, motionless, approximately 50 cm above the motion sensor.

COLLECT DATA

24. Allow the mass to hang motionless from the spring, and then use the meter stick to note the height of the mass; this height is the equilibrium position of the mass and spring system.

25. Raise the mass 10 cm vertically from the equilibrium position and release it to oscillate freely.

26. After the mass has begun oscillating, start recording data, and then stop after you have recorded at least 10 complete oscillation cycles.

27. Use the tools on your data collection system to determine the time needed to complete 10 oscillations. Record this value into Table 3 in the Data Analysis section.

28. Record the spring constant for the first spring in Table 3 (spring constant values will be provided by your teacher).

29. Repeat the same data collection steps with the other two springs. Record the time for 10 complete cycles and the spring constant for each spring into Table 3.

Part 4 – Mass and Period

SET UP

30. Choose one spring with a spring constant greater than 6 N/m, but less than 10 N/m, and attach it to the horizontal support rod so it hangs beyond the edge of the lab table. Place a small piece of tape over the spring where it attaches to the rod to prevent it from sliding.

31. Hang 200 g of mass from the spring, and then adjust the position of the motion sensor so it is directly under the hanging mass with the front of the sensor aimed up at the bottom of the mass.

32. Adjust the height of the mass and spring so that the mass hangs, motionless, approximately 50 cm above the motion sensor.

COLLECT DATA

33. Allow the mass hanging from the spring to become motionless, and then use the meter stick to note the height of the mass; this height is the equilibrium position of the mass and spring system.

34. Raise the mass 10 cm vertically from the equilibrium position and release it to oscillate freely.

35. After the mass has begun oscillating, start recording data, and then stop after you have recorded at least 10 complete oscillation cycles.

36. Use the tools on your data collection system to determine the time needed to complete 10 oscillations. Record this value and the amount of mass hanging from the spring in Table 4 in the Data Analysis section.

37. Repeat the same data collection steps 4 more times, increasing the amount of hanging mass by 100 g each trial. Record the time for 10 complete cycles and the hanging mass value for each trial into Table 4.

Data Analysis

Part 1 – Displacement and Period

Table 1: Period and displacement data for a mass and spring system

Trial	Time for 10 Cycles (s)	Initial Vertical Displacement (cm)	Average Period (s)
1			
2			
3			

1. Calculate the average period for each Part 1 trial in Table 1. Record your results for each trial in Table 1.

$$\text{Average period} = \frac{\text{Time for 10 cycles}}{10}$$

❓ 2. Did changing the displacement of the mass affect the period of the mass and spring system? Justify your answer.

____

____

____

Part 2 – Length and Period

Table 2: Period of two springs with the same spring constant but different length

Spring	Time for 10 Cycles (s)	Spring Length (cm)	Average Period (s)
1			
2			

3. Calculate the average period for each spring. Record your results in Table 2.

❓ 4. Did changing the length of the spring affect the period of the mass and spring system? Justify your answer.

____

____

____

Part 3 – Spring Constant and Period

Table 3: Period of three springs with the same length but different spring constant

Spring	Time for 10 Cycles (s)	Spring Constant (N/m)	Average Period (s)	$\frac{1}{\sqrt{\text{Spring Constant}}}$ $[(N/m)^{-1/2}]$
1				
2				
3				

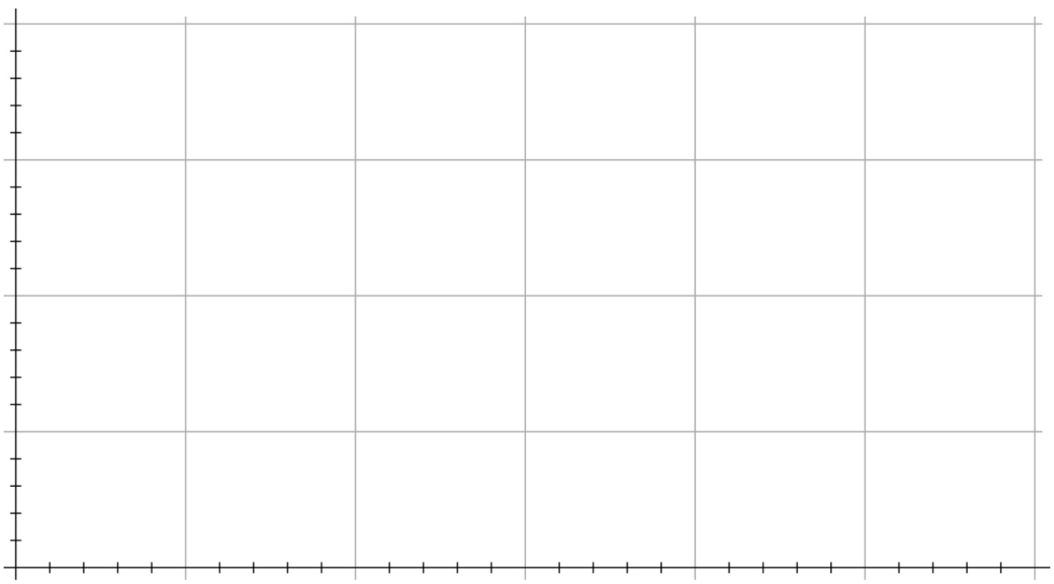
5. Calculate the average period for each spring. Record your results for each trial in Table 3.

6. Did changing the spring constant affect the period of the mass and spring system? Justify your answer.

7. Calculate $1/\sqrt{\text{spring constant}}$ for each spring in Table 3. Record the results in Table 3.

8. Plot a graph of *average period* versus $1/\sqrt{\textit{spring constant}}$ in the blank Graph 1 axes. Be sure to label both axes with the correct scale and units.

Graph 1: Average period versus $1/\sqrt{\text{spring constant}}$ for three mass and spring systems with constant length and mass



9. Based on Graph 1, what is the relationship between period and spring constant for an oscillating mass and spring system (proportional, inverse, squared, et cetera)? Justify your answer.

Part 4 – Mass and Period

Table 4: Period of a mass and spring system with varying mass

Trial	Time for 10 Cycles (s)	Hanging Mass (kg)	Average Period (s)	$\sqrt{\text{Hanging Mass}}$ ($kg^{1/2}$)
1				
2				
3				
4				
5				

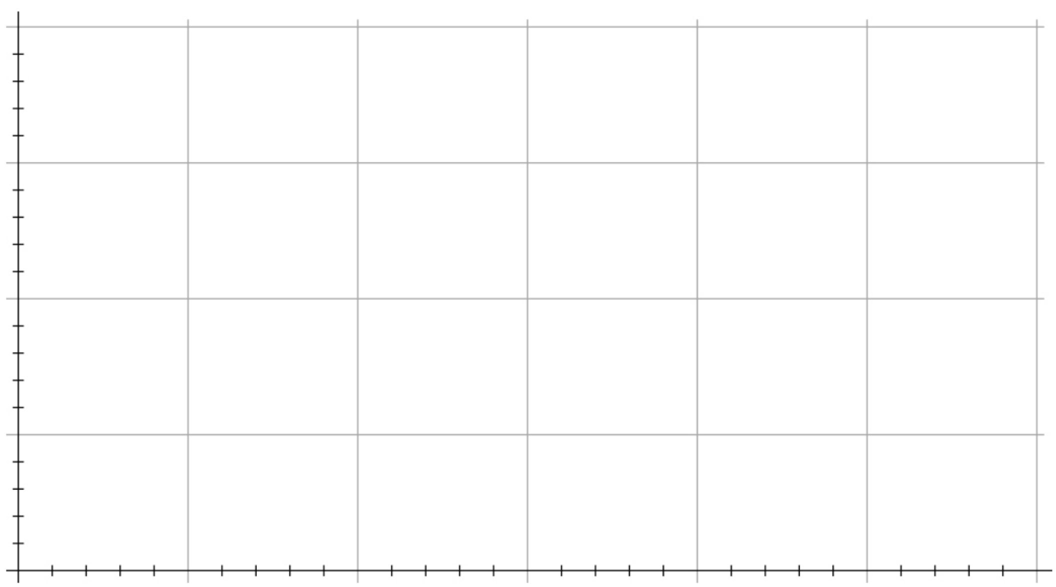
10. Calculate the average period for each spring. Record your results for each trial in Table 4.

11. Did changing the mass affect the period of the mass and spring system? Justify your answer.

12. Calculate $\sqrt{\text{hanging mass}}$ for each trial in Table 4. Record the results in Table 4.

13. Plot a graph of *average period* versus $\sqrt{\textit{hanging mass}}$ in Graph 2. Be sure to label both axes with the correct scale and units.

Graph 2: Average period versus $\sqrt{\text{hanging mass}}$ for a mass and spring system



14. Based on Graph 2, what is the relationship between period and mass for an oscillating mass and spring system (proportional, inverse, squared, et cetera)? Justify your answer.

Analysis Questions

1. For each part of your experiment, list each variable involved and state whether it was held constant, increased, or decreased.

2. In your experiment, what variables (physical properties) affected the period of a mass and spring system and how did they affect the period?

3. The mathematical equation describing the period T_s of a mass and spring system is:

$$T_s = 2\pi\sqrt{\frac{m}{k}} \tag{2}$$

where k is the spring constant of the spring, and m is the amount of hanging mass. Does your data support this mathematical relationship? Justify your answer.

Synthesis Questions

1. The motion of oscillating mass and spring systems follow cyclical patterns, so their motion is often described using sinusoidal functions with an angular velocity ω:

$$\omega = \frac{2\pi}{T_s} \tag{3}$$

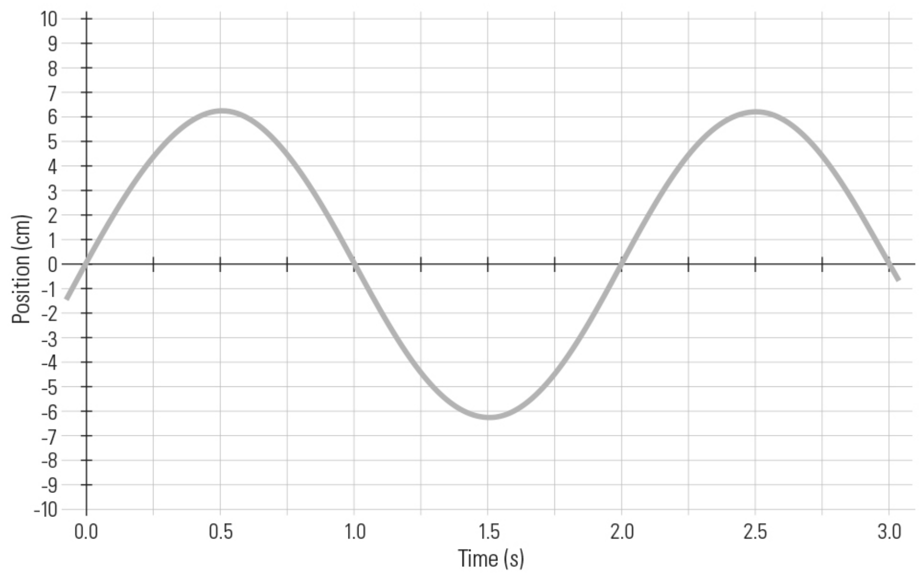
What is the period for a mass and spring system whose angular velocity is 6.28 rad/s? Show calculations and all work.

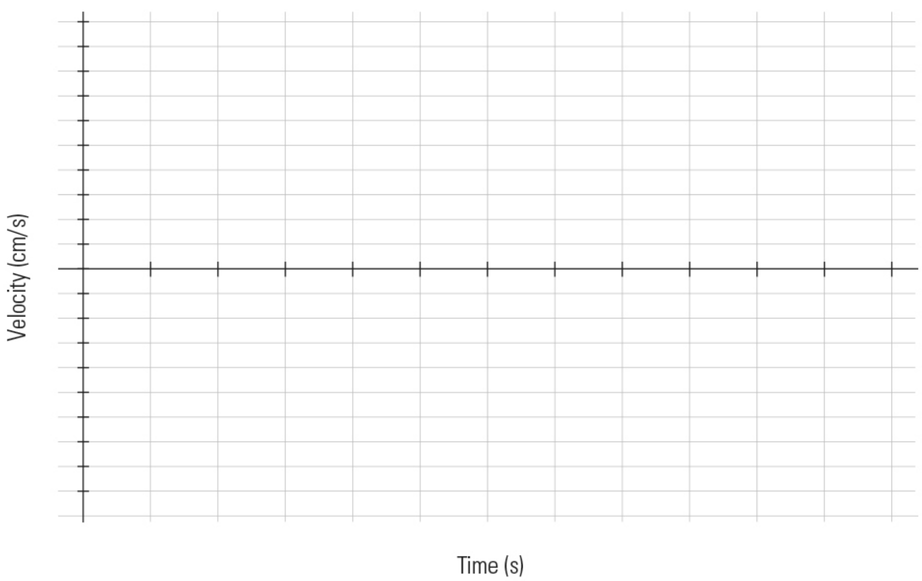
2. Use Equations 2 and 3 to derive a new expression for ω using just mass m and the spring constant k. Show your work here.

❓ 3. The position versus time graph below shows the motion of an oscillating mass and spring system. This graph can be described using the equation:

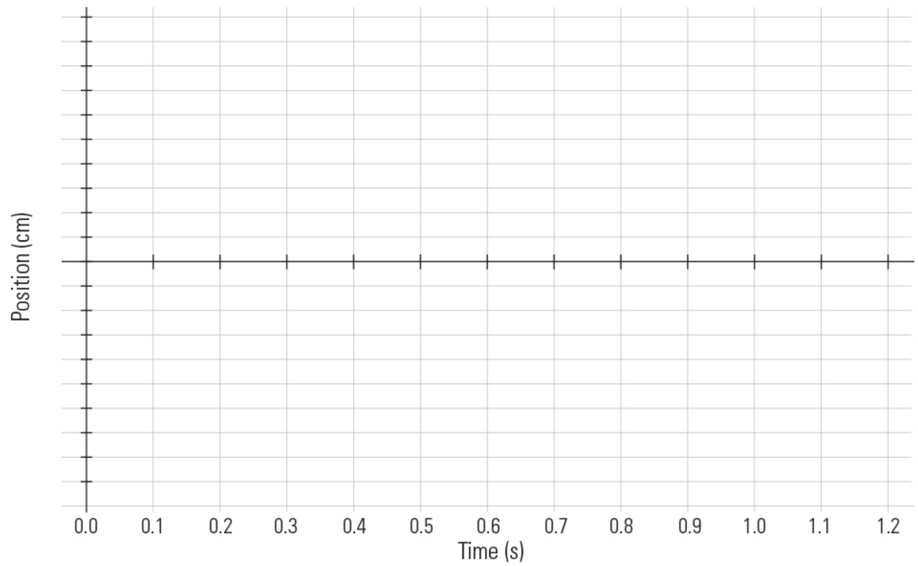
$$x(t) = A\sin\left(\frac{2\pi}{T_s}t\right)$$

where A is the maximum displacement of the mass from equilibrium (both positive and negative). Use your knowledge of the graphical connection between position versus time and velocity versus time graphs to sketch the system's corresponding velocity versus time graph in the blank axes below. Be sure to label both axes with a correct scale.





4. Sketch the position versus time graph for an oscillating mass and spring system whose position at time $t = 0$ is equal to its maximum displacement of 8.0 cm, and takes 0.80 seconds to complete one cycle of motion. Sketch as much of the graph that will fit in the blank axes below, and identify on your sketch the points at which the system has maximum velocity.



5. If the spring in the previous question has a spring constant of 25 N/m, what is the value of the mass? Show calculations and all work.

6. A given spring has a spring constant k and period T_s. If you doubled the mass, what would the new period T_s' be? (Show all work, and put T_s' in terms of T_s).

13. SIMPLE PENDULUM

Connections to the AP® Physics 1 Curriculum*

The lab activity correlates to the following pieces of the AP® Physics 1 framework:

Big Idea	3	Enduring Understanding	B	Essential Knowledge	3

Learning Objective 1: The student is able to predict which properties determine the motion of a simple harmonic oscillator and what the dependence of the motion is on those properties.
Science Practices: 6.4, 7.2

Learning Objective 2: The student is able to design a plan and collect data in order to ascertain the characteristics of the motion of a system undergoing oscillatory motion caused by a restoring force.
Science Practices: 4.2

Learning Objective 3: The student can analyze data to identify qualitative or quantitative relationships between given values and variables (i.e., force, displacement, acceleration, velocity, period of motion, frequency, spring constant, string length, mass) associated with objects in oscillatory motion to use that data to determine the value of an unknown.
Science Practices: 2.2, 5.1

Time Requirement

Preparation Time: 10 minutes

Lab Activity: 45 minutes

Prerequisites

Students should be familiar with the following concepts:

- The basic concept of periodic motion as a result of a restoring force.
- *Period* is a quantity that describes the time it takes a pendulum to complete one full cycle of motion.

Driving Question | Objective

What variables affect the period of a pendulum? Determine the physical properties of a simple pendulum that affect its period.

Procedural Overview

The Structured version of this lab activity is divided into three parts:

Part 1 – Students build a simple pendulum using a pendulum bob and thread and then measure the period T_p of their pendulum while increasing the horizontal displacement of the pendulum bob in each trial. Ten periods are measured in each trial and averaged. A plot of the average period versus displacement will show a straight horizontal line. From that, students are expected to recognize that the period is constant regardless of the magnitude of initial displacement of the pendulum.

Part 2 – Using the same setup as in Part 1, students measure the period of their pendulum while increasing the mass of the pendulum bob in each trial. Ten periods are measured in each trial and averaged. A plot of the average period versus mass will show a straight horizontal line. From that, students are expected to recognize that period is constant regardless of the magnitude of the pendulum bob mass.

Part 3 – Using the same setup as in Part 1, students measure the period of their pendulum while increasing the pendulum arm length (length of thread used). Ten periods are measured in each trial and averaged. A plot of the average period versus pendulum arm length will show a curved, or non-proportional, relationship establishing that the period is affected by the length of the pendulum arm. Students then use their data to calculate $\sqrt{\text{Pendulum arm length}}$. A plot of the average period versus $\sqrt{\text{Pendulum arm length}}$ will show a linear (proportional) relationship. Students are expected to recognize that the period is proportional to the square root of the pendulum arm length and use their data to support the actual mathematical relationship between period and pendulum arm length:

$$T_{\text{p}} = 2\pi\sqrt{\frac{l}{g}}$$

Pre-Lab Discussion and Activity

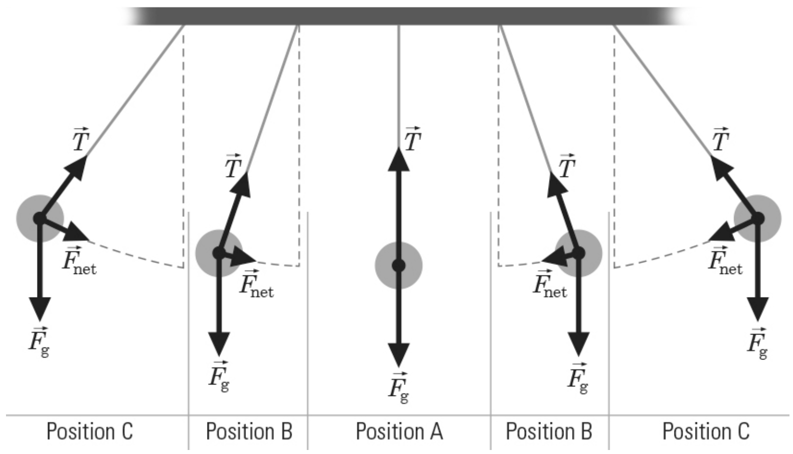
Students may benefit from a brief group discussion, based on the objective of this lab activity, that identifies some of the physical properties of a pendulum that can be chosen as independent variables in their experiment. It is recommended to list each variable identified by the class on the board. Some physical properties include (but are not limited to):

- Mass of the pendulum bob
- Volume of the pendulum bob
- Initial displacement of the pendulum bob
- Direction of the pendulum bob displacement
- Magnitude of the swing
- Pendulum bob shape (e.g. square versus round)
- Thickness of the pendulum arm
- Length of the pendulum arm
- Density of the pendulum arm
- Pendulum arm shape (e.g. square versus round)

Before students can understand the objective of this lab activity, they must first know the definition of *pendulum period*:

Pendulum period: The quantity that describes the time it takes a pendulum to complete one full cycle of motion.

Student may not be familiar with the term *cycle of motion*. Explain to students the motion of a simple pendulum and how its motion arises from the restoring force of gravity. Use the following diagram and descriptions to help identify the forces associated with a pendulum's motion and the cycle of motion that results:



Position A: This is the equilibrium position of the pendulum. If the pendulum were to sit motionless, this is where the tension *T* in the string and the force from gravity *mg* acting on the pendulum bob are equal, making the net force *F* acting on the bob zero.

Position B: As the pendulum is displaced from its equilibrium position, the tension in the string decreases while the force from gravity stays constant. The result: a non-zero net force *F* directed back toward the equilibrium position. This is the restoring force directing the pendulum bob back toward equilibrium.

Position C: The farther from the equilibrium position the pendulum bob is displaced, the greater the restoring force (net force) *F* directed toward the equilibrium position.

If left to swing freely under these forces, the pendulum would experience cyclical motion in which the pendulum would swing to the left and to the right, repeating the same path each time. The greater the magnitude of swing in either direction, the greater the restoring force directed toward the equilibrium.

The term *cycle of motion* describes the motion associated with one complete back-and-forth cycle, starting and ending at the same position, which, assuming the simple pendulum is a closed system, will repeat indefinitely. The time it takes for the pendulum to complete this cycle is known as the *pendulum period*.

Materials and Equipment

- Data collection system
- PASCO Smart Gate photogate[1]
- PASCO Photogate Pendulum Set[2]
- PASCO Pendulum Clamp[3]
- Balance, 0.1-g resolution, 2,000-g capacity (1 per class)
- Meter stick
- Table clamp or large base
- Support rod, 60-cm or taller
- Thread
- Scissors

Probeware Resources

Below are web-link and QR codes that will direct you to instructional video resources for individual pieces of PASCO probeware, sensors, and other hardware used in the lab activity. These same links and codes are provided to students in their activity handouts.

[1]www.pasco.com/ap21	[2]www.pasco.com/ap10	[3]www.pasco.com/ap15
PASCO Smart Gate	PASCO Photogate Pendulum Set	PASCO Pendulum Clamp

Teacher Tips

Tip 1 – Using Older Photogates

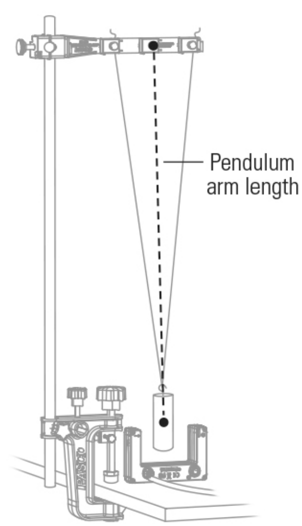
- The setup and procedure outlined in the Structured version of this lab activity utilizes a PASCO Smart Gate but an older PASCO photogate will work seamlessly in its place. If your data collection system has digital ports, the digital adapter is not needed: the photogate can be connected directly to the digital ports on the data collection system.

Tip 2 – Keep Displacement Under 15°

- For students to record consistent data, all trials should be conducted keeping the angular displacement of the pendulum less than 15°. The equation describing the constant period of a simple pendulum was derived using the assumption that the maximum angular displacement of the pendulum is small. Violating this may cause measurements to drift as the displacement angle decreases during data collection.
- Students can prevent large angular displacements by increasing the length of their pendulum arm. The longer the pendulum arm, the better.

Tip 3 – Double-String Pendulum

- For simplicity, the procedure outlined in the structured version of this lab activity describes building a simple pendulum using a bob hung from a single piece of string. Although this is adequate to record consistent and accurate data, the path of the pendulum may drift, making it difficult to keep its swing between the arms of the photogate. Students can correct this by hanging the pendulum from two equal-length strings similar to the picture on the right.



Tip 4 – Measuring Pendulum Arm Length

- When students measure pendulum arm length, they should measure the distance from the point at which the string attaches to the pendulum clamp to the center of mass of their pendulum bob (or where they believe the center of mass is). If students are using a double string pendulum, they should measure to the bob's center of mass from the point half way between where the strings attach to the pendulum clamp.

Tip 5 – Avoid a Physical Pendulum

- Students performing the Guided Inquiry or Student Designed version of this lab activity should avoid using different pendulum arm materials as their independent variable. Using a pendulum arm material that is relatively too massive compared to the pendulum bob will cause the pendulum to no longer be "simple," but rather, it becomes a *physical pendulum* which has a different, and more complicated, relationship than that shown in Equation 1.

Sample Data

Below are sample data, acquired using the experimental setup and procedure outlined in the Structured version of the lab activity, and answers to questions in the Data Analysis section.

Data Analysis

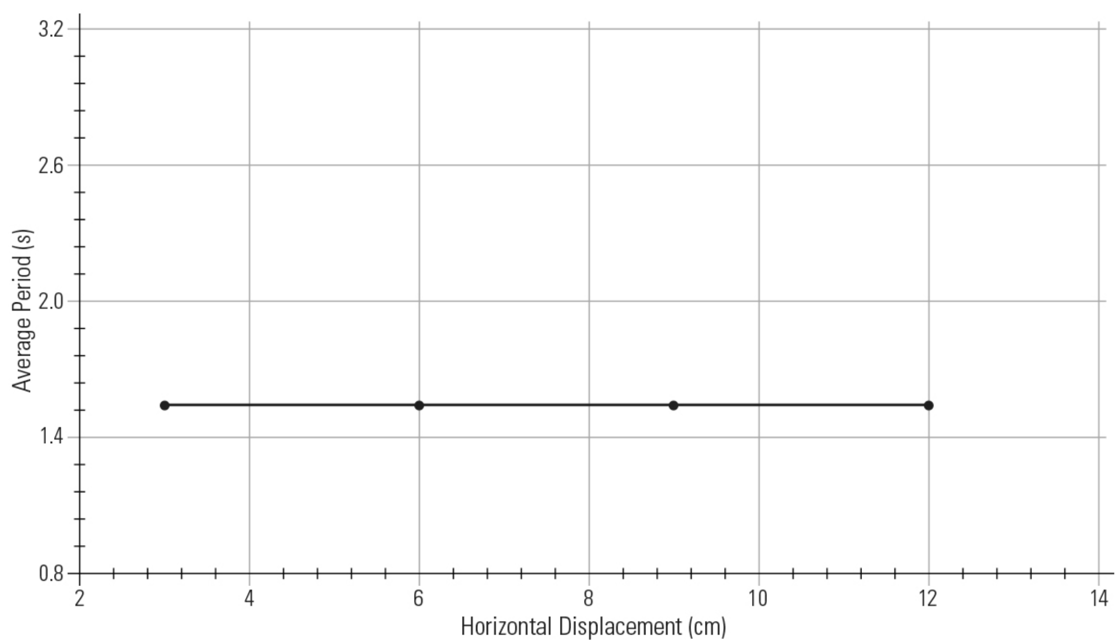
PART 1 – DISPLACEMENT AND PERIOD

Table 1: Period of a pendulum with varying horizontal displacement

Trial	Horizontal Displacement (cm)	Average Period (s)
1	3	1.53
2	6	1.53
3	9	1.53
4	12	1.53

1. Plot a graph of *average period* versus *horizontal displacement* in Graph 1. Be sure to label both axes with the correct scale and units.

Graph 1: Period versus displacement of a simple pendulum with constant length and mass



2. Did changing the displacement of the pendulum bob affect the period of the simple pendulum? Justify your answer.

   No. The period of a simple pendulum should not be affected by any changes to the starting angular displacement, unless that displacement exceeds approximately 15°.

   Students should conclude that the period is unaffected by changes to initial angular displacement by observing the flat horizontal shape of the average period versus displacement graph.

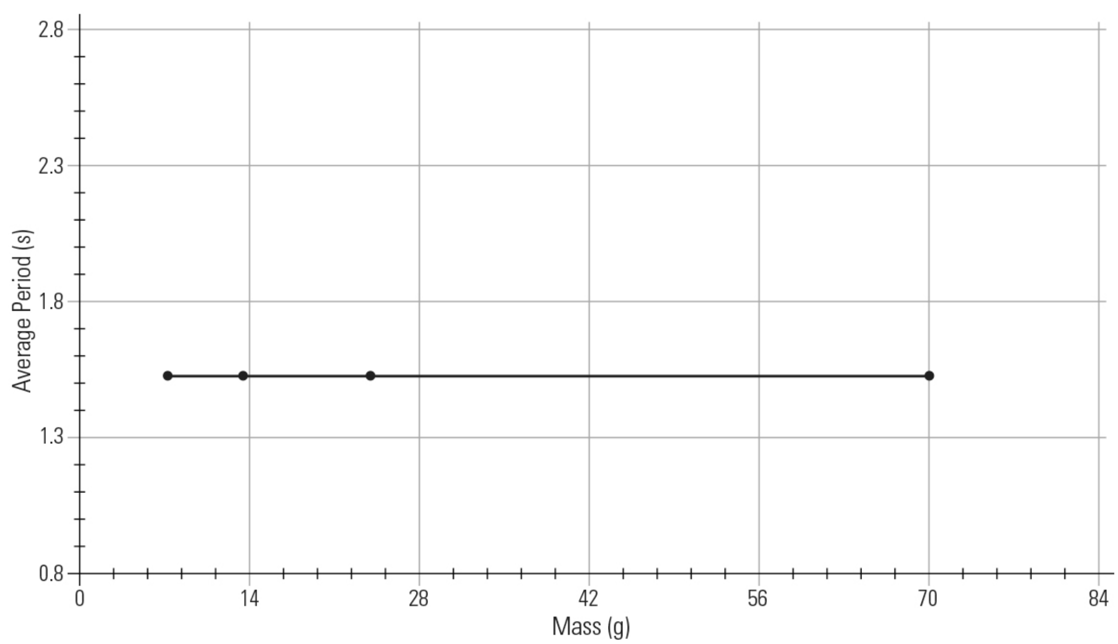
PART 2 – MASS AND PERIOD

Table 2: Period of a pendulum with varying mass

Trial	Bob Mass (g)	Average Period (s)
1	7.3	1.53
2	13.5	1.53
3	24.0	1.53
4	70.0	1.53

3. Plot a graph of *average period* versus *bob mass* in Graph 2. Be sure to label both axes with the correct scale and units.

Graph 2: Period versus mass of a simple pendulum with constant length and displacement



4. Did changing the mass of the pendulum bob affect the period of the simple pendulum? Justify your answer.

   No. The period of a simple pendulum should not be affected by any changes to the mass of the pendulum bob. Students should determine this by observing the flat horizontal shape of their average period versus pendulum bob mass graph.

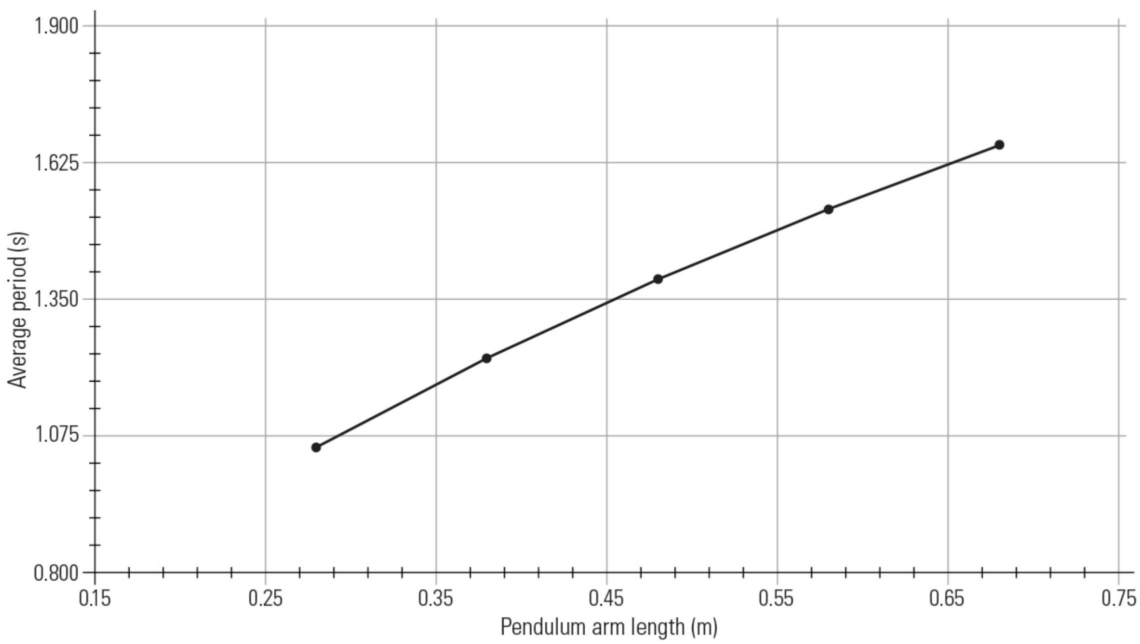
PART 3 – ARM LENGTH AND PERIOD

Table 3: Period of a pendulum with varying arm length

Trial	Pendulum Arm Length (m)	Average Period (s)	$\sqrt{\text{Pendulum Arm Length}}$ ($m^{1/2}$)
1	0.680	1.66	0.825
2	0.580	1.53	0.762
3	0.480	1.39	0.693
4	0.380	1.24	0.616
5	0.280	1.06	0.529

5. Plot a graph of *average period* versus *pendulum arm length* in Graph 3. Be sure to label both axes with the correct scale and units.

Graph 3: Period versus arm length of a simple pendulum with constant displacement and mass



6. Did changing the length of the pendulum arm affect the period of the simple pendulum? Justify your answer.

   Yes. The period of a simple pendulum should ONLY be affected by changes to the length of the pendulum arm. Students should conclude this by observing that their graph of average period versus pendulum arm length is not a flat horizontal line. Students may also note that the graph is non-linear, indicating that period and pendulum arm length are not proportional to each other.

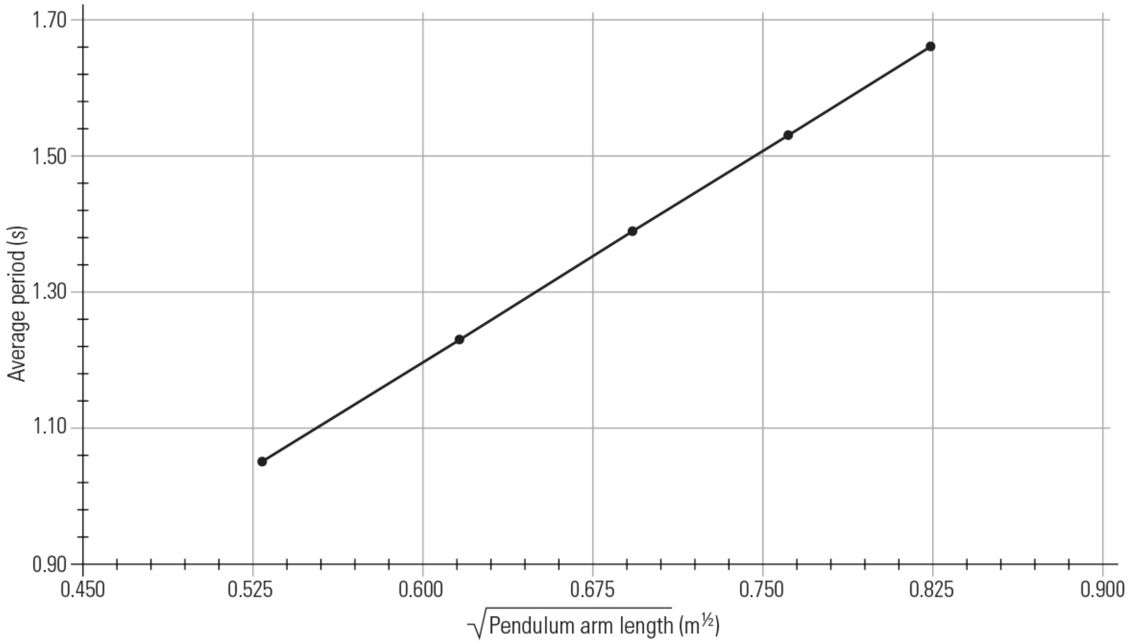
7. Calculate the square root of the pendulum arm length from Table 3. Record the result for each trial in Table 3.

   Calculation using sample data for Trial 1:

   $\sqrt{\text{Pendulum arm length}} = \sqrt{0.680\text{ m}} = 0.825\text{ m}^{1/2}$

8. Plot a graph of *average period* versus $\sqrt{\textit{pendulum arm length}}$ in Graph 4. Be sure to label both axes with the correct scale and units.

Graph 4: Average period versus $\sqrt{\text{pendulum arm length}}$ of a simple pendulum with constant displacement and mass



9. Based on Graph 4, what is the relationship between period and pendulum arm length for a simple pendulum (proportional, inverse, squared, et cetera)? Justify your answer.

   Students' graphs of average period versus the square root of pendulum arm length should be linear, which indicates that the period is proportional to the square root of pendulum arm length.

Guided Inquiry Questions

Below are sample responses to the Guiding Questions found in the Guided Inquiry version of this lab activity.

1. How do you plan on constructing your simple pendulum? What will the bob be made out of, what will the pendulum arm be made out of, and how will you anchor the pendulum arm?

   Students should choose a relatively massive, but uniform, object to use as their pendulum bob. PASCO's photogate pendulum set offers four cylindrical uniform pendulum bobs with varying mass. String or thread should be used as the pendulum arm, anchored to the PASCO pendulum clamp using the string clips attached to the clamp. Do not use string or other pendulum arms with high mass per unit length.

2. What are three physical properties of your pendulum that can be changed, that you believe will affect the period of the pendulum?

   Students answers will vary. Below is a list of possible responses:

   - Mass of the pendulum bob
   - Volume of the pendulum bob
   - Initial displacement of the pendulum bob
   - Direction of the pendulum bob displacement
   - Magnitude of the swing
   - Shape of the pendulum bob (e.g. square versus round)
   - Thickness of the pendulum arm
   - Length of the pendulum arm
   - Density of the pendulum arm
   - Shape of the pendulum arm (e.g. square versus round)

3. How do you plan on changing each of these properties within your experiment? Explain the process for changing each one.

   All descriptions should include details regarding how each property is varied either directly or indirectly. The independent variables chosen should be changed one at a time while all other variables are held constant: for example, vary the mass of the pendulum bob in each trial while measuring the pendulum period, keeping the pendulum arm length, initial angular displacement, and the size (volume) of the bob constant in all trials.

4. How would you assemble the equipment from the materials provided in preparation for making measurements? Explain the important points regarding your setup, and explain how you plan to measure the period of the pendulum.

   Setups will vary. Pendulum period can be measured directly using a PASCO photogate or other timing device. If students choose to use a stopwatch to make period measurements, they should pay close attention to the beginning and end of the pendulum's motion cycle, being careful to start and stop at the exact moment the cycle starts and completes, respectively. To achieve greater accuracy in measurement when using a stopwatch, students can measure the time it takes to complete several or many motion cycles and then divide that time by the number of cycles.

5. The experimental process generally involves changing one variable, keeping others constant while recording data. For this experiment, which variables will you change and which variables will be held constant?

   Student responses will depend on which variables were chosen as independent variables in their experiment (refer to the Guided Inquiry Question above). Each independent variable chosen should be changed one at a time while all other variables are held constant. Examples of other variables that should be held constant are listed in the sample response above.

6. When making measurements of period, the pendulum should not be displaced by more than ~15° before releasing it. Displacements greater than 15° may cause the period to be inconsistent. How does this affect your experiment design?

   To accommodate this requirement, students may need to use a longer pendulum arm to allow for a smaller angular displacement while achieving a greater absolute horizontal displacement. Using a longer pendulum will also produce longer period measurements, which will help produce more accurate data based on the timing device used to measure the period.

Assessment Questions: Sample Responses

Sample responses to the Analysis and Synthesis questions found in each version of the lab activity:

Analysis Questions

1. For each part of your experiment, list each variable involved and state whether it was held constant, increased, or decreased.

   Answers for the Structured version of this activity:

   Part 1—Independent variable: initial horizontal displacement (increased); Dependent variable: pendulum period (stayed constant); Variables held constant: pendulum arm length, bob mass, and bob shape and size.

   Part 2—Independent variable: pendulum bob mass (increased); Dependent variable: pendulum period (stayed constant); Variables held constant: pendulum arm length, bob shape and size, and initial angular displacement.

   Part 3—Independent variable: pendulum arm length (decreased); Dependent variable: pendulum period (decreased); Variables held constant: initial angular displacement, bob mass, and bob shape and size.

   Answers for the Guided Inquiry and Student Designed versions of this activity will vary depending on the independent variables tested.

2. In your experiment, what variables (physical properties) affected the period of a simple pendulum and how did they affect the period?

   The only variable that affected the period of our simple pendulum was the length of the pendulum arm. As the pendulum arm length decreased, so did the period of the pendulum.

3. The mathematical equation describing the period T_p of a pendulum is:

$$T_p = 2\pi\sqrt{\frac{l}{g}} \tag{2}$$

where l is the length of the pendulum arm and g is earth's gravitational acceleration constant. Does your data support this mathematical relationship? Justify your answer.

Our graph supports this relationship: the average period versus the square root of the pendulum arm length is linear, which indicates that the period is proportional to the square root of pendulum arm length, the mathematical relationship shown in Equation 2.

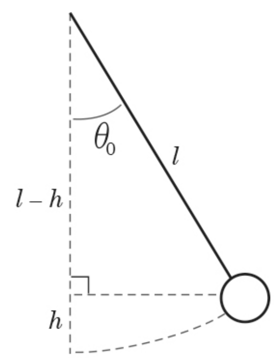
Synthesis Questions

1. In 1851, French physicist Leon Foucault created an enormous pendulum designed to demonstrate the rotation of the earth. The pendulum consisted of a 28.0 kg bob hung from a 67.0 m cable in the Pantheon in Paris, France. Given this information, what is the period of oscillation for the pendulum? Show your work.

$$T_p = 2\pi\sqrt{\frac{l}{g}}$$

$$T_p = 2\pi\sqrt{\frac{67.0\text{ m}}{9.8\text{ m/s}^2}} = 16\text{ s}$$

2. If the maximum displacement θ_0 of Foucault's pendulum bob while in motion is 0.0149 rad, what is the maximum linear speed of the bob during its motion? What is the maximum momentum of the bob? Show your work.



$$mgh = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh}$$

$$\cos\theta = \frac{l-h}{l};\ h = l - l\cos\theta$$

Replacing h and solving for the maximum linear speed:

$$v = \sqrt{2g(l - l\cos\theta)}$$

$$v = \sqrt{2(9.8\text{ m/s}^2)((67.0\text{ m}) - (67.0\text{ m})\cos(0.0149))} = 0.38\text{ m/s}$$

Then the maximum momentum of the bob is

$$p = mv = (28.0\text{ kg})(0.38\text{ m/s}) = 11\text{ N}\cdot\text{s}$$

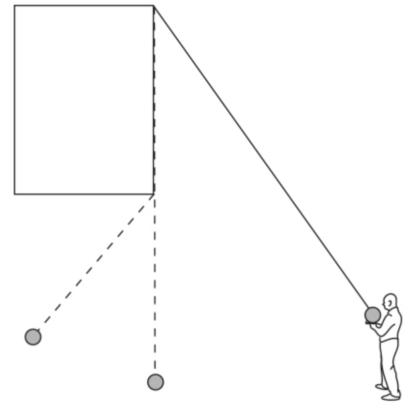
3. One of two identical grandfather pendulum clocks from Earth is placed on Mars. If the grandfather clock on Mars is 23 minutes behind the grandfather clock on Earth after 1 hour, what is the acceleration due to gravity on Mars?

The grandfather clock on Mars has advanced 37 minutes while the clock on Earth has advanced 60 minutes, so the period of the pendulum on Earth must be less than the period of the pendulum on Mars:

$$\frac{37\text{ min}}{60\text{ min}} = \frac{T_{pE}}{T_{pM}}$$

$$0.62 = \frac{2\pi\sqrt{\frac{l}{g_E}}}{2\pi\sqrt{\frac{l}{g_M}}} = \sqrt{\frac{g_M}{g_E}}$$

$$g_M = (0.62)^2 g_E = (0.38)(9.8\text{ m/s}^2) = 3.7\text{ m/s}^2$$



4. The figure to the right shows a person holding a 4-kg pendulum bob hanging from a 6.0-m long cable. The cable is attached to the side of an over-hanging wall 3.0 m up from the bottom of the wall. If the person lets go of the bob, how long will it take for it to swing back to him?

T_{p1} is the period of the pendulum hanging from a 6.0 m cable; T_{p2} is the period of the pendulum once it hits the wall, so then the pendulum is hanging from a 3.0 m cable, but since the pendulum only completes 1/2 of T_{p1} and 1/2 of T_{p2}, $T_{p\ total}$ is calculated as follows:

$$T_{\text{p total}} = \frac{1}{2}\left(T_{\text{p1}} + T_{\text{p2}}\right)$$

$$T_{\text{p total}} = \frac{\pi}{\sqrt{g}}\left(\sqrt{l_1} + \sqrt{l_2}\right)$$

$$T_{\text{p total}} = \frac{\pi}{\sqrt{9.8\ \text{m/s}^2}}\left(\sqrt{6.0\ \text{m}} + \sqrt{3.0\ \text{m}}\right) = 4.2\ \text{s}$$

Extended Inquiry Suggestions

A good challenge for students after they have completed the lab activity is to have them use their average period data to determine an experimental value for earth's gravitational acceleration. The simplest way to do this is for students to apply a linear curve fit regression to their graph of average period versus square root of pendulum arm length. The slope of this graph will be equal to $\text{slope} = 2\pi/\sqrt{g}$, which makes $g = 4\pi^2/\text{slope}^2$.

Similarly, students can be given a pendulum of unknown length and asked to determine its length using only measurements of period. Inserting period measurements into the following calculation yields an experimental value for pendulum arm length:

$$l = \frac{gT_{\text{p}}^{\ 2}}{4\pi^2}$$

13. SIMPLE PENDULUM

STRUCTURED

Driving Question | Objective

What variables affect the period of a pendulum? Determine the physical properties of a simple pendulum that affect its period.

Materials and Equipment

- Data collection system
- PASCO Smart Gate photogate[1]
- PASCO Photogate Pendulum Set[2]
- PASCO Pendulum Clamp[3]
- Balance, 0.1-g resolution, 2,000-g capacity (1 per class)
- Meter stick
- Table clamp or large base
- Support rod, 60-cm or taller
- Thread
- Scissors

[1]www.pasco.com/ap21 — PASCO Smart Gate

[2]www.pasco.com/ap10 — PASCO Photogate Pendulum Set

[3]www.pasco.com/ap15 — PASCO Pendulum Clamp

Background

How is a playground swing like a pendulum? Its motion is periodic; the swing's motion is repeated in equal intervals of time. What produces this repetitive motion? The motion of both a swing and a pendulum is the product of a variable force acting on them, called a *linear restoring force*. This force is proportional to the horizontal distance x the swing is displaced from its equilibrium position.

$$\left|\vec{F}\right| = -k\left|\vec{x}\right|$$
$$\left|\vec{F}\right| = -\left(\frac{mg}{l}\right)\left|\vec{x}\right| \qquad (1)$$

For a pendulum, m is the mass of the pendulum "bob," g is earth's gravitational acceleration constant, and l is the length of the pendulum arm.

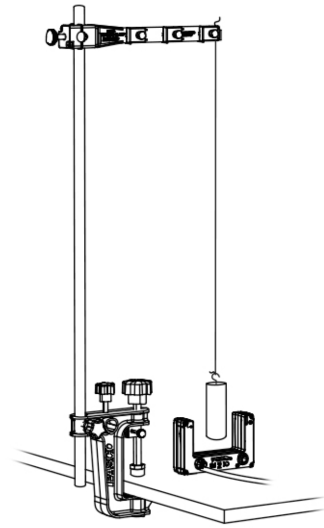
The equilibrium position for a pendulum is defined as the point at which the net force (and torque) acting on the pendulum bob is zero. For the swing, the equilibrium position is the point at which the force from gravity is counteracted completely by the tension from the swing's chains. This is where the swing hangs motionless below its point of anchor. When a swing or pendulum is displaced from its equilibrium position, the restoring force then acts on the swing, inducing motion back toward the equilibrium position. For a pendulum, the speed of the bob increases as it approaches the equilibrium position, at which point it achieves maximum speed.

Once the bob passes through the equilibrium position, its speed decrease as it moves up, away from the equilibrium position, until it stops at the peak of its swing. At this point the bob begins to fall back toward the equilibrium position, again subject to the same restoring force, and the changing speed cycle starts over.

Procedure

Part 1 – Displacement and Period

SET UP



1. Mount the pendulum clamp near the top of the rod and then cut a length of thread about 10 cm longer than the distance between the pendulum clamp and the top of the lab table.

2. Choose one of the four pendulum bobs from the PASCO Photogate Pendulum Set (all have identical volume but different mass) and tie one of the loose ends of the thread to the hook on the bob.

3. Hang the pendulum from the third anchor point on the clamp (farthest from the rod): loosen the anchor's thumbscrew and run the thread under the anchor. Tighten the thumbscrew to hold the thread in place.

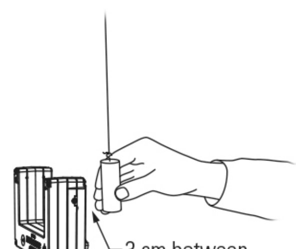
4. Adjust the pendulum arm length (thread length) so the bob hangs approximately 4 cm above the lab table.

5. With the pendulum bob hanging motionless, place the photogate on the lab table with the arms of the photogate pointed upward, directly under the pendulum bob. The pendulum bob should swing freely between the arms on the photogate.

6. Connect the photogate to your data collection system.

7. Configure the data collection system to measure the period of a pendulum using the photogate and then create a table display with the period measurement as one of the columns.

COLLECT DATA



8. Use your hand to pull the pendulum bob back, displacing it a horizontal distance of 3 cm from its equilibrium position. Use the meter stick to measure the horizontal displacement.

9. Begin recording data, and then release the pendulum bob so it swings freely through the photogate.

10. Stop recording data when the data collection system has recorded 10 period measurements.

11. Use the tools on your data collection system to determine the average of the 10 period data points. Record this average value as well as the horizontal displacement into Table 1 in the Data Analysis section.

12. Repeat the same collect data steps 3 more times, increasing the horizontal displacement by an additional 3 cm each trial. Record your average period and displacement values for each trial into Table 1.

Part 2 – Mass and Period

COLLECT DATA

13. Use the Part 1 setup for Part 2.

14. Remove the pendulum bob currently attached to the thread and then measure the individual mass of all four pendulum bobs. Record the mass in order from smallest to largest in Table 2 in the Data Analysis section.

15. Attach the pendulum bob with the smallest mass to the thread.

16. Use your hand to pull the pendulum bob back, displacing it a horizontal distance of 6 cm from its equilibrium position. Use the meter stick to measure the horizontal displacement.

17. Begin recording data, and then release the pendulum bob so it swings freely through the photogate.

18. Stop recording data when the data collection system has recorded 10 period measurements.

19. Use the tools on your data collection system to determine the average of the 10 period data points. Record this value next to its corresponding pendulum bob mass in Table 2.

20. Repeat the same data collection steps 3 more times, keeping the horizontal displacement constant for each trial and changing the pendulum bob, using a bob with increasing mass each time. Record the average period for each trial in Table 2.

Part 3 – Length and Period

COLLECT DATA

21. Use the Part 1 setup for Part 3. Use any of the four pendulum bobs, but use the same pendulum bob for each Part 3 trial.

22. Use the meter stick to measure the length of the pendulum arm. Record this length in Table 3.

23. Use your hand to pull the pendulum bob back, displacing it a horizontal distance of 6 cm from its equilibrium position. Use the meter stick to measure the horizontal displacement.

24. Begin recording data, and then release the pendulum bob so it swings freely through the photogate.

25. Stop recording data when the data collection system has recorded 10 period measurements.

26. Use the tools on your data collection system to determine the average of the 10 period data points. Record this value next to its corresponding pendulum arm length in Table 3.

27. Repeat the same collect data steps 4 more times, keeping the horizontal displacement constant for each trial and shortening the length of the pendulum arm by 10 cm each time. Record your average period for each trial into Table 3.

    NOTE: To shorten the pendulum arm length, loosen the anchor thumbscrew on the pendulum clamp and gently pull the loose end of the thread upward under the anchor. Tighten the anchor thumbscrew again to hold the thread in place and then lower the pendulum clamp, with the thread and bob attached, so that the bob hangs about 4 cm above the lab table in each trial.

Data Analysis

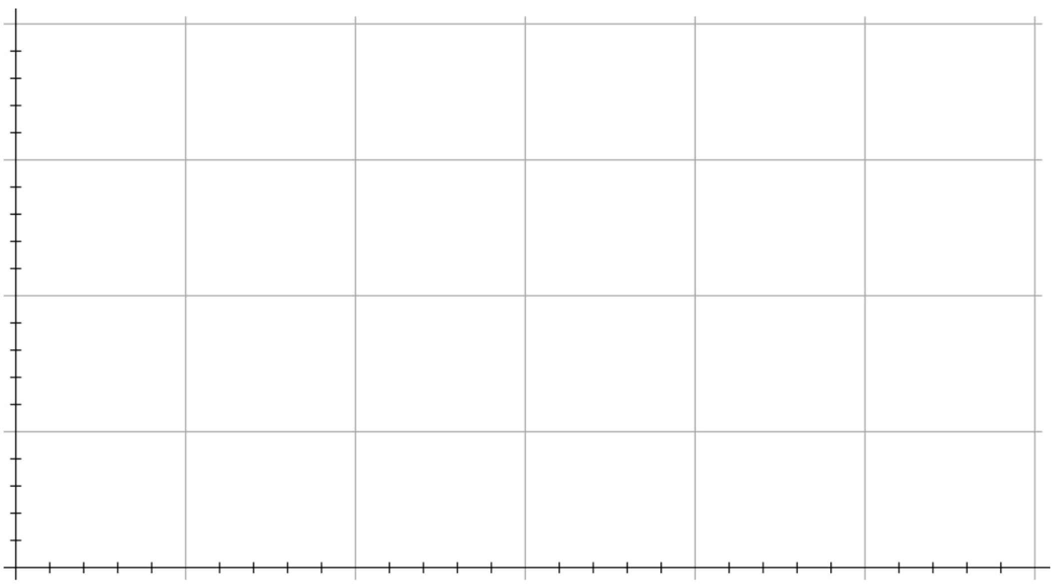
Part 1 – Displacement and Period

Table 1: Period of a pendulum with varying horizontal displacement

Trial	Horizontal Displacement (cm)	Average Period (s)
1		
2		
3		
4		

1. Plot a graph of *average period* versus *horizontal displacement* in Graph 1. Be sure to label both axes with the correct scale and units.

Graph 1: Period versus displacement of a simple pendulum with constant length and mass



2. Did changing the displacement of the pendulum bob affect the period of the simple pendulum? Justify your answer.

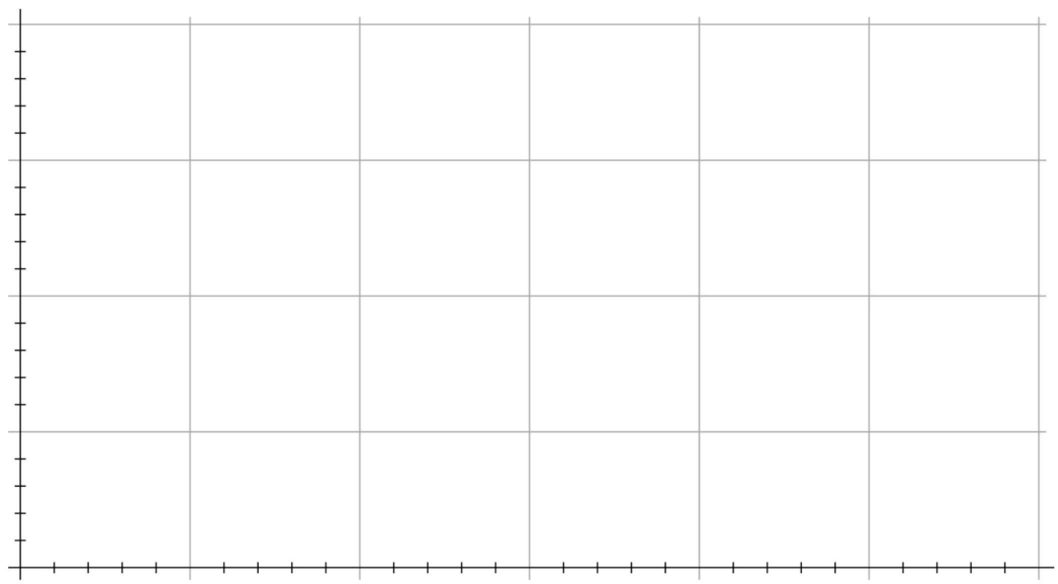
Part 2 – Mass and Period

Table 2: Period of a pendulum with varying mass

Trial	Bob Mass (g)	Average Period (s)
1		
2		
3		
4		

3. Plot a graph of *average period* versus *bob mass* in Graph 2. Be sure to label both axes with the correct scale and units.

Graph 2: Period versus mass of a simple pendulum with constant length and displacement



4. Did changing the mass of the pendulum bob affect the period of the simple pendulum? Justify your answer.

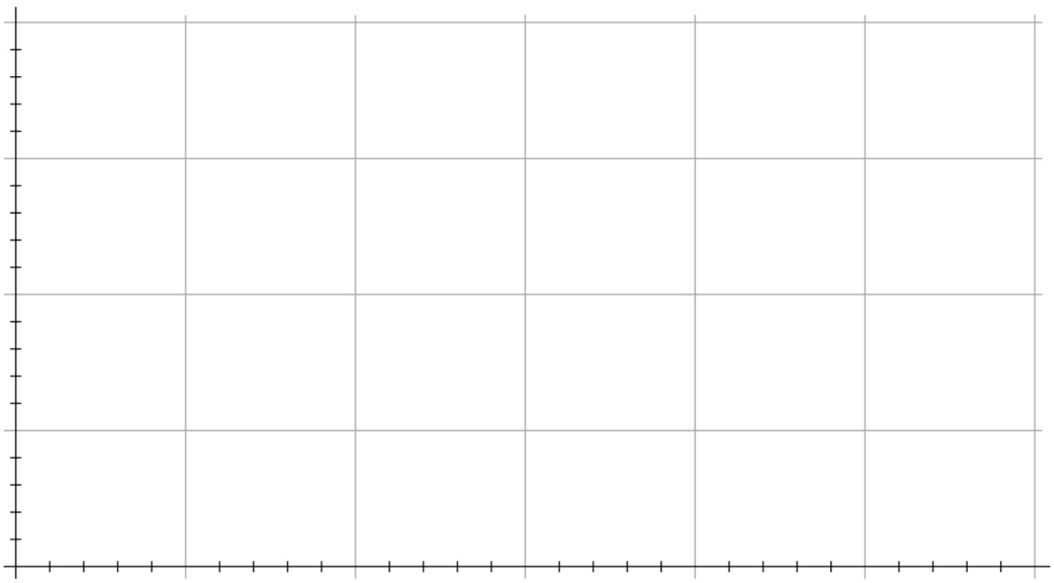
Part 3 – Arm Length and Period

Table 3: Period of a pendulum with varying arm length

Trial	Pendulum Arm Length (cm)	Average Period (s)	$\sqrt{\text{Pendulum Arm Length}}$ ($m^{1/2}$)
1			
2			
3			
4			
5			

5. Plot a graph of *average period* versus *pendulum arm length* in Graph 3. Be sure to label both axes with the correct scale and units.

Graph 3: Period versus arm length of a simple pendulum with constant displacement and mass



6. Did changing the length of the pendulum arm affect the period of the simple pendulum? Justify your answer.

___

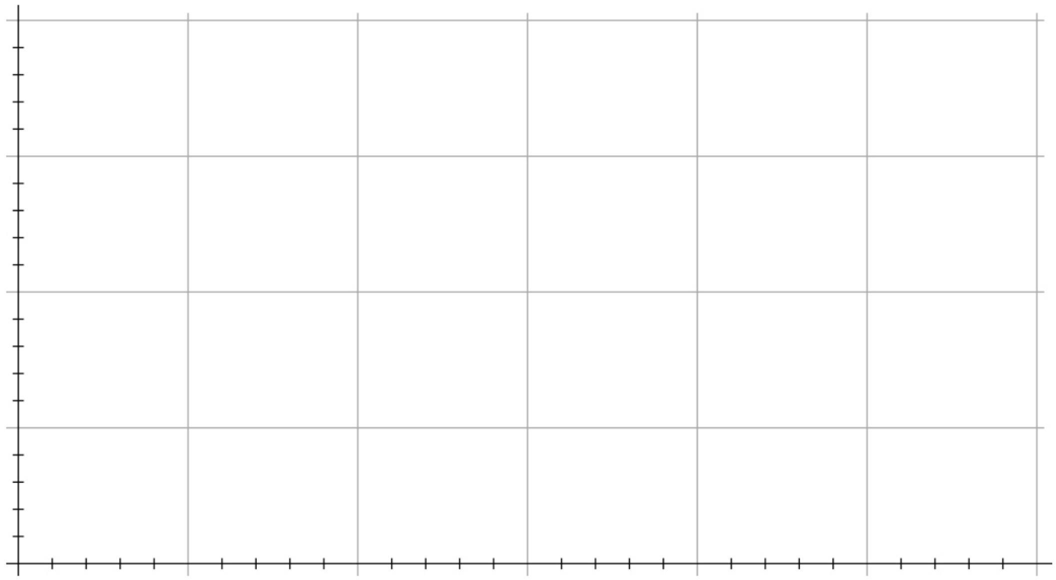
___

___

7. Calculate the square root of the pendulum arm length from Table 3. Record the result for each trial in Table 3.

8. Plot a graph of *average period* versus $\sqrt{\textit{pendulum arm length}}$ in Graph 4. Be sure to label both axes with the correct scale and units.

Graph 4: Average period versus $\sqrt{\text{arm length}}$ of a simple pendulum with constant displacement and mass



9. Based on Graph 4, what is the relationship between period and pendulum arm length for a simple pendulum (proportional, inverse, squared, et cetera)? Justify your answer.

Analysis Questions

1. For each part of your experiment, list each variable involved and state whether it was held constant, increased, or decreased.

2. In your experiment, what variables (physical properties) affected the period of a simple pendulum and how did they affect the period?

❓ 3. The mathematical equation describing the period T_p of a pendulum is:

$$T_p = 2\pi\sqrt{\frac{l}{g}} \qquad (2)$$

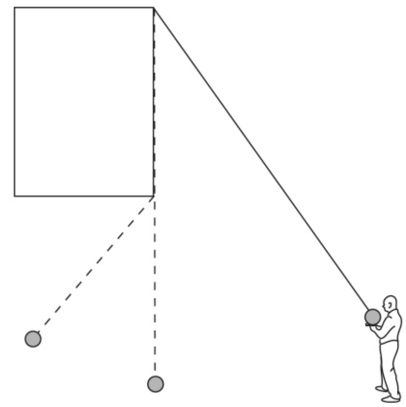
where l is the length of the pendulum arm and g is earth's gravitational acceleration constant. Does your data support this mathematical relationship? Justify your answer.

Synthesis Questions

❓ 1. In 1851, French physicist Leon Foucault created an enormous pendulum designed to demonstrate the rotation of the earth. The pendulum consisted of a 28.0 kg bob hung from a 67.0 m cable in the Pantheon in Paris, France. Given this information, what is the period of oscillation for the pendulum? Show your work.

❓ 2. If the maximum displacement θ_0 of Foucault's pendulum bob while in motion is 0.0149 rad, what is the maximum linear speed of the bob during its motion? What is the maximum momentum of the bob? Show your work.

❓ 3. One of two identical grandfather pendulum clocks from Earth is placed on Mars. If the grandfather clock on Mars is 23 minutes behind the grandfather clock on Earth after 1 hour, what is the acceleration due to gravity on Mars?



❷ 4. The figure to the right shows a person holding a 4 kg pendulum bob hanging from a 6-m long cable. The cable is attached to the side of an over-hanging wall 3 m up from the bottom of the wall. If the person lets go of the bob, how long will it take for it to swing back to him?

14. RESONANCE AND STANDING WAVES

Connections to the AP® Physics 1 Curriculum*

The lab activity correlates to the following pieces of the AP® Physics 1 framework:

Big Idea 6 Enduring Understanding D Essential Knowledge 3

Learning Objective 4: The student is able to describe representations and models of situations in which standing waves result from the addition of incident and reflected waves confined to a region.
Science Practices: 1.2

Big Idea 6 Enduring Understanding D Essential Knowledge 4

Learning Objective 1: The student is able to challenge with evidence the claim that the wavelengths of standing waves are determined by the frequency of the source regardless of the size of the region.
Science Practices: 1.5, 6.1

Learning Objective 2: The student is able to calculate wavelengths and frequencies (if given wave speed) of standing waves based on boundary conditions and length of region within which the wave is confined, and calculate numerical values of wavelengths and frequencies. Examples should include musical instruments.
Science Practices: 2.2

Time Requirement

Preparation Time: 30 minutes

Lab Activity: 45 minutes

Prerequisites

Students should be familiar with the following concepts:

- Properties of sound waves including the equations relating wavelength, wave speed in air, and frequency.
- How standing sound waves (harmonics) are generated inside a tube with one closed end; the positions at which nodes and antinodes exist at different harmonics; and the equation describing the frequency of sound required to generate those harmonics given a tube with fixed length.

Driving Question | Objective

How can standing waves be used to determine the speed of sound in air? Use sound waves traveling into a tube with one closed end and the principles of resonance and standing waves to experimentally determine the speed of sound in air.

Procedural Overview

In the Structured version of this lab activity, students use a tube with one closed end and adjustable length to identify the length of tube required to establish the first harmonic (one standing wave with one-quarter wavelength inside the tube) for 5 different sound wave frequencies. Students then plot a graph of sound wave frequency versus inverse tube length. This graph displays a linear relationship to which students apply a line of best fit whose slope is equal to one-fourth the speed of sound in air. Using the slope, students calculate the speed of sound in air.

Pre-Lab Discussion and Activity

Begin the discussion by asking students about the properties of sound waves and how they are similar to those waves generated on a string:

- Both propagate through a medium
- Both have wave speeds dependent on the medium
- Both are periodic
- Both have areas of high and low amplitude (energy)
- Both are subject to superposition

Explore with students the behavior of longitudinal waves versus transverse waves, and how superposition occurs in either wave type when two waves interfere with each other.

Expand the conversation by exploring longitudinal sound waves traveling down a tube with one closed end: as waves continuously enter the tube, waves reflected from the closed end of the tube interfere with incoming waves and superposition results, creating regions of varying wave amplitude as the waves traverse the tube. Explain to students that standing waves are established inside the tube at certain frequencies (resonant frequencies), that is, when the wavelength and tube length are spatially synchronized.

In this case, the reflected waves are in phase with the incoming waves, producing regions inside the tube known as *antinodes* that oscillate from zero to maximum amplitude (positive and negative), and regions that have constant zero amplitude, known as *nodes*. Such a standing wave is known as a *harmonic*.

Inquire of students what is physically happening to the air inside a tube in which standing waves are established. Ask students how the pressure inside the tube behaves at different regions of the tube when the antinodes of standing waves are driving against the walls of the tube. Explain that this reverberating pressure is known as *resonance*.

Now explain to students that the fundamental frequencies associated with tubes having one closed end are dependent on the creation of standing waves with a node at the closed end and an antinode at the open end. Given this information, students should be able to derive the equations for the harmonic frequencies in the pre-lab questions below.

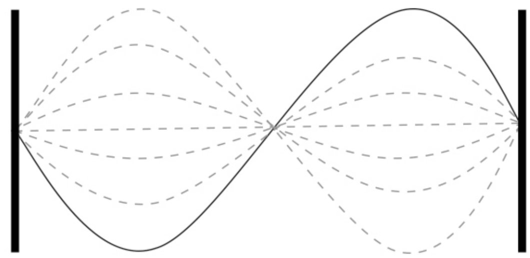
Pre-Lab Questions

❷ 1. How are the frequency of a wave and its wavelength related? What is the equation relating the two?

$f = \frac{v}{\lambda}$, where f is the frequency, λ is the wavelength, and v is the speed of the wave traveling through a medium.

❷ 2. The string in the picture has standing waves established on it with three nodes and two antinodes. If the distance between the fixed ends of the string is 1 m, what is the wavelength of the wave that has created these standing waves?

The wavelength equals the distance between three nodes, which is 1 m.



❷ 3. A resonant frequency of a tube with one closed end is one which will produce standing waves inside the tube with a node at the closed end of the tube and an antinode at the open end of the tube. How many wavelengths of the original driving wave are present in a closed-end tube with one node and one antinode in it?

The distance between one node and one antinode is equal to 1/4 of the original wavelength.

❷ 4. The previous question referred to a standing wave pattern with one node and one antinode. Given that the tube will only resonate when the standing wave pattern has a node at the closed end and an antinode at the open end (with any integer number of nodes and antinodes between them), write an equation that generalizes the number of wavelengths present in a closed-end tube when it is resonating.

To have a node at the closed end and an antinode at the open end (1/4 of one wavelength), the amount of wavelengths present must be

$n\frac{\lambda}{4}$ where n is equal to an odd integer: $n = 1, 3, 5, \ldots$

❷ 5. If the length of the tube is l, what is the result of the previous question equal to in terms of l?

The result of the previous question represents the total distance from the node at the closed end of the tube to the antinode at the open end of the tube; therefore,

$l = n\frac{\lambda}{4}$ where n is equal to an odd integer: $n = 1, 3, 5, \ldots$

❷ 6. Using the answers to the above questions, write an expression for the resonant frequencies of a closed-end tube as a function of speed and tube length.

Combining the equations: $f = \frac{v}{\lambda}$ and $l = n\frac{\lambda}{4}$,

$l = n\frac{\left(v/f\right)}{4}$ where n is equal to an odd integer: $n = 1, 3, 5, \ldots$

$f = n\frac{v}{4l}$ where n is equal to an odd integer: $n = 1, 3, 5, \ldots$

Materials and Equipment

- PASCO Resonance Air Column[1]
- Meter stick
- Tuning forks (5), with frequencies between 200 Hz and 600 Hz

Probeware Resources

Below are web-link and QR codes that will direct you to instructional video resources for individual pieces of PASCO probeware, sensors, and other hardware used in the lab activity. These same links and codes are provided to students in their activity handouts.

[1]www.pasco.com/ap23

PASCO Resonance Air Column

Safety

Follow these important safety precautions in addition to your regular classroom procedures:

- Do not strike any tuning forks on a rigid surface as this may damage the tuning fork.

Lab Preparation

These preparation instructions are to determine the frequency of the tuning forks that students will use in their experiments. If your tuning forks have known frequency values, this section can be disregarded.

1. Before students perform the Structured version of this lab activity, the teacher must determine the frequency of each tuning fork that will be used. This information will need to be provided to students when performing the activity. If your tuning forks have known frequency values, no further preparation is necessary.

   Although there are many methods for determining the frequency of a tuning fork, the procedure outlined here is simple and will help minimize preparation time.

   a. Place a resonance air column flat on a lab bench, insert the piston, and slide the piston all the way down the tube until the front edge of the piston is just inside the opposite end of the tube.

   b. Strike the tuning fork on a soft surface and then hold it over the open end of the air column next to the piston.

   c. Slide the piston backward until it is at a point where the air column resonates loudest. Hold the piston in place and use one of the resonance column markers to mark the position of the front edge of the piston.

   NOTE: You may have to repeat this step several times to determine exactly where the first harmonic is established.

   d. Use the meter stick to measure the length l of the tube, from its opening to the marker, and then apply this measurement in the following equation to solve for the frequency f:

   $$f = \frac{343\ \text{m/s}}{4l}$$

Teacher Tips

Tip 1 – Common Students Misconceptions

- Students often assume that standing waves are always present inside the tube (regardless of the tube length) as long as the tuning fork is vibrating. The standing waves inside the tube will only form when the conditions for tube length and tuning fork frequency are met.
- Students overlook the fact that the amplitude of the antinodes inside the tube is greater than the maximum amplitude of the original wave by a factor of two due to the superposition of both the incoming and reflected waves.
- The normal mode frequencies are not dependent on the cross-sectional area of the tube, only its length.

Tip 2 – Using a Tuning Fork

- The procedure outlined in the Structured version of this lab activity calls for students to use a tuning fork to generate a constant frequency sound wave, but students will find that the amplitude of the sound wave generate by the tuning fork will be greatest immediately after striking it and will quickly attenuate. Students may mistake the decreasing amplitude for an incorrect tube length. Have student test the same tube length several times immediately after striking the tuning fork to ensure they've located the proper length.

Tip 3 – Using a Speaker or other Sound Source

- Students may choose to use any sound wave source, including a speaker. This is perfectly acceptable as long as the driving frequency to the speaker is easily controlled and can be held constant if necessary. Students will need to mount their sound source near the open end of their tube.

Sample Data

Below are sample data, acquired using the experimental setup and procedure outlined in the Structured version of the lab activity, and answers to any questions found within the Data Analysis section.

Data Analysis

Table 1: Tuning fork frequency and tube length at the first harmonic for a tube with one closed end

Tuning Fork	Frequency (Hz)	Tube Length (m)	1/Tube Length (m^{-1})
1	256.0	0.331	3.02
2	320.0	0.261	3.83
3	384.0	0.220	4.55
4	426.7	0.196	5.10
5	512.0	0.165	6.06

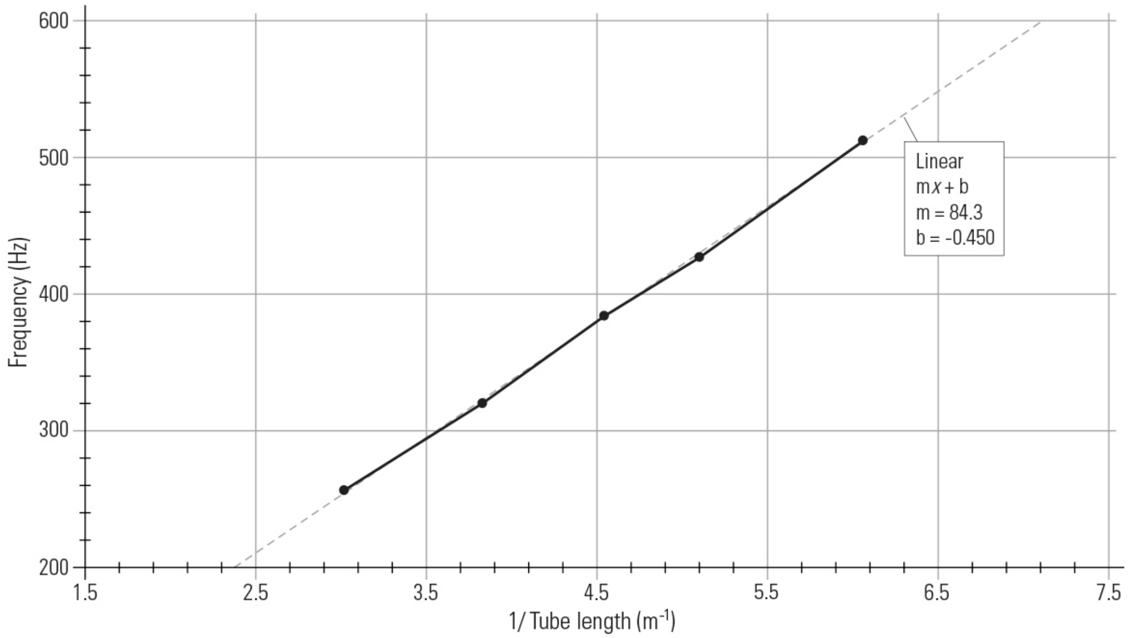
1. Calculate the inverse tube length (1/tube length) for each trial. Record your results for each trial in Table 1.

   Calculation using sample data for Tuning Fork 1:

   $$\text{Inverse tube length} = \frac{1}{0.331\text{ m}} = 3.02\text{ m}$$

2. Plot a graph of *frequency* versus *1/tube length* in the blank Graph 1 axes. Be sure to label both axes with the correct scale and units.

Graph 1: Frequency versus 1/tube length for a tube with one closed end resonating at the first harmonic



3. Draw a line of best fit through your data in Graph 1. Record the equation for the best fit line here:

   Best fit line equation (first harmonic): $f = (84.3\text{ m/s})\frac{1}{l} - 0.45\text{ s}^{-1}$

4. Use the slope from your best fit line, and the equation below, to determine your experimental value for the speed of sound (waves) v_s in air:

   $$\text{slope} = n\frac{v_s}{4},\ \text{where } n = 1$$

   Calculation using sample data from Graph 1:

   $v_s = 4(\text{slope}) = 4(84.3\text{ m/s}) = 337\text{ m/s}$

Guided Inquiry Questions

Below are sample responses to the Guiding Questions found in the Guided Inquiry version of this lab activity.

❓ 1. What is the mathematical equation relating the frequency of a sound wave that is generating a standing wave in a tube with one closed end, the length of the tube, and the speed of sound in air?

   For a resonating tube with one closed end:

   $$f_n = n\frac{v_s}{4l}$$

   where f_n is the frequency of the sound generating the standing waves, n is an odd integer, v_s is the speed of sound in air, and l is the length of the tube.

❓ 2. Assuming you will use the equation from the previous question and a graphical method to determine the speed of sound in air, which two variables would need to be plotted to produce a linear graph relating tube length to frequency?

Based on the equation from the previous question, a linear relationship exists between frequency f_n and inverse length $1/l$. These two variables would need to be plotted to display a linear relationship.

❓ 3. Based on your answers to the two previous questions, what variable would the slope of that linear graph be equal to?

The slope of a graph of frequency versus inverse tube length for a tube with one closed end resonating at the first harmonic in each trial is one-fourth the speed of sound in air, v_s.

❓ 4. From this slope, how can you determine a theoretical value for the speed of sound in air?

Students must multiply the slope of their frequency versus inverse tube length graphs by 4. The result will be equal to an experimental value for the speed of sound in air.

❓ 5. How do you plan to manipulate the two variables in your experiment while recording data?

Students can choose any suitable method for changing the length of the tube they are using as long as the opposite end remains closed when making measurements. Students using the PASCO Resonance Air Column will slide the piston down the tube, which effectively changes the length of the tube.

Students can also choose any method for changing the frequency of sound entering the tube as long as the frequency remains constant during each trial and has suitable amplitude to be heard. Using multiple tuning forks with different frequencies or a speaker and signal generator with adjustable frequency are two such methods.

❓ 6. How do you plan to measure these two variables in your experiment?

Students should use a meter stick to measure the length of the resonating air columns in their experiment after adjusting the air column to the point where the loudest tone from the tuning fork or sound wave generator is heard.

Students using tuning forks should be able to read the frequency of each fork on the handle of the fork. If the tuning fork frequencies are unknown, refer to the lab preparation section above for instruction on how to determine the frequency of each tuning fork.

Students using speakers or other sound wave generators should read each driving frequency value from the frequency generators.

❓ 7. In your experiment, how will you know when a standing wave has been established inside the tube? What will be the indication?

Students will know when standing waves have been established as this will cause the tube to resonate, and the amplification of the sound waves entering the tube will be greatest (loudest).

Assessment Questions: Sample Responses

Sample responses to the Analysis and Synthesis questions found in each version of the lab activity:

Analysis Questions

❓ 1. What is your experimental value for the speed of sound in air, and how did you determine this value from your data?

Based on the sample data, $v_s = 337\text{ m/s}$.

Students following the procedure outlined in the Structured version of this lab activity will determine experimental values for the speed of sound in air from the slope of a graph of frequency versus inverse tube length:

$$v_s = 4(\text{slope})$$

❷ 2. If the accepted value for the speed of sound in air at standard temperature and pressure is 343 m/s, calculate the percent error in your measurement.

$$\text{Percent error} = \left|\frac{\text{Theoretical} - \text{Measured}}{\text{Theoretical}}\right| \times 100$$

Calculation using the sample data:

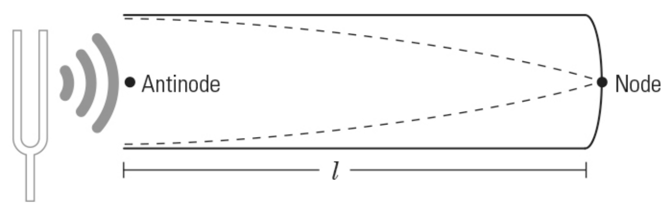
$$\text{Percent error} = \left|\frac{343 \text{ m/s} - 337 \text{ m/s}}{343 \text{ m/s}}\right| \times 100 = 1.75\%$$

❷ 3. What are factors that may have caused error in your measured value for the speed of sound?

Most of the error in student data will be from improperly identifying the length of the tube that produces the loudest sound at the given harmonic. This difficulty arises because the tuning fork becomes quieter after it is struck.

The accepted value for the speed of sound in air is at sea level at standard temperature (0 °C) and pressure, but students may not be performing their experiments at sea level, nor at normal temperature and pressure.

❷ 4. Imagine that the cylinder below represents the column of air you used in your experiment. Sketch the standing wave inside the column, showing where the nodes and antinodes were located. Be specific.



❷ 5. How would your data have been different if you had used a column of air with a greater diameter or a smaller diameter? Justify your answer.

The data would not have been different, as the equation describing the resonant frequency of a tube with one closed end is dependent only on wave speed and length, not diameter.

Synthesis Questions

❷ 1. Calculate the length of tube with one closed end that has a fundamental frequency equal to 512 Hz. Show your work.

$$f_1 = \frac{v_s}{4l}$$

$$l = \frac{v_s}{4f_1} = \frac{343 \text{ m/s}}{4(512 \text{ s}^{-1})} = 0.167 \text{ m}$$

❷ 2. What frequency sound wave would you need to drive down a 19.1-cm tube with one closed end so that the tube resonates with 3 standing wave nodes inside the column? Show your work.

3 nodes = 5th harmonic or 5th fundamental frequency

$$f_1 = \frac{v_s}{4l} = \frac{343 \text{ m/s}}{4(0.191 \text{ m})} = 449 \text{ Hz}$$

$$f_5 = 5 \times 449 \text{ Hz} = 2{,}250 \text{ Hz}$$

❓ 3. Assuming you had the ability to adjust the length of the tube from the previous question, then without changing the driving frequency, how much longer would you have to make the tube to reach the next resonant mode? How many standing wave antinodes are present inside the tube at the next resonant mode?

$$\Delta l = l_7 - l_5 = l_7 - 0.191\text{ m} = \frac{7v_s}{4f} - 0.191\text{ m}$$

$$\Delta l = \frac{7(343\text{ m/s})}{4(2{,}230\text{ s}^{-1})} - 0.191\text{ m} = 0.0782\text{ m}$$

The 7th harmonic has 4 antinodes.

Extended Inquiry Suggestions

Explore with students the difference between standing waves in a tube with one closed end and the standing wave pattern in a tube with both ends open. Explain using the same derivation as shown in this lab's pre-lab discussion, but indicate that the standing wave pattern for an open-ended tube has an antinode at each end of the tube, unlike the closed-end tube with a node at one end. This will prompt a discussion about how the open-ended tube has 1/4 more of the original wavelength in its standing wave pattern than the closed-end tube, thus changing the normal mode frequency equation to have a factor of 1/2 rather than 1/4.

Use this to segue to the string model, which involves the same normal mode equation as the open-ended tube model. Indicate to students how the actual standing wave patterns are physically different between the open-ended tube and the string models, but are mathematically the same in terms of their fundamental frequencies. Inquire how the standing wave pattern inside an air column might change if the speed of sound was different, and then ask students to predict how this would apply to waves on or in a string. Ask students what factors affect the speed of waves on or in a string, and then convey the equation for the resonant frequencies of a taut string:

$$f_1 = \frac{n}{2l}\sqrt{\frac{T}{\mu}}$$

where T is the tension in the string and μ is the mass per unit length.

NAME PERIOD DATE

14. RESONANCE AND STANDING WAVES

STRUCTURED

Driving Question | Objective

How can standing waves be used to determine the speed of sound in air? Use sound waves traveling into a tube with one closed end and the principles of resonance and standing waves to experimentally determine the speed of sound in air.

Materials and Equipment

- PASCO Resonance Air Column[1]
- Meter stick
- Tuning forks (5), with frequencies between 200 Hz and 600 Hz

[1]www.pasco.com/ap23

PASCO Resonance Air Column

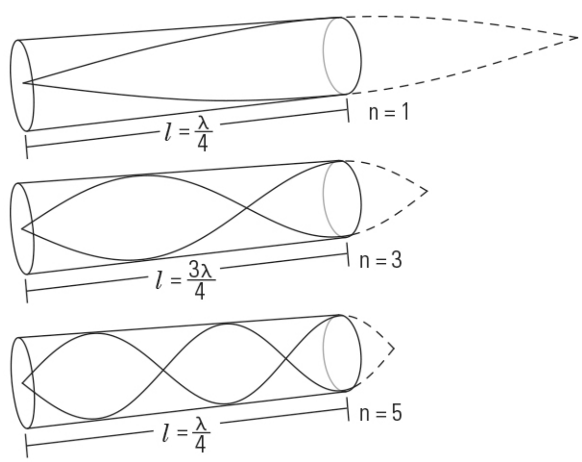
Background

As sound waves travel down a column of air with one end closed, they will interfere with earlier waves from the same source that were reflected from the closed end of the column.

As they interfere, standing waves are produced in the column; however, these standing waves can only form if certain conditions involving the frequency of the sound wave and length of the column are met. The fundamental mathematical relationship is:

$$f_n = n\frac{v_s}{4l} \qquad n = 1, 3, 5, \ldots \tag{1}$$

where f_n is the natural frequency (first harmonic) of the air column, v_s is the speed of sound in air, l is the length of the column, and n is an odd integer value representing the number of quarter wavelength standing waves in the column.



For a closed-end column of air, resonance occurs when an odd number of quarter wavelength standing waves have formed within the column. For sound waves, this resonance will manifest itself as an amplification of the original driving frequency. This is the reason a pipe organ can be heard from far away.

From Equation 1 it would be easy to assume that a pipe organ could operate with just one pipe, considering that one length of pipe can resonate at many different tones and frequencies. This is true; however, the frequency at which one pipe will resonate *must* be an integer multiple of the first harmonic for that pipe, similar to Eq.1. To produce flat and sharp notes, the organ must have pipes of very specific length.

RELEVANT EQUATIONS

$$f_n = n\frac{v_s}{4l} \qquad n = 1, 3, 5, \ldots \qquad (1)$$

This equation states that the frequency at which standing waves (harmonics) are established in a tube with one closed end is an odd integer multiple of the wave speed divided by 4 times the length of the tube.

Safety

Follow these important safety precautions in addition to your regular classroom procedures:

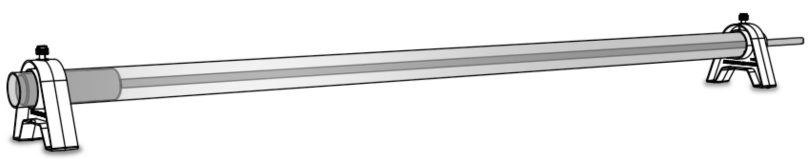
- Do not strike any tuning forks on a rigid surface as this may damage the tuning fork.

Procedure

SET UP

1. Place the resonance air column on the lab table and adjust its feet so that one foot is near each end of the tube. Attach one of the column's marker clips to the tube.

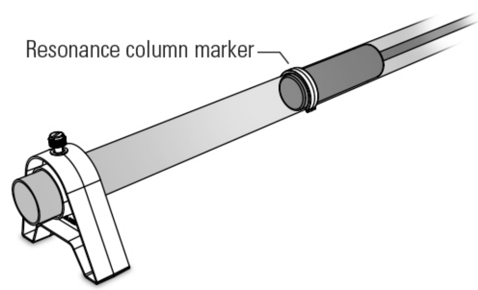
2. Insert the piston into one of the tube's open ends, and then slide the piston all the way down the tube until the front edge of the piston is just inside the opposite end of the tube.



COLLECT DATA

3. Choose the tuning fork with the lowest frequency of the five, and then strike the tuning fork against a soft surface such as a rubber mallet or rubber stopper and hold it over the open end of the tube.

4. While the tuning fork is still vibrating, slide the piston down the tube until the sound resonating from the tube is loudest. This is the point at which a 1/4-wavelength standing wave has been established in the pipe (first harmonic).



5. When the tube resonates the loudest, hold the piston in place and use one of the resonance column markers to mark the position of the front edge of the piston.

NOTE: You may have to repeat the previous steps several times to determine exactly where the first harmonic is established.

6. Use the meter stick to measure the length of the tube from its opening to the marker. Record this length and the frequency of the tuning fork used in Table 1 in the Data Analysis section.

   NOTE: The tuning fork frequency is written on the handle of each tuning fork. If not, consult your teacher to determine the frequency of the tuning fork you are using.

7. Repeat the previous data collection steps using the four remaining tuning forks in order of frequency from lowest to highest. Record each tube length measurement and tuning fork frequency into Table 1.

Data Analysis

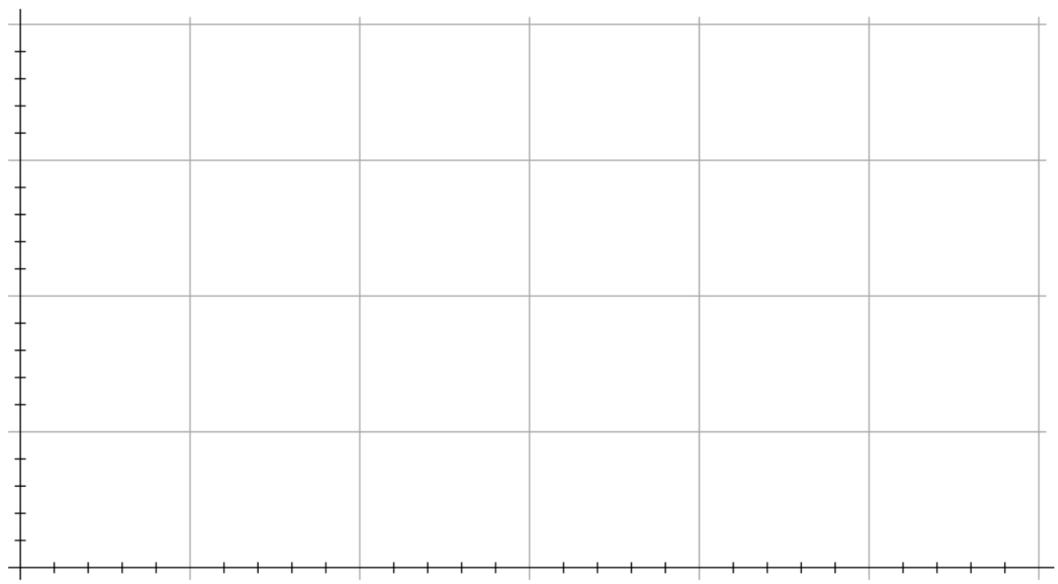
Table 1: Tuning fork frequency and tube length at the first harmonic for a tube with one closed end

Tuning Fork	Frequency (Hz)	Tube Length (m)	1/Tube Length (m^{-1})
1			
2			
3			
4			
5			

1. Calculate the inverse tube length (1/tube length) for each trial. Record your results for each trial in Table 1.

2. Plot a graph of *frequency* versus *1/tube length* in the blank Graph 1 axes. Be sure to label both axes with the correct scale and units.

Graph 1: Frequency versus 1/tube length for a tube with one closed end resonating at the first harmonic



3. Draw a line of best fit through your data in Graph 1. Record the equation for the best fit line here:

   Best fit line equation (first harmonic): ______________________________

4. Use the slope from your best fit line, and the equation below, to determine your experimental value for the speed of sound (waves) v_s in air:

   $\text{slope} = n\frac{v_s}{4}$ where $n = 1$

Analysis Questions

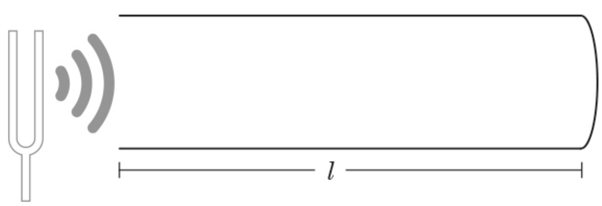
1. What is your experimental value for the speed of sound in air, and how did you determine this value from your data?

2. If the accepted value for the speed of sound in air at standard temperature and pressure is 340 m/s, calculate the percent error in your measurement.

$$\text{Percent error} = \left|\frac{\text{Theoretical} - \text{Measured}}{\text{Theoretical}}\right| \times 100$$

3. What are factors that may have caused error in your measured value for the speed of sound?

4. Imagine that the cylinder below represents the column of air you used in your experiment. Sketch the standing wave inside the column, showing where the nodes and antinodes were located. Be specific.



5. How would your data have been different if you had used a column of air that had a greater diameter or a smaller diameter? Justify your answer.

Synthesis Questions

1. Calculate the length of tube with one closed end that has a fundamental frequency equal to 512 Hz. Show your work.

2. What frequency sound wave would you need to drive down a 19.1-cm tube with one closed end so that the tube resonates with 3 standing wave nodes inside the column? Show your work.

3. Assuming you had the ability to adjust the length of the tube from the previous question, then without changing the driving frequency, how much longer would you have to make the tube to reach the next resonant mode? How many standing wave antinodes are present inside the tube at the next resonant mode?

15. DC CIRCUITS

Connections to the AP® Physics 1 Curriculum*

The lab activity correlates to the following pieces of the AP® Physics 1 framework:

Big Idea	1	Enduring Understanding	B	Essential Knowledge	1

Learning Objective 2: The student is able make predictions, using the conservation of electric charge, about the sign and relative quantity of net charge of objects or systems after various charging processes, including conservation of charge in simple circuits.
Science Practices: 6.4, 7.2

Big Idea	5	Enduring Understanding	B	Essential Knowledge	9

Learning Objective 2: The student is able to apply the conservation of energy concepts to the design of an experiment that will demonstrate the validity of Kirchhoff's loop rule ($\Sigma\Delta V = 0$) in a circuit with only a battery and resistors either in series or in, at most, one pair of parallel branches.
Science Practices: 4.2, 6.4, 7.2

Learning Objective 3: The student is able to apply the conservation of energy (Kirchhoff's loop rule) in calculations involving the total electric potential difference for complete circuit loops with only a single battery and resistors in series and/or in, at most, one parallel branch.
Science Practices: 2.2, 6.4, 7.2

Big Idea	5	Enduring Understanding	C	Essential Knowledge	3

Learning Objective 1: The student is able to apply the conservation of electric charge (Kirchhoff's junction rule) to the comparison of electric currents in various segments of an electrical circuit with a single battery and resistors in series and in, at most, one parallel branch and predict how those values would change if configurations of the circuit are changed.
Science Practices: 6.4, 7.2

Learning Objective 2: The student is able to design an investigation of an electrical circuit with one or more resistors in which evidence of conservation of electric charge can be collected and analyzed.
Science Practices: 4.1, 4.2, 5.1

Time Requirement

Preparation Time: 10 minutes

Lab Activity: 50 minutes

Prerequisites

Students should be familiar with the following concepts:

- Ohm's Law states that the current through a circuit is equal to the voltage across the circuit divided by the resistance in the circuit.
- For resistors in series, the equivalent resistance is equal to the sum of the resistances.
- For resistors in parallel, the inverse of the equivalent resistance is equal to the sum of the inverse of each individual resistance.
- Resistors are coded with a series of colored bands that indicate their resistance, in ohms, along with the tolerance.
- For measured values, the absolute uncertainty is given as a range above and below the accepted value, and the relative uncertainty is given as a percent.

Driving Question | Objective

Are the total charge and total energy in an electrical circuit conserved? Does the sum of the voltage drops across each circuit component equal zero (conservation of energy); does the sum of currents going into a circuit junction equal the sum of currents going out of the junction (conservation of charge)? Construct simple resistor circuits with resistors in series or in parallel, or both (with at most one parallel loop of resistors), to demonstrate the validity of Kirchhoff's loop rule (conservation of energy), and Kirchhoff's junction rule (conservation of charge).

Procedural Overview

In the Structured version of this lab activity, students construct a series circuit and a parallel circuit, using one battery and three resistors, ranging in resistance from 4.7 Ω to 33 Ω, and measure the current going through each circuit and the voltage drop across each circuit. They then measure the voltage drop, or change in electrical potential difference, across each element in the series circuit, and the current through each element in the parallel circuit. Using these measurements, students determine the behavior of the current at the junctions, as well as the change in potential across the series components within the circuit.

Students analyze their data using Ohm's Law and equivalent resistance, and examine their results for evidence that supports the conservation of electric charge and conservation of energy. Students are expected to demonstrate that Kirchhoff's loop and junction rules are valid for the analysis of DC circuits in series and parallel configurations.

Pre-Lab Discussion and Activity

These are suggested pre-lab questions. Students may complete the questions the day or night before the lab as homework, or the questions may be used for a class discussion before beginning the lab.

PRE-LAB QUESTIONS

❓ 1. What is the equation for Ohm's Law?

$\Delta V = IR$

❓ 2. If the resistance in a circuit is held constant and the voltage is doubled, what will happen to the current in the circuit?

According to Ohm's Law, the current will double.

3. In this lab you use a voltage–current sensor. How should the leads of the voltage sensor be placed in a circuit? How should the leads of the current sensor be placed in a circuit?

   The voltage sensor leads should be connected in parallel with the element whose voltage is being measured. The current sensor leads should be connected in series with the path whose current is being measured, which means the circuit must be broken and the leads inserted into the circuit at that point.

4. What is the equation to find the equivalent resistance when adding resistors in series?

$$R_s = R_1 + R_2 + \ldots + R_n \tag{3}$$

5. What is the equation to find the equivalent resistance when adding resistors in parallel?

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n} \tag{4}$$

Materials and Equipment

- Data collection system (2)
- PASCO Voltage–Current Sensor[1]
- 4-mm banana plug patch cord[1] (2)
- 4-mm banana plug patch cord alligator clip[1] (4)
- PASCO AC/DC Electronics Laboratory[2]
- Resistor, 4.7-Ω
- Resistor, 33-Ω
- Resistor, 10-Ω
- Battery, D-cell

Probeware Resources

Below are web-link and QR codes that will direct you to instructional video resources for individual pieces of PASCO probeware, sensors, and other hardware used in the lab activity. These same links and codes are provided to students in their activity handouts.

[1]www.pasco.com/ap19

PASCO Voltage-Current Sensor

[2]www.pasco.com/ap04

PASCO AC/DC Electronics Laboratory

Safety

Follow this important safety precaution in addition to your regular classroom procedures:

- Do not connect the terminals of a battery without a load; this will cause a short circuit.
- Use caution when stripping insulation from wires. Both the tool used and the ends of the wires are sharp.

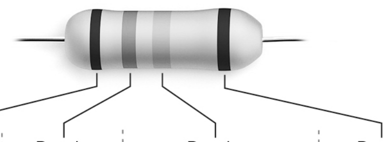
Teacher Tips

Tip 1 – Resistor Color Codes

- The resistors that come in the PASCO AC/DC Electronics Laboratory are coded with colored bands. The first three bands indicate the resistance value, with the first band indicating the first digit, the second band indicating the second digit, and the third band the multiplier (a power of 10). The fourth band indicates the tolerance. For example, the 33-Ω resistor's bands are orange, orange, and black, where the black band indicates that one multiplies the 33 by 10^0, or 1. The fourth, the gold band, indicates that the resistor has a tolerance of 5%, or about 1.7 Ω.

- This guide is provided for students in each *Structured*, *Guided Inquiry*, and *Student Designed* student handout to save time for those unfamiliar with the resistor band color code.

4-Band Resistor



Band 1 *first digit*	Band 2 *second digit*	Band 3 *multiplier*	Band 4 *tolerance*
0 black	0 black	black x 1	none ±20%
1 brown	1 brown	brown x 10	silver ±10%
2 red	2 red	red x 100	gold ±5%
3 orange	3 orange	orange x 1,000	red ±2%
4 yellow	4 yellow	yellow x 10,000	brown ±1%
5 green	5 green	green x 100,000	
6 blue	6 blue	blue x 1,000,000	
7 violet	7 violet	silver x .01	
8 gray	8 gray	gold x .1	
9 white	9 white		

Example

yellow	violet	brown	gold
4	7	10	5%

[4] [7] × 10 = 470 Ω (5% tolerance)

Tip 2 – Circuit Components

- Any circuit components kit may be used for this lab experiment. If you do not have the PASCO AC/DC Electronics Laboratory, you can substitute breadboards and a selection of jumper wires and resistors obtained from an electronics or hobby shop. This lab can also be completed with a D-cell battery, approximately 50 cm of insulated wire (20 or 22 AWG), tape (such as masking or Blue Tape), and resistors ranging from 2 Ω to 100 Ω. If the resistors lack color bands, use an ohmmeter to determine the resistance of each resistor before student use. Make this information available to students.
- If components other than those included in the PASCO AC/DC Electronics Laboratory are used, the values of the resistors may be different than those given in the Structured version. This means that the data tables and many of the answers will be different from those provided and will need to be recalculated using the values of the resistors supplied to the students.
- Provide students with wire strippers or scissors if they need to cut wire to specific lengths.
- Caution students to use the wire strippers and scissors correctly. If necessary, demonstrate how to strip insulation from the ends of wire.
- Make at least two D-cell batteries available to each student group if students will be doing the Guided Inquiry (GI) or Student Designed (SD) versions of the lab; students may choose to vary the voltage in their circuits as a means to verify equivalent resistance. If students will be doing the Structured (S) version of the lab, each group will only need one D-cell battery.

Tip 3 – Time Constraints

- If time is an issue, this lab experiment may be divided into multiple parts. For example, students carrying out the Structured version could do Part 1 on the first day, and Part 2 on a second day.

Sample Data

Below are sample data, acquired using the experimental setup and procedure outlined in the Structured version of the lab activity, and answers to questions in the Data Analysis section.

Data Analysis

PART 1 – RESISTORS IN SERIES

Table 1: Circuit with three resistors in series

Parameter	Measured Value
Voltage across battery	1.540 V
Voltage across R_1 (4.7 Ω)	0.153 V
Voltage across R_2 (10 Ω)	0.323 V
Voltage across R_3 (33 Ω)	1.060 V
Current through entire circuit	0.030 A

1. For the series circuit you tested in Part 1, how did the sum of the voltage drops across each individual resistor compare to the voltage drop across all three resistors?

   The sum of the voltage drop across each individual resistor should be equal to the voltage drop across all three resistors together.

2. Would changing the order of the resistors in series have affected the measurements you made in Part 1? If yes, explain how. If no, explain why not.

   The measurements in Part 1 would not have changed if the order of the resistors was changed. The current through the entire circuit, the voltage across the battery, and the voltage across each resistor would remain the same. The current must pass through each circuit component to traverse the circuit. Kirchhoff's loop rule states that the sum of the voltage drops around the loop must equal zero, so the voltage drop across the resistors is the same amount regardless of the order in which they are placed in the circuit.

3. Use Ohm's Law to calculate the current that should be present in the series circuit when the switch was closed. How does this compare to the current you measured? What is the percent error between the theoretical and measured values?

$$\text{Percent error} = \left|\frac{\text{Theoretical} - \text{Measured}}{\text{Theoretical}}\right| \times 100$$

   Calculation of the current:

$$I = \frac{\Delta V}{R_s}$$

$$I = \frac{\Delta V}{R_1 + R_2 + R_3}$$

$$I = \frac{1.5\ \text{V}}{(4.7\ \Omega + 10\ \Omega + 33\ \Omega)}$$

$$I = 0.031\ \text{V}$$

   The current measured in the circuit was 0.030 A

$$\text{Percent error} = \left|\frac{\text{Theoretical} - \text{Measured}}{\text{Theoretical}}\right| \times 100$$

$$\text{Percent error} = \left|\frac{0.031\ \text{A} - 0.030\ \text{A}}{0.031\ \text{A}}\right| \times 100$$

$$\text{Percent error} = 3.2\%$$

PART 2 – RESISTORS IN PARALLEL

Table 2: Circuit with three resistors in parallel

Parameter	Measured Value
Voltage across entire circuit	1.540 V
Current through entire circuit (I_{out})	0.534 A
Current through entire circuit (I_{in})	0.534 A
Current through R_1 (4.7 Ω)	0.332 A
Current through R_2 (10 Ω)	0.151 A
Current through R_3 (33 Ω)	0.050 A

4. How did the current you measured coming out of the power source compare to the current going back in to the power source? Does your data support the conservation of charge? If yes, explain how. If no, explain why not.

   The current going out of the battery should be equal to the current going in to the battery. The data should support conservation of charge, showing that no charge was lost anywhere in the circuit. Charge was not "used up;" it traveled around the closed loop from highest potential back to lowest potential.

5. For the parallel circuit you tested in Part 2, how did the sum of the currents through each individual resistor compare to the total current in the circuit?

   Using the sample data for Part 2: The sum of the measured currents through each individual resistor was 0.533 A. The total current through the circuit was measured to be 0.534 A. This is about 0.19% difference.

6. Use the equation for determining the equivalent resistance of the resistors in parallel to calculate the theoretical equivalent resistance of all three resistors. Then use the equivalent resistance and Ohm's Law to calculate the current that should be present in the series circuit when the switch was closed.

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_p} = \frac{1}{4.7\ \Omega} + \frac{1}{10\ \Omega} + \frac{1}{33\ \Omega}$$

$$\frac{1}{R_p} = 0.34\ \Omega^{-1}$$

$$R_p = 2.9\ \Omega$$

   Calculation using sample data for Part 2:

$$I = \frac{\Delta V}{R_p}$$

$$I = \frac{1.54\ V}{2.9\ \Omega}$$

$$I = 0.53\ \text{A}$$

7. Based on your answer to the previous question and Equation 4 in the Background section, compare the theoretical current calculated for the parallel circuit to the current you measured in the circuit in Part 2. What is the percent error between the theoretical and measured values?

   The theoretical value for current in the circuit was calculated to be 0.53 A. Using the sample data, the measured current was 0.534 A. The percent difference, 0.75% is small; the measured current is almost identical to the calculated current.

$$\text{Percent error} = \left|\frac{\text{Theoretical} - \text{Measured}}{\text{Theoretical}}\right| \times 100$$

$$\text{Percent error} = \left|\frac{0.53\ \text{A} - 0.534\ \text{A}}{0.53\ \text{A}}\right| \times 100 = 0.75\%$$

Guided Inquiry Questions

Below are sample responses to the Guiding Questions found in the Guided Inquiry version of this lab activity.

1. How would you connect circuit components so that all of the charge flowing through the circuit passes through each component? How would you connect circuit components so that the drop in electrical potential difference is the same across each component?

   Connect the components in series so the same charge flows through each component. Connect the components in parallel so the drop in voltage across each component is the same.

2. How would you connect circuit components so that you can determine what happens to both current and voltage at a junction in the circuit?

   Connect the components in parallel so the charge divides before flowing through the components. The charge flows back together at the junction at the end of the parallel branch. Both the current and voltage drop of the circuit can be measured at that junction.

3. In a circuit with several resistors, how can you verify experimentally what the equivalent resistance of the circuit is? In what part of the circuit would you place the leads of the voltage and current sensor?

   Change the voltage by using one, and then two, batteries. For each voltage source, measure voltage and current to ensure that the equivalent resistance behaves like a regular resistor according to Ohm's Law.

   The voltage sensor leads should be placed in parallel across a component to be tested. The current sensor leads need to be placed into the circuit in series.

4. How many ways can you combine five resistors in one circuit, if you can connect the components either in series or in parallel?

   Answers will vary. Sample answer: Five resistors can be connected with all five in series, or all five in parallel. They can be connected with two in series followed by three in parallel; with three in series followed by two in parallel; with two in parallel followed by three in series; with three in parallel followed by two in series. They can also be connected with a parallel branch of four resistors in series and one resistor in series on either side of the parallel branch.

5. How can you record your circuit designs so another person could follow your steps to reproduce the same circuits?

   One way to record the circuit design is to take pictures of each circuit with a camera or phone. Another way is to draw a circuit diagram for each circuit constructed.

Assessment Questions: Sample Responses

Sample responses to the Analysis and Synthesis questions found in each version of the lab activity:

Analysis Questions

1. Kirchhoff's loop rule states that the sum of the voltages around a closed loop is zero. Which type of circuit best illustrates this rule? What assumptions did you make about the sign of the voltage drop across the components in the circuit you tested? How does your data support the loop rule?

   The series circuit best illustrates the loop rule. Students could assume that the voltage drop across a resistor was negative, while there was a voltage gain across the battery (positive voltage). Data should show that the total voltage drop across the circuit is equal to the sum of the individual voltage drops across each resistor in the circuit, which should be equal to the voltage of the voltage source. This is another way of stating Kirchhoff's loop rule.

❷ 2. Kirchhoff's junction rule states that the total current entering a junction equals the sum of the currents leaving the junction. Which type of circuit best illustrates this rule? What assumptions did you make about the sign of the currents into and out of a junction? How does your data support the junction rule?

The parallel circuit best illustrates the junction rule. In one case, students assumed that the sign of the current entering a junction was positive, and the current exiting the same junction was also positive, so they could compare the sums of each. Data should show that the total current entering a common point in a circuit is equal to the sum of the currents exiting the branches of the circuit from that point. This is another way of expressing the junction rule.

❷ 3. If you had doubled the source voltage in your experiment, how would it have affected the circuits?

Doubling the voltage would double the current in all of the circuits both series and parallel circuits. The equivalent resistance R_s or R_p would remain the same, whether the resistors were in series or parallel, so using Ohm's law, $I = \Delta V/R$, the current would double if the source voltage doubled.

❷ 4. In this experiment, what property of the circuits you worked with were you unable to measure directly? How might this affect your results?

The resistance cannot be measured directly with a voltage–current sensor, so students must use the theoretical values indicated by the color code on each resistor for their calculations. The resistors should be within a tolerance of 5% (this may vary, depending on the resistors provided), but this uncertainty would contribute to the difference between the theoretical values students calculate and the values they measure for current and voltage in each circuit.

❷ 5. If several resistors of different resistances are connected in series to a voltage source of known value, how can the current present in the circuit be determined theoretically?

Sum the resistors according to Equation 3 to find the equivalent resistance. Then use Ohm's Law to determine the current, given the resistance and the voltage.

❷ 6. What is the minimum and maximum resistance for each individual resistor you used in the circuits you worked with? If you combined a 10-Ω and a 33-Ω resistor in series, what are the possible ways that the minimum and maximum resistances sum to give an equivalent resistance? Give your answer to the last part in qualitative terms.

These resistors have a 5% tolerance, so the maximum and minimum resistances are:

Maximum: 4.7 Ω + 0.24 Ω = 4.94 Ω; Minimum: 4.7 Ω – 0.24 Ω = 4.46 Ω.

Maximum: 10 Ω + 0.5 Ω = 9.5 Ω; Minimum: 10 Ω – 0.5 Ω = 10.5 Ω

Maximum: 33 Ω + 1.65 Ω = 34.64 Ω; Minimum: 33 Ω – 1.65 Ω = 31.35 Ω

There are four possible combinations of the 10-Ω and 33-Ω resistors:

The minimum resistance of the 10-Ω resistor could combine with the minimum resistance of the 33-Ω resistor.

The maximum resistance of the 10-Ω resistor could combine with the minimum resistance of the 33-Ω resistor.

The minimum resistance of the 10-Ω resistor could combine with the maximum resistance of the 33-Ω resistor.

The maximum resistance of the 10-Ω resistor could combine with the maximum resistance of the 33-Ω resistor.

❷ 7. Suppose you built a circuit with one battery and two resistors in series. One of the resistors was 50 Ω with 5% tolerance, and the other was 5 Ω with 10% tolerance. Which resistor has the greater absolute error? Which resistor has the greater relative error? Justify your answer.

The 50-Ω resistor has the greater absolute error because it can vary from 47.5 Ω to 52.5 Ω, a 2.5-Ω difference from the nominal value, while the 5-Ω resistor varies from 4.5 Ω to 5.5 Ω, a 0.5-Ω difference from the nominal value. The 5-Ω resistor has the greater relative error because 10% error is greater than 5% error.

❷ 8. In a DC circuit with one battery, what must be true of the resistors if there are five of them in the circuit and a current sensor measures the current through each of the resistors to be 0.025 A?

The resistors must be connected to each other in series with only one loop for the current to pass through.

Synthesis Questions

1. The voltage sensor is connected in parallel to a circuit component for testing. For example, you connected the leads of the voltage sensor *across* one or more of the resistors, creating a separate path for the current to travel. Based on your data, would you expect the resistance of the voltage sensor to be larger or smaller than the resistance of the circuit tested? Justify your answer.

   Based on my data from series and parallel circuits, I would expect the resistance of the voltage sensor to be much greater than the resistance of the circuits I tested because the current would flow through the voltage sensor if the resistance were similar to the circuits. If current were present in the voltage sensor, the resistance in the circuit would be reduced and the voltage sensor would no longer accurately report the voltage in the circuit.

2. Engineers who design electronics often need to divide either current or voltage in complex circuits. Based on your data, how would you place circuit elements to make a "voltage divider" in a circuit and a "current divider" in a circuit?

   Resistors in parallel are used to divide current inversely proportionally to the value of the resistors. Resistors in series are used to divide the voltage proportionally to the values of the resistors, because there is the same current through each resistor.

3. A student built a parallel circuit to record voltage and current data, but has yet to connect the last of five resistors to the circuit board, so one of the five branches of the circuit is still open. The currents that should be present theoretically in the branches are 1.0 A, 2.0 A, 2.5 A, 2.1 A, and 0.5 A. If the student begins recording data that shows a total current of 5.6 A with only four of five branches connected, which branch is open? Justify your answer.

   The sum of the theoretical current values is (1.0 + 2.0 + 2.5 + 2.1 + 0.5) A = 8.1 A. If the current was divided among five branches, there would be 8.1 A present in the circuit. Since the current is divided among only four branches, 8.1 A – 5.6 A = 2.5 A means the branch with the theoretical 2.5 A current is open.

4. Suppose a circuit is built with a variable-voltage DC power source set to 20 V and one junction that divides the circuit into 100 parallel branches, each with a resistor of n Ω. Without knowing the value of the resistors, is it possible to make only one current measurement and be able to predict the current at any point in the circuit? If yes, explain where in the circuit you would measure the current, and how Kirchhoff's rules are relevant to your prediction. If no, explain why not.

   It would be possible to make one current measurement in this circuit; the current sensor leads could be placed above or below the voltage source, or in any of the parallel branches because the values of the resistors, although unknown, are all equal. Kirchhoff's junction rule (conservation of charge) tells us that the total amount of current in the circuit is constant, no matter how many parallel branches it is divided into. The sum of the current flowing out of the branches is equal to the current flowing into the branches.

Extended Inquiry Suggestions

Have students investigate circuits with a combination of resistors in series and parallel. Students can construct a circuit with, for example, two or three resistors in series followed by a parallel branch with two resistors, and perform a circuit analysis on this more complex circuit to determine if Kirchhoff's Laws hold for the combination they devise.

NAME PERIOD DATE

15. DC CIRCUITS

STRUCTURED

Driving Question | Objective

Are the total charge and total energy in an electrical circuit conserved? Does the sum of the voltage drops across each circuit component equal zero (conservation of energy); does the sum of currents going into a circuit junction equal the sum of currents going out of the junction (conservation of charge)? Construct simple resistor circuits with resistors in series or in parallel, or both (with at most one parallel loop of resistors), to demonstrate the validity of Kirchhoff's loop rule (conservation of energy), and Kirchhoff's junction rule (conservation of charge).

Materials and Equipment

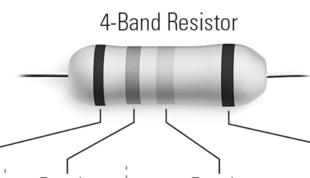
- Data collection system (2)
- PASCO Voltage–Current Sensor[1]
- 4-mm banana plug patch cord[1] (2)
- 4-mm banana plug patch cord alligator clip[1] (4)
- PASCO AC/DC Electronics Laboratory[2]
- Resistor, 4.7-Ω
- Resistor, 33-Ω
- Resistor, 10-Ω
- Battery, D-cell

[1]www.pasco.com/ap19 — PASCO Voltage-Current Sensor

[2]www.pasco.com/ap04 — PASCO AC/DC Electronics Laboratory

Resistor Value Color Code

The resistors that come in the AC/DC Electronics Laboratory are coded with colored bands. The first three bands indicate the resistance value, with the first band indicating the first digit, the second band indicating the second digit, and the third band the multiplier (a power of 10). The fourth band indicates the tolerance. For example, the 33-Ω resistor's bands are orange, orange, and black, where the black band indicates that one multiplies the 33 by 10^0, or 1. The fourth, the gold band, indicates that the resistor has a tolerance of 5%, or about 1.7 Ω.



Band 1 *first digit*	Band 2 *second digit*	Band 3 *multiplier*	Band 4 *tolerance*
0 black	0 black	black x 1	none ±20%
1 brown	1 brown	brown x 10	silver ±10%
2 red	2 red	red x 100	gold ±5%
3 orange	3 orange	orange x 1,000	red ±2%
4 yellow	4 yellow	yellow x 10,000	brown ±1%
5 green	5 green	green x 100,000	
6 blue	6 blue	blue x 1,000,000	
7 violet	7 violet	silver x .01	
8 gray	8 gray	gold x .1	
9 white	9 white		
Example: yellow 4	violet 7	brown 10	gold 5%

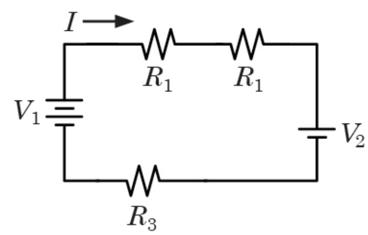
Example: [4] [7] × 10 = 470 Ω (5% tolerance)

Background

Kirchhoff's rules are the basis of all circuit analysis. These rules, sometimes called laws or theorems, are based on the fact that both charge and energy are conserved.

Kirchhoff's loop rule, or Kirchhoff's Voltage Law (KVL), arises from the fact that charges moving around a closed loop and returning to their original starting position have zero change in their potential energy. The sum of all the potential differences in the circuit loop is zero. For the circuit shown,

Kirchhoff's Loop Rule



$V_1 - IR_1 - IR_2 - V_2 - IR_3 = 0$

$$\sum \Delta V = \Delta V_1 + \Delta V_2 + \Delta V_{R_1} + \Delta V_{R_2} + \Delta V_{R_3} = 0$$

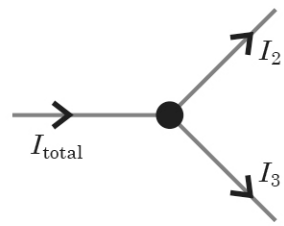
More generally, Kirchhoff's loop rule can be written as:

$$\sum \Delta V = 0 \qquad (1)$$

From the equation form of the loop rule, notice that at least one of the changes in potential energy must be negative in order for the total to sum to zero. For this reason, it is important to explicitly identify each potential difference as positive or negative.

Kirchhoff's junction rule, also known as Kirchhoff's Current Law (KCL), arises from charge conservation; since current is the flow of individual charges, current is conserved in a circuit. This means that the total amount of current that enters a junction must equal the total current that leaves the junction.

Kirchhoff's Junction Rule



$I_{total} = I_2 + I_3$

This can be stated as $I_{total} = I_2 + I_3$ for the diagram. More generally, Kirchhoff's junction rule can be written as:

$$\sum I_{in} = \sum I_{out} \qquad (2)$$

In this lab activity, you will use Kirchhoff's loop rule and junction rule to analyze the voltage across and current through resistors in series, in parallel, and in combination series–parallel circuits. You will use your data to make conclusions about the junction rule and loop rule in DC circuits, and use your data to support those conclusions.

Relevant Equations

$$\sum \Delta V = 0 \qquad (1)$$

$$\sum I_{in} = \sum I_{out} \qquad (2)$$

$$R_s = R_1 + R_2 + \ldots + R_n \qquad (3)$$

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n} \qquad (4)$$

Safety

Follow this important safety precaution in addition to your regular classroom procedures:

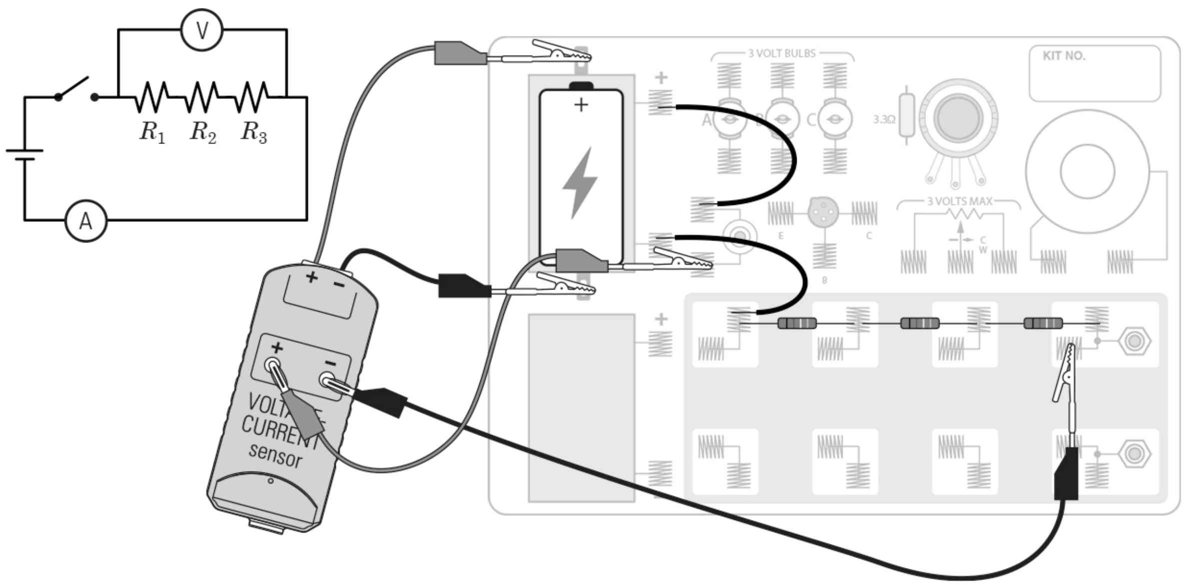
- Do not connect the terminals of a battery without a load; this will cause a short circuit.

Procedure

Part 1 – Resistors in Series

SET UP

1. Assemble the circuit as shown, using the 4.7 Ω, 10 Ω, and 33 Ω resistors. Put the D-cell battery into the top battery holder (closest to the push button switch).



2. Connect the voltage sensor leads across the battery in your circuit (in parallel, red to positive; black to negative), and the current sensor leads in series anywhere in your circuit.

   NOTE: The voltage–current sensor measures both the magnitude and direction of voltage and current.

3. Connect the voltage–current sensor to the data collection system, and then create two digits displays: one showing voltage, and the second showing current.

COLLECT DATA

4. Begin recording data, and then press the push-button switch to close the circuit, allowing current to flow for several seconds.

5. Record the voltage across the battery and current through the circuit in Table 1 in the Data Analysis section.

6. Remove the voltage sensor leads from the battery and connect them in parallel across the 4.7-Ω resistor, being mindful of the polarity of the circuit (red to positive; black to negative).

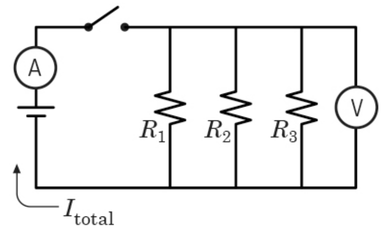
7. Press the push-button switch to close the circuit again for several seconds.

8. Record the voltage across the 4.7 Ω-resistor in Table 1.

9. Repeat the same data collection steps for the 10-Ω and 33-Ω resistors. Record all the voltages and currents in Table 1 in the Data Analysis section.

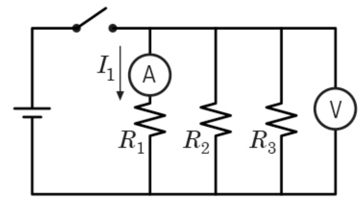
Part 2 – Resistors in Parallel

SET UP



10. Assemble the circuit as shown, using the 4.7-Ω, 10-Ω, and 33-Ω resistors. Connect the voltage sensor leads in parallel across all three resistors and connect the current sensor leads in series on the positive side of the battery.

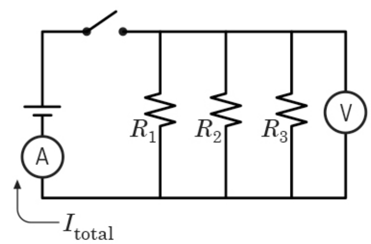
COLLECT DATA



11. Press the push-button switch to close the circuit, and then record the voltage and current in Table 2 in the Data Analysis section.

12. Switch the current sensor leads in the circuit to measure the current through the 4.7-Ω resistor (see diagram).

13. Press the push-button switch to close the circuit, and then record the current through the 4.7-Ω resistor in Table 2.



14. Repeat the same data collection steps for the 10-Ω and 33-Ω resistors. Record each current in Table 2.

15. Position the current sensor leads to measure current in the circuit on the negative side of the battery (see diagram). Record the current in Table 2.

Data Analysis

Part 1 – Resistors in Series

Table 1: Circuit with three resistors in series

Parameter	Measured Value
Voltage across battery	
Voltage across R_1 (4.7 Ω)	
Voltage across R_2 (10 Ω)	
Voltage across R_3 (33 Ω)	
Current through entire circuit	

❓ 1. For the series circuit you tested in Part 1, how did the sum of the voltage drops across each individual resistor compare to the voltage drop across all three resistors?

2. Would changing the order of the resistors in series have affected the measurements you made in Part 1? If yes, explain how. If no, explain why not.

3. Use Ohm's Law to calculate the current that should be present in the series circuit when the switch was closed. How does this compare to the current you measured? What is the percent error between the theoretical and measured values?

$$\text{Percent error} = \left|\frac{\text{Theoretical} - \text{Measured}}{\text{Theoretical}}\right| \times 100$$

Part 2 – Resistors in Parallel

Table 2: Circuit with three resistors in parallel

Parameter	Measured Value
Voltage across entire circuit	
Current through entire circuit (I_{out})	
Current through entire circuit (I_{in})	
Current through R_1 (4.7 Ω)	
Current through R_2 (10 Ω)	
Current through R_3 (33 Ω)	

4. How did the current you measured coming out of the power source compare to the current going back in to the power source? Does your data support the conservation of charge? If yes, explain how. If no, explain why not.

5. For the parallel circuit you tested in Part 2, how did the sum of the currents through each individual resistor compare to the total current in the circuit?

❷ 6. Use the equation for determining the equivalent resistance of the resistors in parallel to calculate the theoretical equivalent resistance of all three resistors. Then use the equivalent resistance and Ohm's Law to calculate the current that should be present in the series circuit when the switch was closed.

❷ 7. Based on your answer to the previous question and Equation 4 in the Background section, compare the theoretical current calculated for the parallel circuit to the current you measured in the circuit in Part 2. What is the percent error between the theoretical and measured values?

Analysis Questions

❷ 1. Kirchhoff's loop rule states that the sum of the voltages around a closed loop is zero. Which type of circuit best illustrates this rule? What assumptions did you make about the sign of the voltage drop across the components in the circuit you tested? How does your data support the loop rule?

❷ 2. Kirchhoff's junction rule states that the total current entering a junction equals the sum of the currents leaving the junction. Which type of circuit best illustrates this rule? What assumptions did you make about the sign of the currents into and out of a junction? How does your data support the junction rule?

❷ 3. If you had doubled the source voltage in your experiment, how would it have affected the circuits?

❓ 4. In this experiment, what property of the circuits you worked with were you unable to measure directly? How might this affect your results?

❓ 5. If several resistors of different resistances are connected in series to a voltage source of known value, how can the current present in the circuit be determined theoretically?

❓ 6. What is the minimum and maximum resistance for each individual resistor you used in the circuits you worked with? If you combined a 10-Ω and a 33-Ω resistor in series, what are the possible ways that the minimum and maximum resistances sum to give an equivalent resistance? Give your answer to the last part in qualitative terms.

❓ 7. Suppose you built a circuit with one battery and two resistors in series. One of the resistors was 50 Ω with 5% tolerance, and the other was 5 Ω with 10% tolerance. Which resistor has the greater absolute error? Which resistor has the greater relative error? Justify your answer.

❓ 8. In a DC circuit with one battery, what must be true of the resistors if there are five of them in the circuit and a current sensor measures the current through each of the resistors to be 0.025 A?

Synthesis Questions

1. The voltage sensor is connected in parallel to a circuit component for testing. For example, you connected the leads of the voltage sensor *across* one or more of the resistors, creating a separate path for the current to travel. Based on your data, would you expect the resistance of the voltage sensor to be larger or smaller than the resistance of the circuit tested? Justify your answer.

2. Engineers who design electronics often need to divide either current or voltage in complex circuits. Based on your data, how would you place circuit elements to make a "voltage divider" in a circuit and a "current divider" in a circuit?

3. A student built a parallel circuit to record voltage and current data, but has yet to connect the last of five resistors to the circuit board, so one of the five branches of the circuit is still open. The currents that should be present theoretically in the branches are 1.0 A, 2.0 A, 2.5 A, 2.1 A, and 0.5 A. If the student begins recording data that shows a total current of 5.6 A with only four of five branches connected, which branch is open? Justify your answer.

4. Suppose a circuit is built with a variable-voltage DC power source set to 20 V and one junction that divides the circuit into 100 parallel branches, each with a resistor of n Ω. Without knowing the value of the resistors, is it possible to make only one current measurement and be able to predict the current at any point in the circuit? If yes, explain where in the circuit you would measure the current, and how Kirchhoff's rules are relevant to your prediction. If no, explain why not.